Remember HW #3 due this Friday (but Monday is OK, if you prefer). It covers Chapters 4 (momentum) and 5 (energy).

Homework study/help sessions (optional):
Bill will be in DRL 2C6 Wednesdays from 4–6pm (today!). Grace will be in DRL 4C2 on Thursdays from 6:30–8:30pm.

For today, you read Ch8 (force). Now we can **finally** talk about forces!

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```mathematica
m1 = 1.0; m2 = 9.0; v1xi = +1.0; v2xi = 0.0;
Reduce[
    m1 v1xi + m2 v2xi == m1 v1xf + m2 v2xf,
    (v1xf - v2xf) == -(v1xi - v2xi)
]
```

```
Out[3]= v2xf == 0.2 && v1xf == -0.8
```
- Hugely important: when two objects interact only with one another:

\[ \Delta p_{1x} = -\Delta p_{2x} \]

\[ \Delta v_{1x}/\Delta v_{2x} = -\frac{m_2}{m_1} \]

\[
\begin{array}{c}
\frac{a_{1x}}{a_{2x}} = -\frac{m_2}{m_1}
\end{array}
\]

- When the medicine ball and I push apart from one another, we both accelerate: in opposite directions, and in inverse proportion to our masses.

- Lifting an object up a height \( \Delta x \) in Earth's gravity changes its gravitational potential energy by

\[ \Delta U^G = mg\Delta x \]

- I usually remember \( U = mgh \) where \( h \) is height

- Basketball: back & forth between \( \frac{1}{2}mv^2 \) and \( mgh \) until mechanical energy is dissipated into thermal energy
Problem: I release a 1 kg ball from rest, from an initial height \( x_i = +5.0 \, \text{m} \) above the ground. (Use \( g \approx 10 \, \text{m/s}^2 \).)

(a) What is the ball’s initial G.P.E.? (Let’s define \( x = 0 \) to be \( U^G = 0 \).) \[50 \, \text{J} \]

(b) What is the ball’s initial K.E.? \[0 \, \text{J} \]

(c) What is the ball’s G.P.E. immediately before it reaches the ground? \[0 \, \text{J} \]

(d) What is the ball’s K.E. immediately before it reaches the ground? \[50 \, \text{J} \]

(e) What is the ball’s speed immediately before it reaches the ground? \[10 \, \text{m/s} \]

(f) If the ball bounces elastically off of the floor, what height will it reach after bouncing? \[5.0 \, \text{m} \]

(g) If instead the ball bounces off of the floor with a restitution coefficient \( e = 0.9 \), what height will it reach after bouncing? \[4.05 \, \text{m} = 0.81 \times 5.0 \, \text{m} \]
Problem: Suppose your friend’s mass is about 50 kg, and she climbs up 30 flights of stairs (that’s about 100 m) to check out a great rooftop view of the city’s architecture.

(a) By how many joules did climbing the stairs change her G.P.E.? 
\[ 50000 \text{ J} \]

(b) Where did this gravitational potential energy come from? I mean what source energy was converted into this G.P.E.? 
\[ \text{Came from metabolising the Kashi cereal she ate for breakfast.} \]
Problem: Suppose your friend’s mass is about 50 kg, and she climbs up 30 flights of stairs (that’s about 100 m) to check out a great rooftop view of the city’s architecture.

(a) By how many joules did climbing the stairs change her G.P.E.? [50000 J]

(b) Where did this gravitational potential energy come from? I mean what source energy was converted into this G.P.E.? [Came from metabolising the Kashi cereal she ate for breakfast.]

(c) How many food Calories did she burn (assuming, unrealistically, that one’s muscles are 100% efficient at converting food into mechanical work)? [12000 chemistry calories = 12 food Calories]
Chapter 8: Force

- Forces **always** come in pairs: when A and B interact,
  \[ \vec{F}_A \text{ on B} = - \vec{F}_B \text{ on A} \]

- “Interaction pairs” have equal magnitude, opposite direction. **Always.** That’s called Newton’s third law. Difficult idea!

- The acceleration of object A is given by vector sum of all of the forces acting **ON** object A, divided by \( m_A \). (Law #2.)
  \[ \vec{a}_A = \frac{1}{m_A} \sum \vec{F}_{\text{on A}} \]

- Object A moves at constant velocity (or stays at rest) if and only if that vector sum (\( \sum \vec{F}_{\text{on A}} \)) equals zero. (Law #1.)
You push with a steady force of 25 N on a 50 kg desk fitted with (ultra-low-friction) casters on its four feet. How long does it take you (starting from rest) to get the desk across a room that is 25 m wide?

(A) 0.71 s  
(B) 1.0 s  
(C) 1.4 s  
(D) 5.0 s  
(E) 7.1 s  
(F) 10 s  
(G) 14 s
**Free-body diagram:** A sort of visual accounting procedure for adding up the forces acting **ON** a given object.  

**FBD for ring:**

- $\vec{a} = 0$
- $\vec{F}_{Er}$
- $\vec{F}_{G}$
- $\vec{F}_{cr}$
- $\vec{F}_{pr}$

**Diagram:**

- Person
- Ring
- Cable
“Think about the familiar example of a basketball dropped from eye level and allowed to bounce a few times. Describe the forces acting on the basketball at its lowest point, as it is in contact with the floor and is changing direction from downward to upward motion.”

► Working with 1-2 nearby people, draw a free-body diagram of the ball at its lowest point (while it is most squished). Include all forces acting ON the ball. Indicate the direction of each force with its vector arrow. Indicate the relative magnitudes of the forces by the lengths of the arrows. Indicate the direction of the ball’s acceleration with an arrow (or a dot).

► When you finish that, draw a second free-body diagram for the ball — this time while the ball is in the air. Will the diagram be different while the ball is rising vs. falling?

► Discuss! I may call on people to describe their diagrams.

(My diagram appears on the next slide.)
Which free-body diagram best represents the forces acting on the basketball at the bottom of its motion?

A
B
C
D
Which free-body diagram best represents the forces acting on the basketball at the *top* of its motion?

A  

B  

C  

D
If I were to draw a free-body diagram for the basketball when it is halfway back down to the ground, that new diagram would be

(A) the same as
(B) slightly different from
(C) very different from

the drawing for the basketball when it is at the top of its motion? (Neglect air resistance.)
Equal and opposite forces?

Consider a car at rest on a road. We can conclude that the downward gravitational pull of Earth on the car and the upward contact force of the road on the car are equal and opposite because

(A) the two forces form an interaction pair.
(B) the net force on the car is zero.
(C) neither: the two forces are not equal and opposite
(D) both (A) and (B)
“Explain briefly in your own words what it means for the interaction between two objects to involve ‘equal and opposite’ forces. Can you illustrate this with an everyday example?”

For instance, if I push against some object O that moves, deforms, or collapses in response to my push, is the force exerted by O on me still equal in magnitude and opposite in direction to the force exerted by me on O?

If every force is paired with an equal and opposite force, why is it ever possible for any object to be accelerated? Don’t they all just cancel each other out?

(I think the next example may help.)
Have you ever spotted the Tropicana juice train?!
vocab: powered “locomotive” pulls the unpowered “cars”
Equal and opposite forces?

An engine ("locomotive") (the first vehicle of the train) pulls a series of train cars. Which is the correct analysis of the situation?

(A) The train moves forward because the locomotive pulls forward slightly harder on the cars than the cars pull backward on the locomotive.

(B) Because action always equals reaction, the locomotive cannot pull the cars — the cars pull backward just as hard as the locomotive pulls forward, so there is no motion.

(C) The locomotive gets the cars to move by giving them a tug during which the force on the cars is momentarily greater than the force exerted by the cars on the locomotive.

(D) The locomotive’s force on the cars is as strong as the force of the cars on the locomotive, but the frictional force by the track on the locomotive is forward and large while the backward frictional force by the track on the cars is small.

(E) The locomotive can pull the cars forward only if its inertia is larger than that of the cars.
Let’s see the effect of including or not including the frictional force of the tracks pushing forward on the wheels of the engine.

I’ll pretend to be the engine!
We stopped here. We’ll come back to the train locomotive vs. cars example Friday.
Only *external* forces can accelerate a system’s CoM

Let’s define “system” to be locomotive+car. Remember that forces internal to system cannot accelerate system’s CoM.

To change the velocity of the CoM, we need a force that is *external* to the system.

(By the way, when you look at the two free-body diagrams on the next page, tell me if you see an “interaction pair” of forces somewhere!)
$\vec{a}_{\text{CoM}} = \sum \frac{\vec{F}_{\text{external}}}{m_{\text{total}}}$

It’s useful to remember that even if the several pieces of a system are behaving in a complicated way, you can find the acceleration of the CoM of the system by considering only the external forces that act on the system.

Once again, a careful choice of “system” boundary often makes the analysis much easier. We’ll see more examples of this next time.
Hooke’s law

- When you pull on a spring, it stretches
- When you stretch a spring, it pulls back on you
- When you compress a spring, it pushes back on you
- For an ideal spring, the pull is proportional to the stretch
- Force by spring, on load is

$$F_x = -k \left( x - x_0 \right)$$

- The constant of proportionality is the “spring constant” $k$, which varies from spring to spring. When we talk later about properties of building materials, we’ll see where $k$ comes from.
- The minus sign indicates that if I move my end of the spring to the right of its relaxed position, the force exerted by the spring on my finger points left.

Let’s look at some examples of springs.
A spring hanging from the ceiling is 1.00 m long when there is no object attached to its free end. When a 4.0 kg brick is attached to the free end, the spring is 1.98 m long. What is the spring constant of the spring?

(A) 5.0 N/m
(B) 10 N/m
(C) 20 N/m
(D) 30 N/m
(E) 40 N/m
Measuring your weight \((F = mg)\) with a spring scale

Most bathroom scales work something like this:

Now suppose I take my bathroom scale on an elevator . . .
A bathroom scale typically uses the compression of a spring to measure the gravitational force \((F = mg)\) exerted by Earth on you, which we call your weight.

Suppose I am standing on such a scale while riding an elevator. With the elevator parked at the bottom floor, the scale reads 700 N. I push the button for the top floor. The door closes. The elevator begins moving upward. At the moment when I can feel that the elevator has begun moving upward, the scale reads

\(\text{(A) a value smaller than 700 N.}\)

\(\text{(B) the same value: 700 N.}\)

\(\text{(C) a value larger than 700 N.}\)

You might want to try drawing a free-body diagram for your body, showing the downward force of gravity, the upward force of the scale pushing on your feet, and your body’s acceleration.
Tension vs. compression

- When a force tries to squish a spring, that is called *compression*, or a compressive force.
- When a force tries to elongate a spring, that is called *tension*, or a tensile force.
- We’ll spend a lot of time next month talking about compression and tension in columns, beams, etc.
- For now, remember that tension is the force trying to pull apart a spring, rope, etc., and compression is the force trying to squeeze a post, a basketball, a mechanical linkage, etc.
Tension in cables

- A large category of physics problems (and even architectural structures, e.g., a suspension bridge) involves two objects connected by a rope, a cable, a chain, etc.
- These things (cables, chains, ropes) can pull but can’t push. Two cables in this figure:
Tension in cables

- Usually the cables in physics problems are considered light enough that you don’t worry about their inertia (we pretend $m = 0$), and stiff enough that you don’t worry about their stretching when you pull on them (we pretend $k = \infty$).

- The cable’s job is just to transmit a force from one end to the other. We call that force the cable’s tension, $T$.

- Cable always pulls on both ends with same magnitude ($T$), though in opposite directions. [Formally: we neglect the cable’s mass, and the cable’s acceleration must be finite.]

- E.g. hang basketball from ceiling. Cable transmits $mg$ to ceiling. Gravity pulls ball down. Tension pulls ball up. Forces on ball add to zero.

- Let’s try an example.
Two blocks of equal mass are pulled to the right by a constant force, which is applied by pulling at the arrow-tip on the right. The blue lines represent two identical sections of rope (which can be considered massless). Both cables are taut, and friction (if any) is the same for both blocks. What is the ratio of $T_1$ to $T_2$?

(A) zero: $T_1 = 0$ and $T_2 \neq 0$.
(B) $T_1 = \frac{1}{2} T_2$
(C) $T_1 = T_2$
(D) $T_1 = 2 T_2$
(E) infinite: $T_2 = 0$ and $T_1 \neq 0$. 
These last few slides we squeezed in as a digression.
Dissipative / incoherent / irreversible

A simple ball / spring model of the atoms in a solid.

This is sometimes a useful picture to keep in your head.
Dissipative / incoherent / irreversible

2D version for simplicity

illustrate “reversible” and “irreversible” deformation with e.g. marbles and egg crate
Dissipative / incoherent / irreversible

I showed you once before my low-tech animation of two objects in a totally inelastic collision. Collision dissipates coherent motion (kinetic energy) into incoherent vibration of atoms (thermal energy)

https://youtu.be/SJIKCmg2Uzug
Here’s a high-speed movie of a (mostly) reversible process a golf ball bouncing off of a wall at 150 mph.

https://www.youtube.com/watch?v=AkB81u5IM3I
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