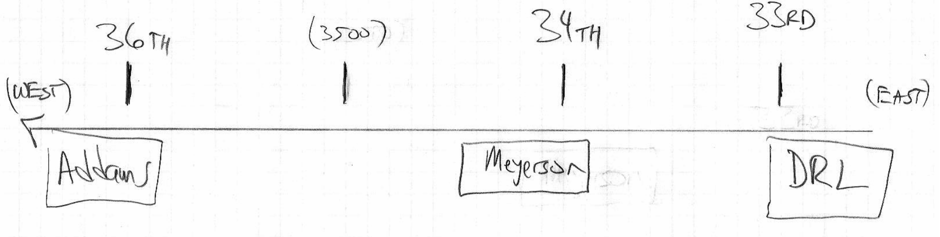
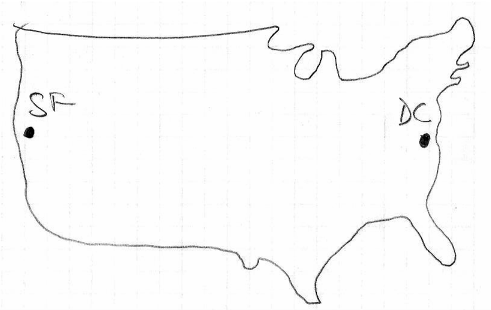
- Course www: `http://positron.hep.upenn.edu/physics8`
- Before second day of class:
- first quickly skim through Mazur chapter 1
- then watch this video (which covers chapter 2)
- then skim through Mazur chapter 2

Vectors

- A **vector** has both a magnitude and a spatial direction, e.g. 5 meters up, 3 miles north, 2 blocks east, etc.
- The **position** $\vec{r}$ is a vector (x, y, z) pointing from the origin $(0, 0, 0)$ to the object's location in space. $\vec{r}$ indicates where the object is with respect to $x = 0$, $y = 0$, $z = 0$.
- You may be familiar with vectors written as triplets (x, y, z), or with arrows, $\vec{r} = (x, y, z)$.
- The **components** of this vector are
  $r_x = x$ (the x component),
  $r_y = y$ (the y component), and
  $r_z = z$ (the z component).
- The **magnitude** of vector $\vec{r}$ is $|\vec{r}| = r = \sqrt{x^2 + y^2 + z^2}$ (but we won't see that until Chapter 10).
- But for the first 9 chapters, we will deal only with the x axis. Once we reach chapter 10, we'll use x and y axes together. So no $\sqrt{x^2 + y^2}$ until then.



- What is the distance (in blocks) between DRL and Addams?
- If you walk in a straight line that starts at DRL and ends at Addams, what is your distance traveled (in blocks)?
- What is your displacement (expressed using blocks and a compass direction)?
- If you start at Addams and end at DRL, what is your displacement?
- What is your distance traveled in that case?
- If you start at Addams, walk to Meyerson, walk back to Addams, then walk to DRL (ending there), what is your displacement?
- What is your distance traveled?



- What is (roughly) the distance between SF and DC?
- If you start in SF and end in DC, what is your displacement?
- Which one is a vector?
- How does the distance between SF and DC relate to the displacement from SF to DC?
- How does the distance between SF and DC relate to the displacement from DC to SF?
- For a journey on which I go in a straight line, never changing direction, how are "distance" and "distance traveled" related?
- For a journey on which I do change direction several times, how can I figure out the distance traveled?

Position, displacement, etc.

- A **vector** has both a magnitude and a spatial direction, e.g. up, north, east, etc.
- The **position** $\vec{r}$ is a vector (x, y, z) pointing from the origin $(0, 0, 0)$ to the object's location in space. $\vec{r}$ indicates where the object is with respect to $x = 0$, $y = 0$, $z = 0$.
- If an object moves from some initial position $\vec{r}_i$ to some final position $\vec{r}_f$, we say its **displacement** (vector) is $\Delta\vec{r} = \vec{r}_f - \vec{r}_i$, pointing from its initial position $\vec{r}_i$ to its final position $\vec{r}_f$.
- The x **component** of the displacement is $x_f - x_i$.
- The **distance** (scalar) between $\vec{r}_i$ and $\vec{r}_f$ is $d = |\Delta\vec{r}| = |\vec{r}_f - \vec{r}_i|$. In one dimension, $d = |x_f - x_i|$.
- We'll be reminded in Chapter 10 that in two dimensions, $d = \sqrt{(x_f - x_i)^2 + (y_f - y_i)^2}$. For now we use 1D.

Position, displacement, etc.

- The **distance** (scalar) between $\vec{r}_i$ and $\vec{r}_f$ is $d = |\Delta\vec{r}| = |\vec{r}_f - \vec{r}_i|$. In one dimension, $d = |x_f - x_i|$.
- If the object does not change direction between $\vec{r}_i$ and $\vec{r}_f$, then the **distance traveled** is the same as d.
- If the object changes direction at (for example) points a,b,c along the way, then the **distance traveled** is

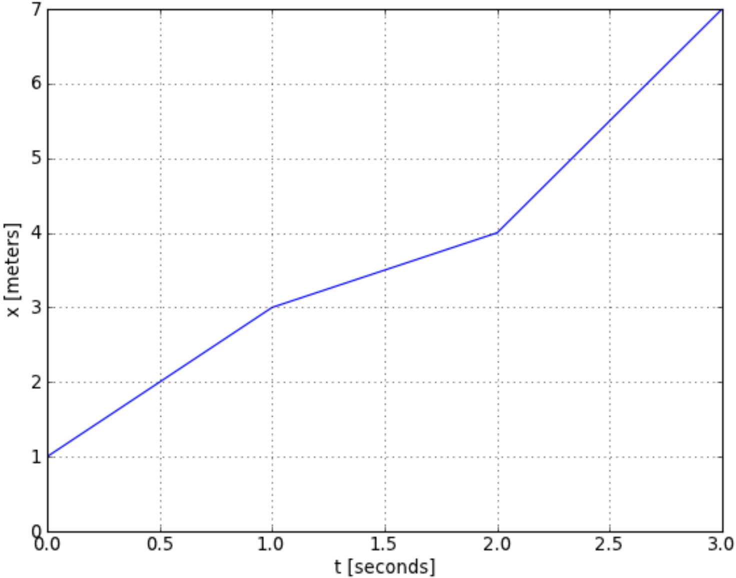
$$d_{\text{traveled}} = |\vec{r}_a - \vec{r}_i| + |\vec{r}_b - \vec{r}_a| + |\vec{r}_c - \vec{r}_b| + |\vec{r}_f - \vec{r}_c|$$

- In one dimension, the distanced traveled for this case (turning at three points a,b,c) would be

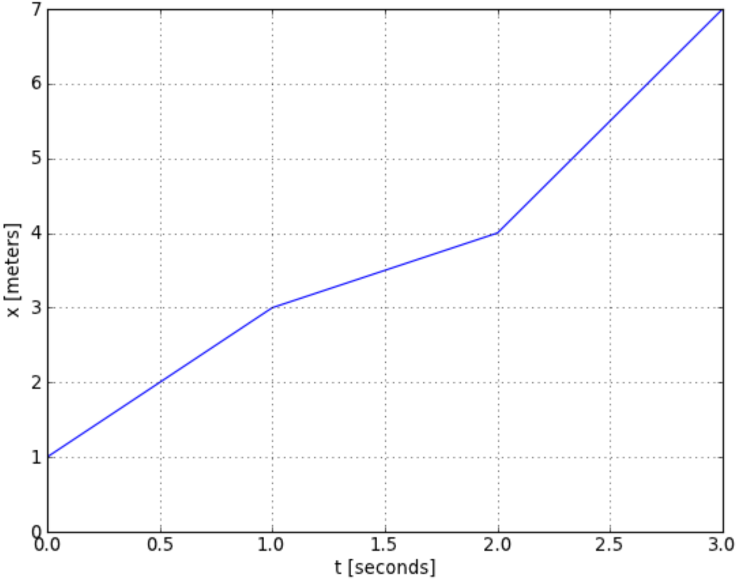
$$d_{\text{traveled}} = |x_a - x_i| + |x_b - x_a| + |x_c - x_b| + |x_f - x_c|$$

- If someone asks you how to get from DRL to 30th Street Station, is it sufficient to say (without pointing), "Go 5 blocks?"
- Is it good enough to say, "Go 2 blocks, then go another 3 blocks?"
- What about "Go 2 blocks north, then go 3 blocks east?"
- Once again, for the first 9 chapters of the textbook, directions will be **either** north/south **OR** east/west **OR** up/down, but we will not (until Chapter 10) work with more than one axis in a given problem.
- (Also, somewhat confusingly, for the first 9 chapters, the one axis that we do work with will always be called **the x axis**, even if it does not point in a direction that you are accustomed to associating wth the x axis.)
- So we won't worry, until Chapter 10, about things like the fact that a bird could travel from DRL to 30th Street Station along a diagonal that is $\sqrt{13}$ blocks long.

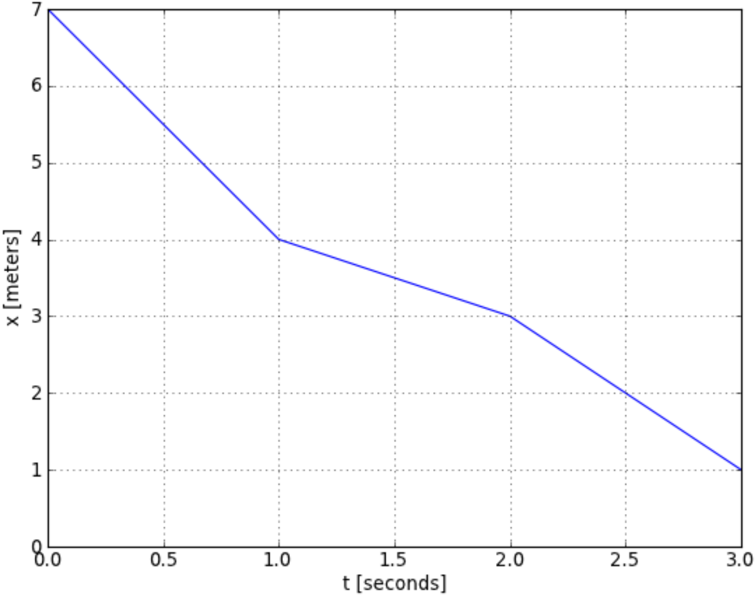
What is the distance traveled from t=0 to t=3s?



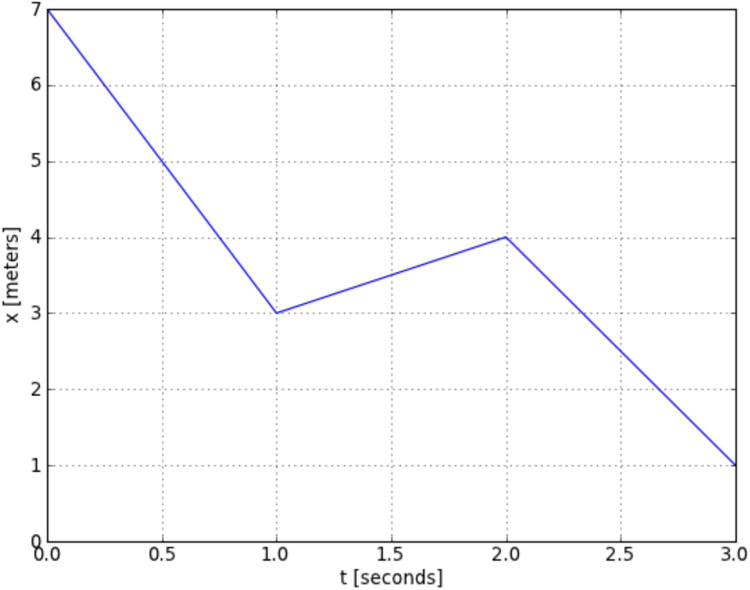
What is the x component of displacement?



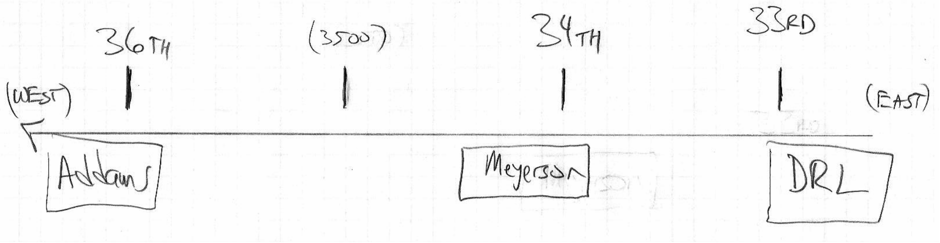
Now what is the x component of displacement?



Now what is the distance traveled?



To keep the math simple, let's pretend that every city block is exactly 100 meters long.



- If I bike directly from DRL to Addams in 100 seconds, what is my average speed?
- What is my average velocity?
- If I walk directly from DRL to Addams in 200 seconds, then bike directly back from Addams to DRL in 100 seconds, what is my average velocity for the journey?
- What is my average speed for the journey?

- What is the relationship between (instantaneous) speed and (instantaneous) velocity?
- What does calculus say about the relationship between speed and distance traveled? (Does one of them equal the rate of change of the other?)
- What does calculus say about the relationship between displacement and velocity? (Does one of them equal the rate of change of the other?)

Velocity and speed

- **Velocity** (a vector) is the rate of change of position with respect to time: $\vec{v} = \frac{\mathrm{d}\vec{r}}{\mathrm{d}t} = (v_x, v_y, v_z) = (\frac{\mathrm{d}x}{\mathrm{d}t}, \frac{\mathrm{d}y}{\mathrm{d}t}, \frac{\mathrm{d}z}{\mathrm{d}t})$
- **speed** $v = |\vec{v}|$ is magnitude (scalar) of velocity (vector)
- In one dimension, speed is $v = |v_x|$, i.e. the absolute value of the x-component of velocity.
- We can talk about velocity at a given instant. Over a finite time interval, we can talk about the **average velocity** during the time from t_i to t_f.

$$\vec{v}_{\mathrm{av}} = \frac{\Delta\vec{r}}{t_f - t_i} \qquad v_{x,\mathrm{av}} = \frac{x_f - x_i}{t_f - t_i}$$

- The **average speed** during the finite time interval from t_i to t_f is the (distance traveled) divided by the (time interval)

$$v_{\mathrm{av}} = \frac{d_{\mathrm{traveled}}}{t_f - t_i}$$

Example 2.9 (modified)

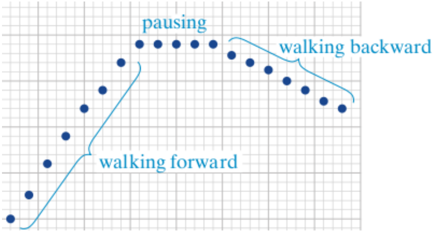
frame #	x (m)	t (s)
1	+1.0	0
2	+1.5	0.33
3	+2.2	0.67
4	+2.8	1.00
5	+3.4	1.33
6	+3.8	1.67
7	+4.4	2.00
8	+4.8	2.33
9	+4.8	2.67
10	+4.8	3.00
11	+4.8	3.33
12	+4.8	3.67
13	+4.6	4.00
14	+4.4	4.33
15	+4.2	4.67
16	+4.0	5.00
17	+3.8	5.33
18	+3.6	5.67
19	+3.4	6.00

Consider Eric's motion between frames 13 and 19 in textbook Figure 2.1. Let's use the values in Table 2.1 to answer to these questions:

(a) What is his average speed over this time interval?

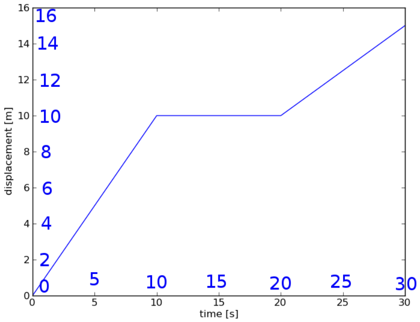
(b) What is the x component of his average velocity over this time interval?

(c) Write the average velocity (during this time interval) in terms of the unit vector $\hat{i}$.



Drawing position (or displacement) vs. time

Which statement best describes the motion depicted by this graph?



(A) I walk 1.0 m/s forward for 10 s. Then I rest 10 s. Then I walk 1.0 m/s backward for 10 s.

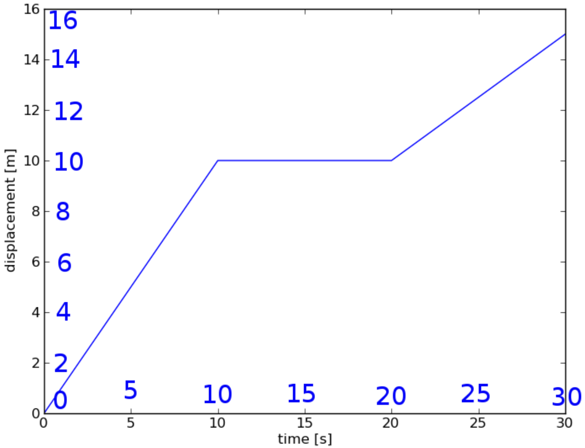
(B) I walk 0.5 m/s forward for 10 s. Then I rest 10 s. Then I walk 1.0 m/s forward for 10 s.

(C) I walk 0.5 m/s forward for 10 s. Then I rest 10 s. Then I walk 0.5 m/s forward for 10 s.

(D) I walk 1.0 m/s forward for 10 s. Then I rest 10 s. Then I walk 0.5 m/s forward for 10 s.

Average velocity

What is my average velocity $\vec{v}_{\rm av}$ during the 30 second interval shown on this graph? (Remember that $\hat{i}$ is the unit vector pointing forward along the x axis, i.e. pointing in the direction in which x increases.)



(A) $+1.0$ m/s $\hat{i}$

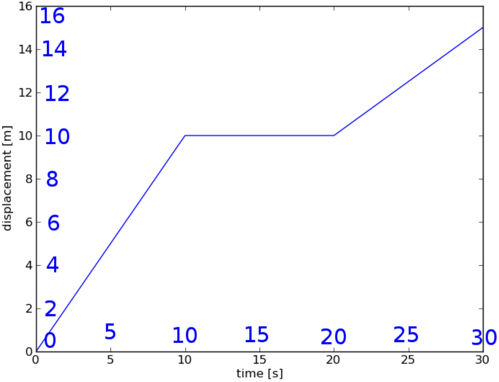
(B) $+0.75$ m/s $\hat{i}$

(C) $+0.5$ m/s $\hat{i}$

(D) -0.25 m/s $\hat{i}$

Instantaneous velocity

What is my instantaneous velocity $\vec{v}$ at time $t = 5\ \mathrm{s}$? What is $\vec{v}$ at time $t = 15\ \mathrm{s}$?



(A) $+1.0$ m/s $\hat{i}$ and
0 m/s $\hat{i}$, respectively

(B) $+0.5$ m/s $\hat{i}$ and
$+1.0$ m/s $\hat{i}$, respectively

(C) $+1.0$ m/s $\hat{i}$ and
$+0.5$ m/s $\hat{i}$, respectively

(D) $+0.5$ m/s $\hat{i}$ and
$+0.5$ m/s $\hat{i}$, respectively

Slope of the x(t) curve

The slope of the curve in the x coordinate of position vs. time graph (graph of $x(t)$ vs. t) for an object's motion gives

(A) the object's speed

(B) the object's acceleration

(C) the object's average velocity

(D) the x component of the object's instantaneous velocity

(E) not covered in today's material

You walk 1.2 km (1200 m) due east from home to a restaurant in 20 min (1200 s), stay there for an hour (3600 s), and then walk back home, taking another 20 min. What is your **average speed** for the trip?

(A) $v_{\mathrm{av}} = 0.0\ \mathrm{m/s}$

(B) $v_{\mathrm{av}} = 0.4\ \mathrm{m/s}$

(C) $v_{\mathrm{av}} = 0.8\ \mathrm{m/s}$

(D) $v_{\mathrm{av}} = 1.0\ \mathrm{m/s}$

(E) $v_{\mathrm{av}} = 2.0\ \mathrm{m/s}$

You walk 1.2 km (1200 m) due east from home to a restaurant in 20 min (1200 s), stay there for an hour (3600 s), and then walk back home, taking another 20 min. What is your **average velocity** for the trip?

(A) $\vec{v}_{\mathrm{av}} = \vec{0}$

(B) $\vec{v}_{\mathrm{av}} = +0.4\ \mathrm{m/s}$ east

(C) $\vec{v}_{\mathrm{av}} = +0.8\ \mathrm{m/s}$ east

(D) $\vec{v}_{\mathrm{av}} = -0.4\ \mathrm{m/s}$ east

(E) $\vec{v}_{\mathrm{av}} = -0.8\ \mathrm{m/s}$ east

You drive an old car on a straight, level highway at 20 m/s for 20 km, and then the car stalls. You leave the car and, continuing in the direction in which you were driving, walk to a friend's house 4 km away, arriving 1000 s after you began walking. What is your average speed during the whole trip?

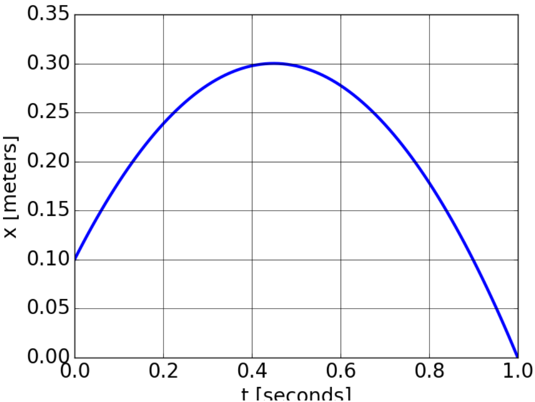
(A) $v_{\mathrm{av}} = 10 \mathrm{~m/s}$

(B) $v_{\mathrm{av}} = 12 \mathrm{~m/s}$

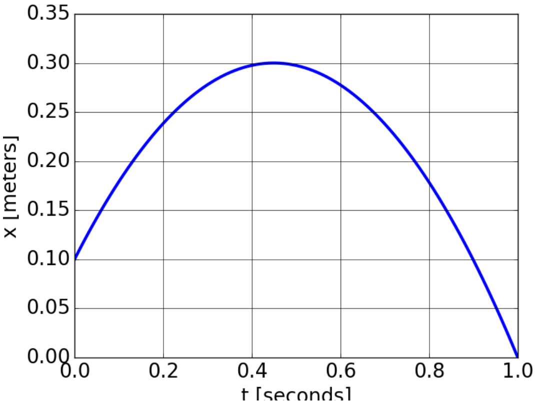
(C) $v_{\mathrm{av}} = 15 \mathrm{~m/s}$

(D) $v_{\mathrm{av}} = 20 \mathrm{~m/s}$

(E) $v_{\mathrm{av}} = 24 \mathrm{~m/s}$



- Where is the object moving forward?
- Where is the object moving backward?
- Where does the speed equal zero?
- Where is the speed largest?
- Where is v_x (the x component of velocity) largest?



For the motion represented in the figure above, what is the object's average velocity between $t = 0$ and $t = 1.0\,\mathrm{s}$?

What is its average speed during this same time interval?

Why is the average speed, for this motion, different from the magnitude of the average velocity?

Unit vectors (yuck)

- We can define **unit vectors** in the x, y, and z directions: $\hat{i} = (1, 0, 0)$, $\hat{j} = (0, 1, 0)$, and $\hat{k} = (0, 0, 1)$.
- Then we can write $\vec{r} = (x, y, z) = x\hat{i} + y\hat{j} + z\hat{k}$.
- It's often convenient to define a coordinate system where the x-axis points east, the y-axis points north, and the z-axis points up, with the origin at some specified location (e.g. the center of the ground floor).
- Then if I'm standing 5 meters east of the origin, my position vector is $+5\,\mathrm{m}\;\hat{i}$, which we could also write as $(+5\,\mathrm{m}, 0, 0)$.
- If I'm 3 m west of the origin, then $\vec{r} = -3\,\mathrm{m}\;\hat{i} = (-3\,\mathrm{m}, 0, 0)$.
- If I'm 2 m north of the origin, then my position is $\vec{r} = +2\,\mathrm{m}\;\hat{j} = (0, +2\,\mathrm{m}, 0)$.
- Most students dislike Mazur's unit-vector notation, so I try to avoid using it. I will instead write, "The displacement is $+5$ meters eastward." I will usually use a word like "east" or "north" or "up" to avoid writing $\hat{i}$ or other unit vectors.

Vectors

- Vectors are very useful on a 2D map ((x, y) or geocode) or in a 3D CAD model (x, y, z).
- For the first 10 chapters of our textbook, all problems will be one-dimensional (we will use the x-axis only), which makes the use of vectors seem contrived at this stage.
- The reason for doing this is so that we can focus on the physics first before reviewing too much math.
- In one dimension, position is $\vec{r} = (x, 0, 0) = x\ \hat{i}$.
- The x component of vector $\vec{v}$ is v_x, and in one dimension $\vec{v} = (v_x, 0, 0) = v_x\ \hat{i}$.
- The x component of vector $\vec{r}$ is x. (Special case notation.)
- In 1D, magnitude of $\vec{r}$ is $|x|$, and magnitude of $\vec{v}$ is $|v_x|$.
- Vectors will seem more natural starting in Chapter 10, when we study motion in a two-dimensional plane.

- **position:** where is it located in space? $\vec{r} = (x, y, z)$
- **displacement:** where is it w.r.t. some earlier position?
- $\Delta\vec{r} = (\Delta x, \Delta y, \Delta z) = \Delta x\hat{\imath} + \Delta y\hat{\jmath} + \Delta z\hat{k}$
- position and displacement are both **vectors**: they have both a direction in space and a magnitude
- **distance** is a scalar (magnitude only, never negative)
- **unit vectors** $\hat{\imath} = (1, 0, 0)$, $\hat{\jmath} = (0, 1, 0)$, $\hat{k} = (0, 0, 1)$ are vectors pointing along x,y,z axes, with "unit" magnitude (length = 1). Until Chapter 10, we use only the x-axis. So $\hat{\imath}$ is the only unit vector introduced in Chapter 2.
- **average velocity** $\vec{v}_{\rm av} = \frac{\Delta\vec{r}}{\Delta t}$: (displacement) / (time interval)
  x-component of $\vec{v}_{\rm av}$ is $v_{x,{\rm av}} = \frac{\Delta x}{\Delta t}$
- **(instantaneous) velocity** $\vec{v} = \frac{{\rm d}\vec{r}}{{\rm d}t} = (\frac{{\rm d}x}{{\rm d}t}, \frac{{\rm d}y}{{\rm d}t}, \frac{{\rm d}z}{{\rm d}t})$
  x-component of $\vec{v}$ is $v_x = \frac{{\rm d}x}{{\rm d}t}$
- velocity is a vector (it has a direction in space),
  speed is a scalar (it has only a magnitude)
- For many people, the hardest part of this reading was getting used to the author's notation.

Reading question:

- What is a vector, and what is it good for?
- By the way, what are two examples of vectors that are focal points of chapter 2?
- Here's what one former student wrote:

"A vector quantity, unlike a scalar quantity, is one that not only has a magnitude but also a direction. An example of an important vector quantity is displacement — unlike distance, displacement takes into account the direction that something has travelled in (i.e. while someone may have run a 400 m distance on a track, their displacement would be 0 since they end up back where they started.)"

By the way: clear and complete answers make me very happy.

Potential sources of confusion from Chapter 2

- It takes a while to get used to the textbook's vector notation. Some people positively hate the book's notation!
  - But the book's notation is extremely self-consistent, even if the many subscripts and superscripts can be annoying.
  - And this book is excellent on the concepts.
- Also, it might take some practice to re-acclimate your brain to reading lots of equations, if it has been several years since your last math course. No worries.
- What is a unit vector? Yuck!
- Using only a single spatial dimension (until Chapter 10) makes the discussion of vectors seem contrived.
- Distinction between displacement & position vectors.
- Difference between average and instantaneous velocity.
- Anything to add to this list?

Next time — onward to chapter 3 (acceleration) ...

- Course www: `http://positron.hep.upenn.edu/physics8`
- Why are we talking about velocity (and next time, acceleration), when architectural structures generally do not move? Answer: to understand force and torque, we need first to discuss motion.