table of contents

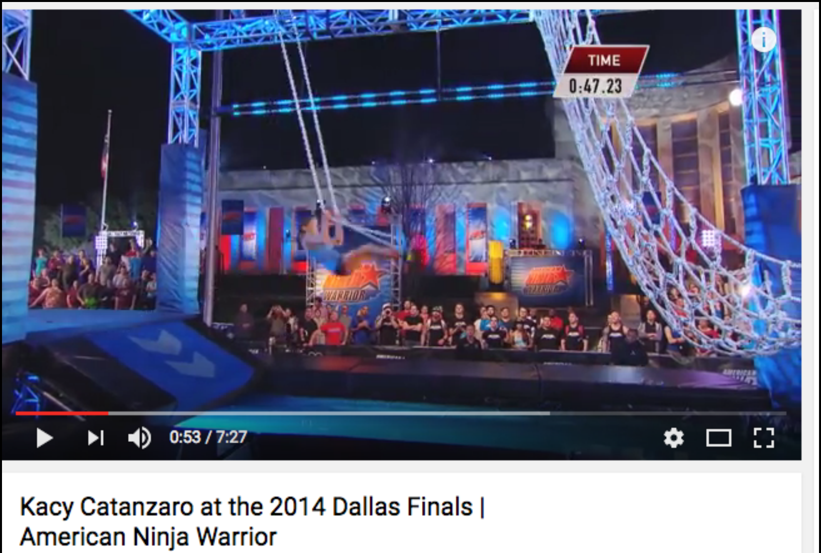
video segment break

- begin video preceding ws01
- Watch this video before first day of class.

Welcome!

- Web page: `http://positron.hep.upenn.edu/physics8`
- Note that this course is offered every **other** year! So Phys 008 will next be offered in fall 2023. (Remind your friends.)
- Physics 8 covers a pretty similar set of topics to other introductory college physics courses, such as Phys 101 (for premeds), Phys 150 (for engineers), Phys 170 (for physics majors). What makes it **Physics for Architects?**
  - About half of you are ARCH students. Having your own course lets us tailor it to your interests and your backgrounds.
  - Once we've covered the basics, we'll spend several weeks applying what we've learned to study topics related to architectural structures: trusses, cables, beams, etc. Fun!
  - Most of you are "visual learners." Lots of lecture demonstrations make the physics concepts memorable.
  - You're used to working together. So we encourage a lot of cooperation and discussion in this course.
  - We know how much time you spend on your studio projects. So we do our best to keep this course low-stress for you.

Physics can give us new insights into the everyday world. We should go through this video a second time at end of semester.



https://www.youtube.com/watch?v=XfZFuw7a13E

https://www.youtube.com/watch?v=XfZFuw7a13E&t=35

- 0:35 — impulse
- 0:43 — rotational inertia, torque
- 0:51 — torque, periodic motion, velocity, projectile motion
- 2:53 — friction, circular motion, projectile motion (173 s)
- 3:30 — center of mass (210 s)
- 6:18 — friction, “normal force” (378 s)

https://www.youtube.com/watch?v=XfZFuw7a13E&t=378

- For a long time, architects have been designing structures to span spaces. Is physics relevant to this pursuit?
- Let's make a **model** of a bridge. (Physics often uses models to simplify problems into a form you can analyze more easily.)

- The two supports are spring scales that read kilograms. The "bridge deck" is an 8 kg wooden plank.

- The two supports are spring scales that read kilograms. The "bridge deck" is an 8 kg wooden plank.
- If I use the two scales to support the plank symmetrically, as shown, from its very ends, what will each scale read?
- (For now, you and your neighbor should discuss, and use your intuition to "guess." As we study forces and torques in Sep/Oct, we'll draw diagrams to analyze more formally.)

(A) Both scales will read 8 kg

(B) Both scales will read 4 kg

(C) The left scale will read more than the right scale

(D) The left scale will read less than the right scale

- The "bridge deck" is an 8 kg wooden plank.
- Now one support stays at the far left, but the right support moves left, so that 1/4 of the plank hangs over the R end.
- What will the left and right scales read now?

(A) 4 kg (L) and 4 kg (R)

(B) (2/3)(8 kg)=5.33 kg (L) and (1/3)(8 kg)=2.67 kg (R)

(C) (1/3)(8 kg)=2.67 kg (L) and (2/3)(8 kg)=5.33 kg (R)

(D) (3/4)(8 kg)=6 kg (L) and (1/4)(8 kg)=2 kg (R)

(E) (1/4)(8 kg)=2 kg (L) and (3/4)(8 kg)=6 kg (R)

(F) 3 kg (L) and 6 kg (R)

- The "bridge deck" is an 8 kg wooden plank.
- Now one support stays at the far left, but the right support moves left, so that 1/2 of the plank hangs over the R end.
- What will the left and right scales read now?

By the way, these two scales report values in kilograms. What does a spring-based scale (like this one) really measure, anyway?

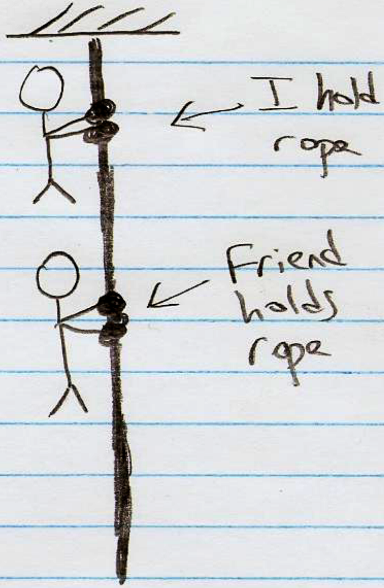
(A) mass

(B) weight

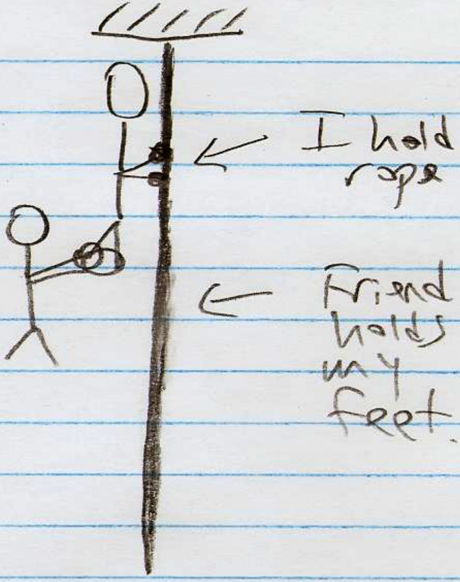
(C) inertia

(D) What's the difference?

- All materials deform (change shape) when you push or pull on them. In November, we'll study how the beams (or joists) beneath a floor bend in response to the "load" (the downward push) imposed by e.g. heavy furniture in the middle of the floor. (Illustrate with ruler.)
- The scale measures how far an internal metal spring bends in response to an object's pushing down on the scale's platform.
- Usually(*) that downward push exerted by the object on the scale is equal to the downward pull that Earth's gravity exerts on the object. We call that downward pull of gravity the object's **weight**. Weight is usually measured in Newtons (a unit of **force**), while mass is usually measured in kilograms.
- (*) Assuming that the object and scale are not accelerating.
- But weight is proportional to mass. The constant of proportionality is smaller on the Moon than it is on Earth.
- By the way, *inertia* is the same thing as mass. It measures an object's tendency to resist being accelerated.
- After a few weeks, this vocabulary will feel much more familar.

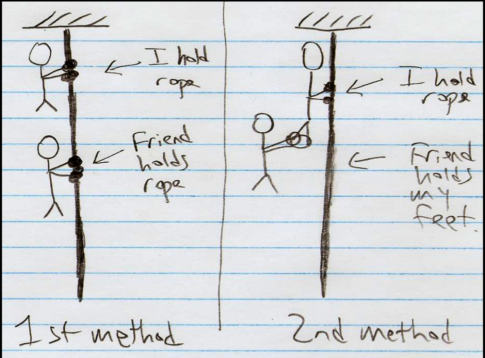


1st method



2nd method

To keep me from falling, the required force between *my* hands and the rope is . . .



(A) The same for both methods: equal to mg ($m =$ my mass)

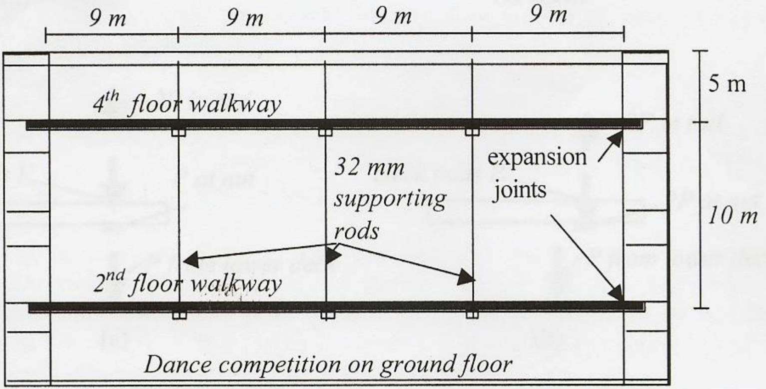
(B) The same for both methods: equal to $2mg$

(C) Twice as much for 1st method ($2mg$ vs. mg)

(D) Twice as much for 2nd method ($2mg$ vs. mg)

Kansas City Hyatt Regency skywalk collapse

On 7^{th} July *1981*, a dance was being held in the lobby of the Hyatt Regency Hotel, Kansas City. As spectators gathered on suspended walkways above the dance floor, the support gave way and the upper walkway fell on the lower walkway, and the two fell onto the crowded dance floor, killing *114* people and injuring over *200*.



For more like this, read *To Engineer is Human* by Henry Petroski.

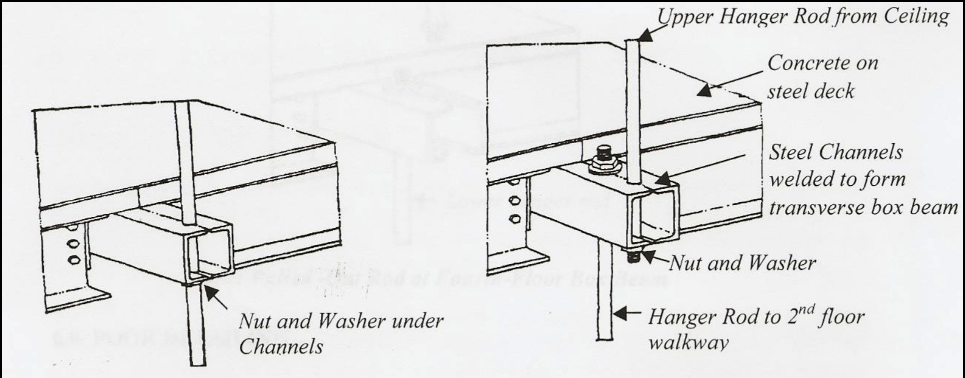


Fig. 5(a): Hyatt Regency Hanger Details As-Designed

Fig. 5(b): Hyatt Regency Hanger Details As-Built

As designed, each of the two skywalks hangs onto the rope with its own hands. As built, the lower skywalk's hands are effectively hanging onto the upper skywalk's feet! So the upper skywalk's grip on the rope feels $2\times$ larger force than in original design. Oops!

Look! A real use for drawing force diagrams!

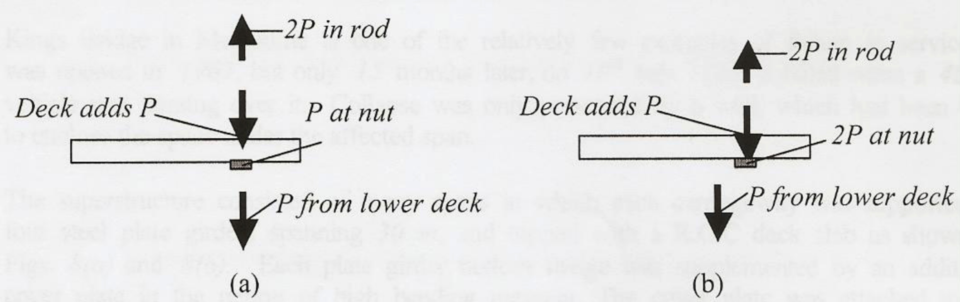


Fig. 6: Free-Body Diagram (a) As Designed (b) As Built

The author uses the symbol P for a "point" force (or point load), as is the custom in engineering. When you see "P" here, pretend it says "F" or "mg" instead.

We'll learn in September how to draw "free-body diagrams" as a graphical method to analyze forces (and then later, torques).

Upper skywalk loses its grip on the "rope"

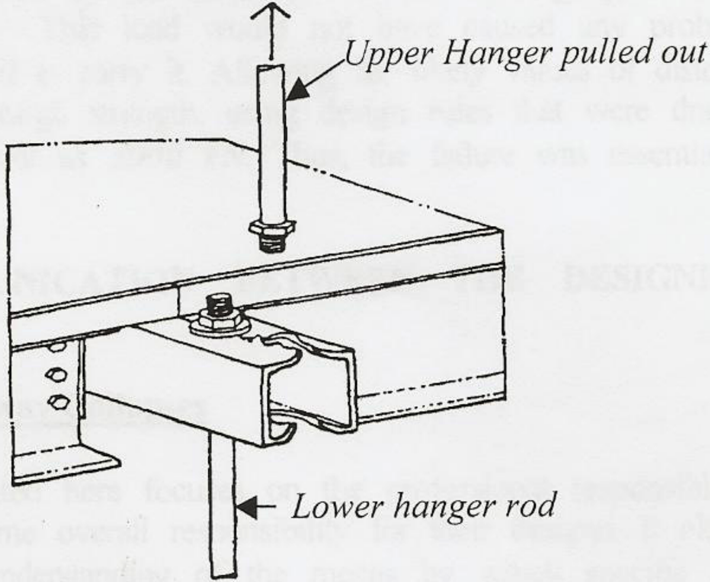
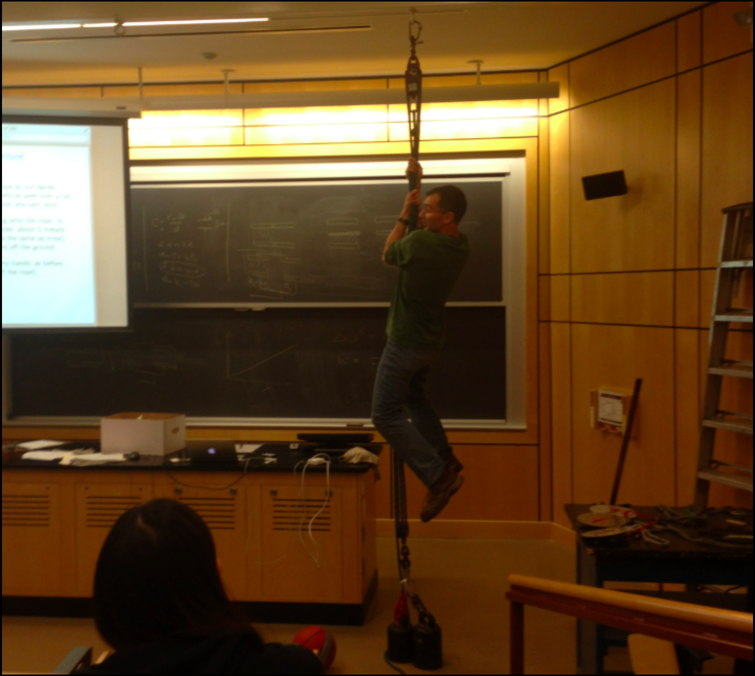


Fig. 7: Pulled -Out Rod at Fourth-Floor Box Beam



Instructor: Bill Ashmanskas (me)

`ashmansk@hep.upenn.edu` office: DRL 1W15
tel. 215-746-8210 mobile: (write on board)
Stop in any time my door is open, or email to set up a time.

I teach physics & electronics here in the Physics Dept., and I design electronics for research projects in both the Radiology and the Physics departments. My design work usually involves writing computer code, but sometimes I use CAD software for Printed Circuit Board design

I grew up in the Boston suburbs, where my dad (a carpenter / framing contractor) built wood-frame houses. Working for him one summer taught me a lot about what holds our homes up! So it felt like destiny that I should teach Physics for Architects.

Course format

- We'll typically have textbook reading / video watching due on Mondays and Wednesdays. That lets us make better use of our classroom time.
- In-class problem-solving should be fun and cooperative. We're here to help you — and your classmates are too.
- Update this!

Why are you here?

- If you're here just to take a college physics course, that's great. Here are my terms: You put in a consistent 10 hours each week (total) on reading, homework, and in-class work; and I'll do my best to make Physics 8 fun, informative, and stress-free for you.
- You may be here because Physics 8/9 is required for the Intensive Major program in architecture at Penn.
- ... or because eventually, to be certified as a practicing architect, you will take exams covering structures, heating/ cooling systems, plumbing, electrical systems, acoustics, etc.
- ... or because making a detailed energy model of a building depends on the physics of heat and light.
- But I think more generally you're here because many of you will someday design things that will exist, will be seen, and will function in the physical world that surrounds us. A better understanding of the physical world will make you a better designer. And if you work with engineers on designs, you can ask better questions if you all speak the same language.

(why you're here . . .)

- So in the fall term we'll focus on *mechanics*, which should prepare you well for the Structures course that many of you will take as seniors.
- While learning (or re-learning) Newton's laws of motion, you'll exercise the mathematical side of your brain.
- By the way, how many of you took a physics course in high school? (A) Yes, (B) No.
- How many of you studied enough calculus to be comfortable with the idea of a derivative (e.g. $\mathrm{d}x/\mathrm{d}t$) as a "rate of change?" (A) Yes, (B) No.
- Prof. Farley, who teaches Structures (ARCH 435/436) here, tells me that he wants students to enter his course with a solid understanding of **forces**, **torques**, **vectors**, and **trigonometry**. A key goal of Physics 8 is to leave you well prepared for his course.

Class meeting format . . .

- Spend a moment thinking of something that you are good at: something you do well enough that you really enjoy the time you spend working on it.
- Now turn to your neighbor and share your ideas on these things you're really good at.
- Ask your neighbor how he or she learned that favorite skill.

Class meeting format . . .

- Ask your neighbor how he or she learned that favorite skill.
- How many of you said . . .
  - (A) By *doing* it / practicing?
  - (B) By working with other people who share that interest?
  - (C) By reading?
  - (D) By asking questions after trying it yourself?
  - (E) . . . other methods? . . .
- How many of you learned to do what you do best by **Listening to lectures?**

Class meeting format . . .

- I want you to learn as much as possible from the time we spend together in class.
- So I plan to make the class meetings as interactive as possible, so that you are actively thinking about, discussing, and doing physics during class time, rather than passively watching me write equations on the blackboard.
- So normally I won't "lecture" very much. Instead, you'll do reading the night before and come to class prepared. In class, I'll summarize the key ideas, and we'll spend much of the hour working on them together.
- As with a coach, a trainer, a piano teacher, etc., I'll provide as much guidance as I can, but I can't do the learning for you.
- Now let's look at the course web page: `http://positron.hep.upenn.edu/physics8`

Online response forms

After skimming Chapter 1 and reading Chapter 2, go to
`http://positron.hep.upenn.edu/wja/jitt/`

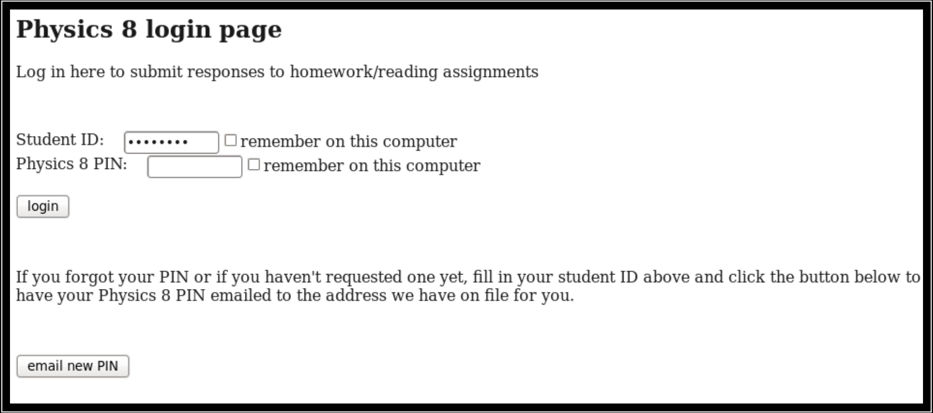
Physics 8/9 reading assignments page

Not signed in

sign in

Monday	Wednesday	Friday
	Aug 26 — *Chapter 1*	Aug 28 — *Chapter 2*
Aug 31 — *Chapter 3*	Sep 2 — *Chapter 4*	Sep 4 — *Homework 1*
	Sep 9 — *Chapter 5*	Sep 11 — *Homework 2*
Sep 14 — *Chapter 6*	Sep 16 — *Chapter 7*	Sep 18 — *Homework 3*
Sep 21 — *Chapter 8*		Sep 25 — *Chapter 9*
Sep 28 — *Chapter 10a*	Sep 30 — *Chapter 10b*	Oct 2 — *Homework 4*
Oct 5 — *Chapter 11a*	Oct 7 — *Chapter 11b*	
Oct 12 — *Chapter 12a*	Oct 14 — *Chapter 12b*	Oct 16 — *Homework 5*
Oct 19 — *Chapter G9*	Oct 21 — *Onouye Chapter 1*	Oct 23 — *Homework 6*
Oct 26 — *Onouye Chapter 2*	Oct 28 — *Onouye Chapter 3*	Oct 30 — *Homework 7*
Nov 2 — *Onouye Chapter 4*	Nov 4 — *Onouye Chapter 5*	Nov 6 — *Homework 8*
Nov 9 — *Onouye Chapter 6*	Nov 11 — *Onouye Chapter 7*	Nov 13 — *Homework 9*
Nov 16 — *Onouye Chapter 8a*	Nov 18 — *Onouye Chapter 8b*	Nov 20 — *Homework 10*
Nov 23 — *Chapter 15a*	Nov 25 — *Chapter 15b*	
Nov 30 — *Chapter "G11" (vibrations)*		Dec 4 — *Homework 11*
Dec 7 — *Practice Exam*		

Click “sign in.” Enter ID number. Click “email new PIN.”



Physics 8 login page

Log in here to submit responses to homework/reading assignments

Student ID: ········· ☐ remember on this computer

Physics 8 PIN: ☐ remember on this computer

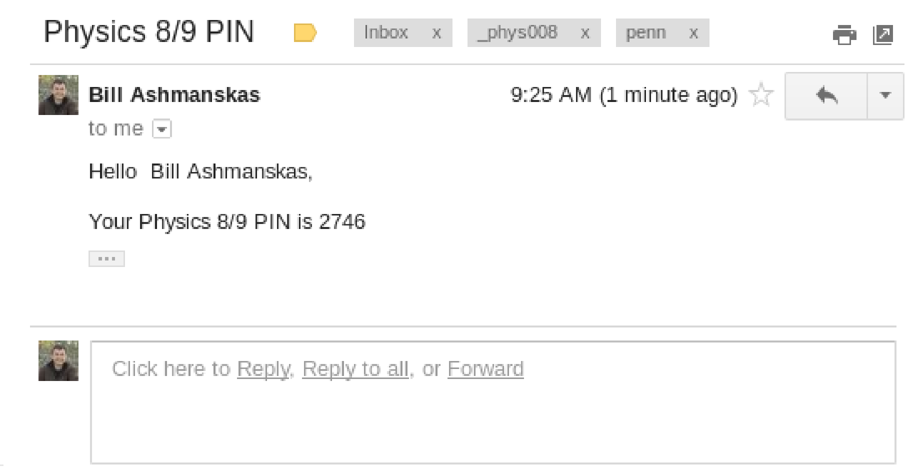
login

If you forgot your PIN or if you haven't requested one yet, fill in your student ID above and click the button below to have your Physics 8 PIN emailed to the address we have on file for you.

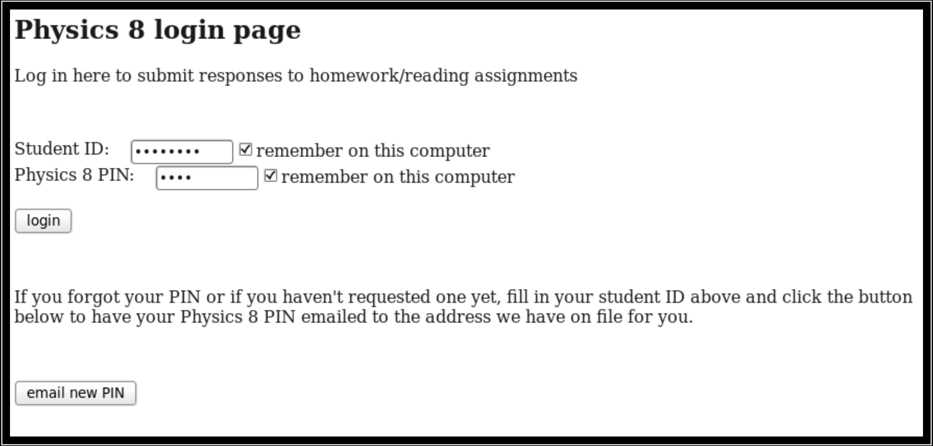
email new PIN

Click "sign in." Enter ID number. Click "email new PIN."

Your PIN should arrive by email.



Now log in. If you click "remember on this computer," then you don't have to bother logging back in next time.



Physics 8 login page

Log in here to submit responses to homework/reading assignments

Student ID: ••••••••• ☑ remember on this computer

Physics 8 PIN: •••• ☑ remember on this computer

login

If you forgot your PIN or if you haven't requested one yet, fill in your student ID above and click the button below to have your Physics 8 PIN emailed to the address we have on file for you.

email new PIN

You should see yourself signed in now. Click on today's assignment.

Physics 8/9 reading assignments page

Signed in as Bill Ashmanskas

sign out | my info

Monday	Wednesday	Friday
	Aug 26 — Chapter 1	*Aug 28 — Chapter 2*
Aug 31 — *Chapter 3*	Sep 2 — *Chapter 4*	Sep 4 — *Homework 1*
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Nov 23 — *Chapter 15a*	Nov 25 — *Chapter 15b*	
Nov 30 — *Chapter "G11" (vibrations)*		Dec 4 — *Homework 11*
Dec 7 — *Practice Exam*		

Fill out the form and then click "submit."

Assignment due 2013-08-30 (Fri Aug 30): Chapter 1

Signed in as Bill Ashmanskas

[sign out] [my info] [assignment list]

Read Chapter 1 before class: online response due by 9am. This online response should take you only about 10 minutes to complete. Your answers will be graded more on effort than on correctness, i.e. mainly convincing me that you did the reading.

To be safe, you might want to compose your answers in a program like Notepad or TextEdit and then paste them in below.

1. What do you hope to learn in Physics 8? Also, if you're willing, tell me something about yourself that will help me to remember who you are. (For instance, "Last summer, I worked for a carpenter friend-of-the-family and learned up-close how wood-frame houses are built.")

```
Hey!  I worked for my dad one summer building wood-frame houses!
```

2. What is the simplest way to convert a quantity given in one unit to the same quantity in a different unit?

```
Multiply by a ratio that equals one, such as (5280 feet) / (1 mile), in such a way that the unwanted units cancel out, and the desired units remain.
```

3. What topic from today's reading assignment did you find most difficult or confusing? Or if you didn't find anything difficult, what topic did you find most interesting?

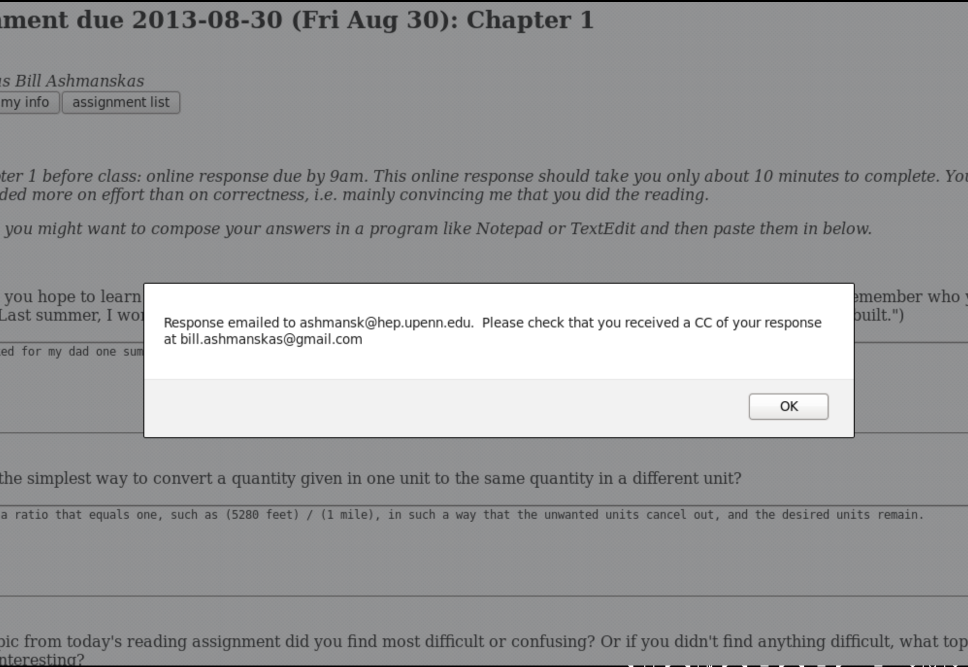
```
I found it interesting that symmetry is such an important idea in physics.  In "developing a feel," I found it weird that anything larger than 3 rounds up to
the next power of 10.  Why does that make sense?
```

4. Roughly how long did it take you to complete today's reading?

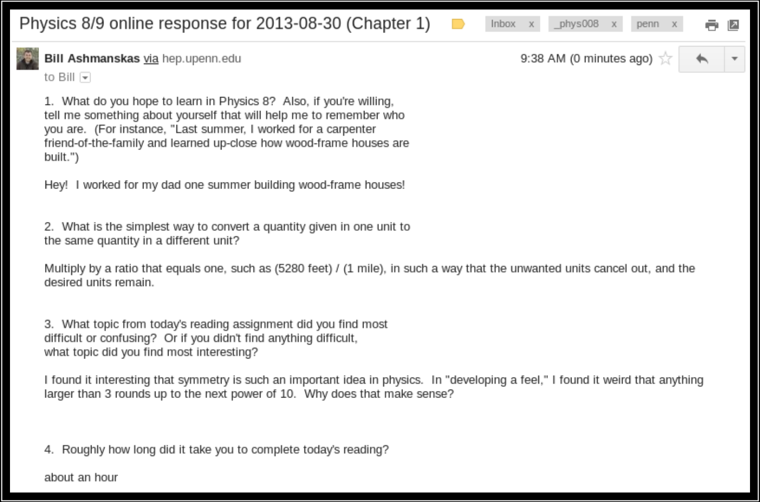
```
about an hour
```

[submit]

Fill out the form and then click "submit."



This should generate an email to me, CC to you. If you have any trouble, just email your answers to ashmansk@hep.upenn.edu



Physics 8/9 online response for 2013-08-30 (Chapter 1)   Inbox x   _phys008 x   penn x

Bill Ashmanskas via hep.upenn.edu   9:38 AM (0 minutes ago)

to Bill

1. What do you hope to learn in Physics 8? Also, if you're willing, tell me something about yourself that will help me to remember who you are. (For instance, "Last summer, I worked for a carpenter friend-of-the-family and learned up-close how wood-frame houses are built.")

Hey! I worked for my dad one summer building wood-frame houses!

2. What is the simplest way to convert a quantity given in one unit to the same quantity in a different unit?

Multiply by a ratio that equals one, such as (5280 feet) / (1 mile), in such a way that the unwanted units cancel out, and the desired units remain.

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4. Roughly how long did it take you to complete today's reading?

about an hour

I do this so that the reading-response emails are automatically formatted in a way that is easy for me to process and search.

You decide to add to your hirise design a rooftop swimming pool, big enough for people to swim laps for exercise. Make an order-of-magnitude estimate of the mass of the water contained in the pool. First guess by pure intuition, then try multiplying out some plausible numbers. [What size cube of water has a mass of a metric tonne?]

(A) 10^3 kg (1 tonne)

(B) 10^4 kg (10 tonnes)

(C) 10^5 kg (10^2 tonnes)

(D) 10^6 kg (10^3 tonnes)

(E) 10^7 kg (10^4 tonnes)

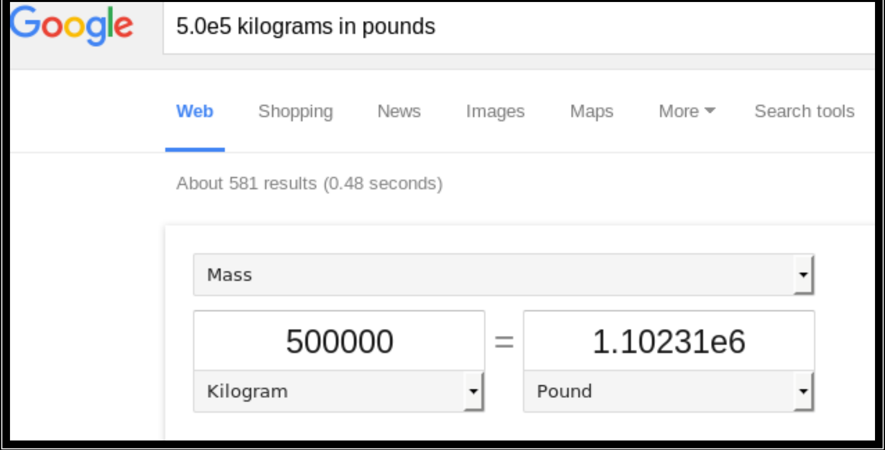
(F) 10^8 kg (10^5 tonnes)

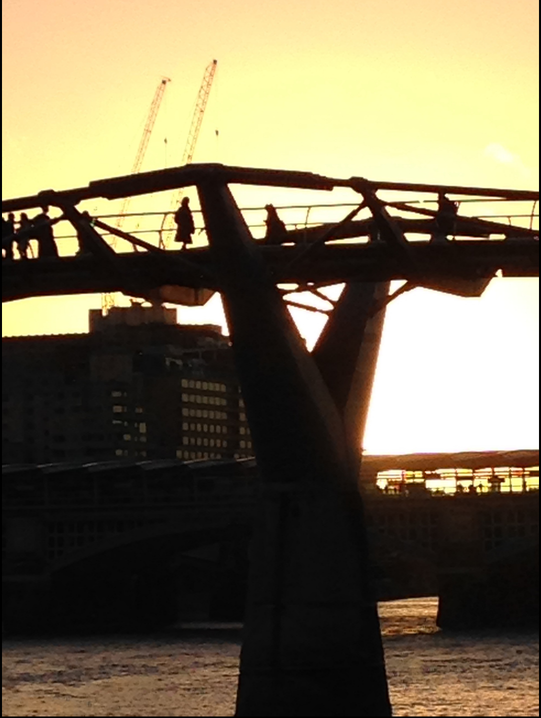
Here's my estimate

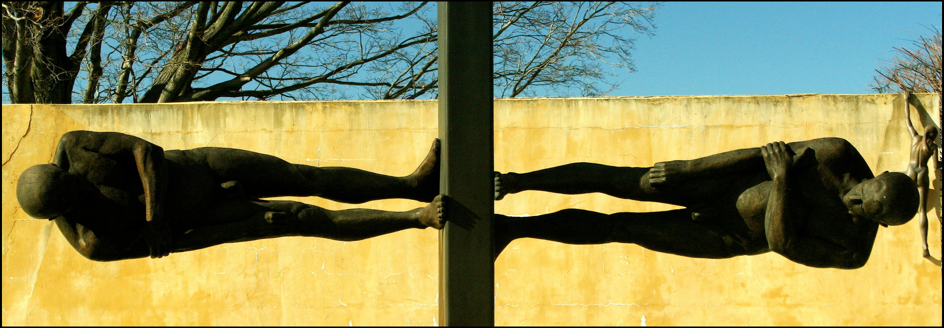
Useful trick: google understands units!

OK, what weight in pounds corresponds to this mass in kilograms?

Another trick: google can convert units!







- Course web page is at `http://positron.hep.upenn.edu/physics8`
- The reading for the first segment of the course is on Canvas. I'll explain later how to purchase or borrow a copy of the textbook for the second segment of the course.
- **To do before next class meeting:**
- Skim Chapter 1 during the long weekend.
- **Watch video then skim Chapter 2 before our 2nd class meeting.**
- Remember to fill out online response forms for both reading assignments at `http://positron.hep.upenn.edu/q008` . (This is linked from Canvas and from course web page, so you don't need to write it down.)
- PDFs of these slides and other handouts can be found at `http://positron.hep.upenn.edu/physics8/files`

video segment break

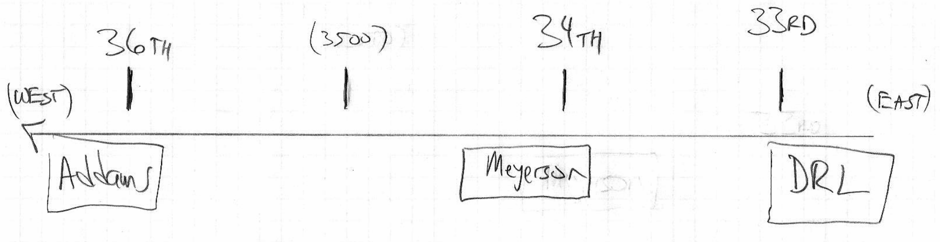
- begin video preceding ws02
- Before second day of class:
- first quickly skim through Mazur chapter 1
- then watch this video (which covers chapter 2)
- then skim through Mazur chapter 2

Physics 8 — Friday, August 30, 2019

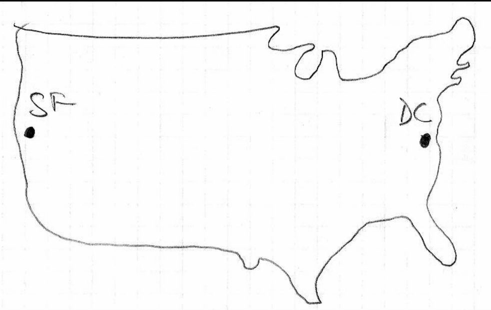
- Course www: `http://positron.hep.upenn.edu/physics8`
- You will skim Mazur Chapter 2 ("motion in one dimension") for today.
- Today's assignment will be first to watch this lecture, then to skim Chapter 2.

Vectors

- A **vector** has both a magnitude and a spatial direction, e.g. up, north, east, etc.
- The **position** $\vec{r}$ is a vector (x, y, z) pointing from the origin $(0, 0, 0)$ to the object's location in space. $\vec{r}$ indicates where the object is with respect to $x = 0$, $y = 0$, $z = 0$.
- You may be familiar with vectors written as triplets (x, y, z), or with arrows, $\vec{r} = (x, y, z)$.
- The **components** of this vector are
  $r_x = x$ (the x component),
  $r_y = y$ (the y component), and
  $r_z = z$ (the z component).
- The **magnitude** of vector $\vec{r}$ is $|\vec{r}| = r = \sqrt{x^2 + y^2 + z^2}$ (but we won't see that until Chapter 10).
- But for the first 9 chapters, we will deal only with the x axis. Once we reach chapter 10, we'll use x and y axes together. So no $\sqrt{x^2 + y^2}$ until then.



- What is the distance (in blocks) between DRL and Addams?
- If you walk in a straight line that starts at DRL and ends at Addams, what is your distance traveled (in blocks)?
- What is your displacement (expressed using blocks and a compass direction)?
- If you start at Addams and end at DRL, what is your displacement?
- What is your distance traveled in that case?
- If you start at Addams, walk to Meyerson, walk back to Addams, then walk to DRL (ending there), what is your displacement?
- What is your distance traveled?



- What is (roughly) the distance between SF and DC?
- If you start in SF and end in DC, what is your displacement?
- Which one is a vector?
- How does the distance between SF and DC relate to the displacement from SF to DC?
- How does the distance between SF and DC relate to the displacement from DC to SF?
- For a journey on which I go in a straight line, never changing direction, how are “distance” and “distance traveled” related?
- For a journey on which I do change direction several times, how can I figure out the distance traveled?

Position, displacement, etc.

- A **vector** has both a magnitude and a spatial direction, e.g. up, north, east, etc.
- The **position** $\vec{r}$ is a vector (x, y, z) pointing from the origin $(0, 0, 0)$ to the object's location in space. $\vec{r}$ indicates where the object is with respect to $x = 0$, $y = 0$, $z = 0$.
- If an object moves from some initial position $\vec{r}_i$ to some final position $\vec{r}_f$, we say its **displacement** (vector) is $\Delta\vec{r} = \vec{r}_f - \vec{r}_i$, pointing from its initial position $\vec{r}_i$ to its final position $\vec{r}_f$.
- The x **component** of the displacement is $x_f - x_i$.
- The **distance** (scalar) between $\vec{r}_i$ and $\vec{r}_f$ is $d = |\Delta\vec{r}| = |\vec{r}_f - \vec{r}_i|$. In one dimension, $d = |x_f - x_i|$.
- We'll be reminded in Chapter 10 that in two dimensions, $d = \sqrt{(x_f - x_i)^2 + (y_f - y_i)^2}$. For now we use 1D.

Position, displacement, etc.

- The **distance** (scalar) between $\vec{r}_i$ and $\vec{r}_f$ is $d = |\Delta\vec{r}| = |\vec{r}_f - \vec{r}_i|$. In one dimension, $d = |x_f - x_i|$.
- If the object does not change direction between $\vec{r}_i$ and $\vec{r}_f$, then the **distance traveled** is the same as d.
- If the object changes direction at (for example) points a,b,c along the way, then the **distance traveled** is

$$d_{\text{traveled}} = |\vec{r}_a - \vec{r}_i| + |\vec{r}_b - \vec{r}_a| + |\vec{r}_c - \vec{r}_b| + |\vec{r}_f - \vec{r}_c|$$

- In one dimension, the distanced traveled for this case (turning at three points a,b,c) would be

$$d_{\text{traveled}} = |x_a - x_i| + |x_b - x_a| + |x_c - x_b| + |x_f - x_c|$$

- If someone asks you how to get from DRL to 30th Street Station, is it sufficient to say (without pointing), "Go 5 blocks?"
- Is it good enough to say, "Go 2 blocks, then go another 3 blocks?"
- What about "Go 2 blocks north, then go 3 blocks east?"
- Once again, for the first 9 chapters of the textbook, directions will be **either** north/south **OR** east/west **OR** up/down, but we will not (until Chapter 10) work with more than one axis in a given problem.
- (Also, somewhat confusingly, for the first 9 chapters, the one axis that we do work with will always be called **the x axis**, even if it does not point in a direction that you are accustomed to associating wth the x axis.)
- So we won't worry, until Chapter 10, about things like the fact that a bird could travel from DRL to 30th Street Station along a diagonal that is $\sqrt{13}$ blocks long.

For next few questions

(I'll copy this to the board.)

(A) $+5$ meters

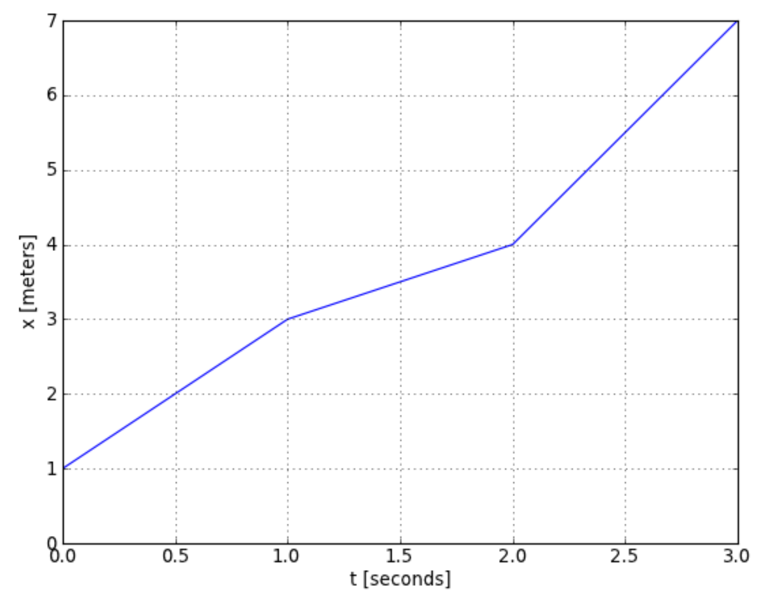
(B) $+6$ meters

(C) $+8$ meters

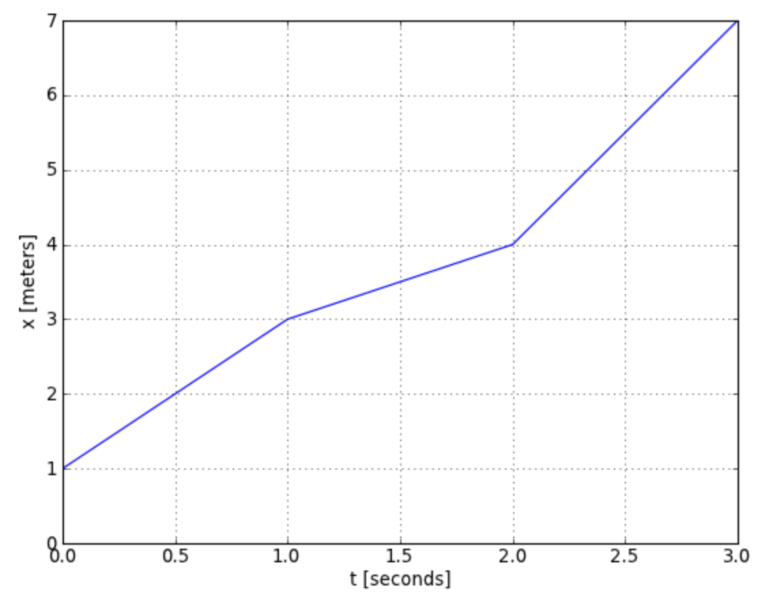
(D) -6 meters

(E) -8 meters

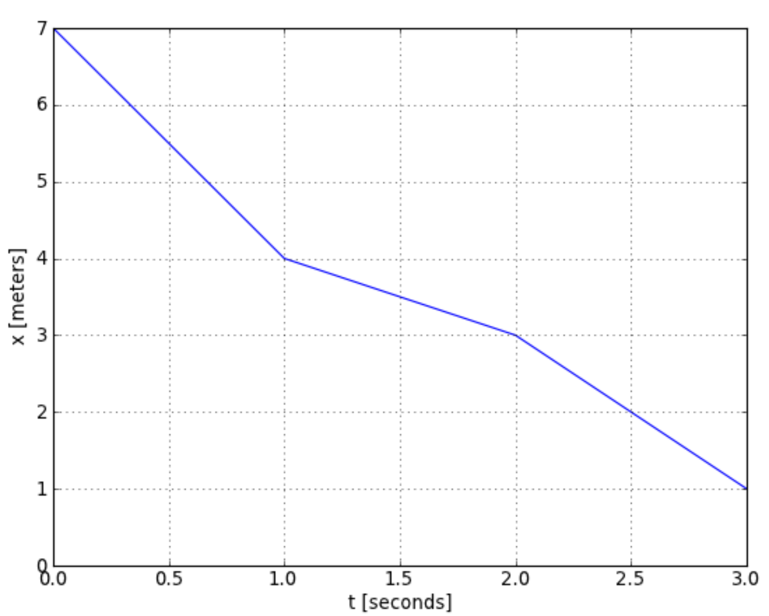
What is the distance traveled from t=0 to t=3s?



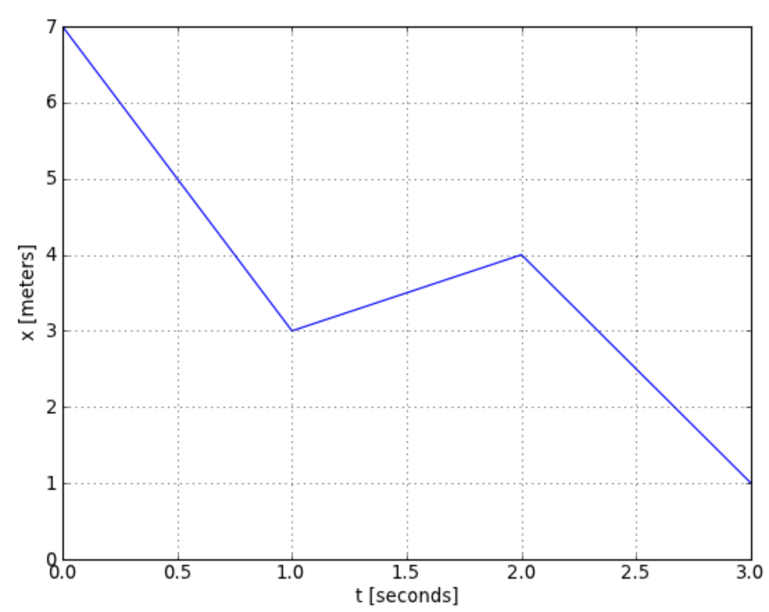
What is the x component of displacement?



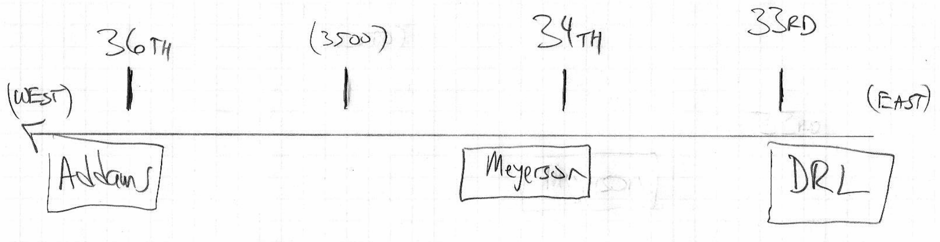
Now what is the x component of displacement?



Now what is the distance traveled?



To keep the math simple, let's pretend that every city block is exactly 100 meters long.



- If I bike directly from DRL to Addams in 100 seconds, what is my average speed?
- What is my average velocity?
- If I walk directly from DRL to Addams in 200 seconds, then bike directly back from Addams to DRL in 100 seconds, what is my average velocity for the journey?
- What is my average speed for the journey?

- What is the relationship between (instantaneous) speed and (instantaneous) velocity?
- What does calculus say about the relationship between speed and distance traveled? (Does one of them equal the rate of change of the other?)
- What does calculus say about the relationship between displacement and velocity? (Does one of them equal the rate of change of the other?)

Velocity and speed

- **Velocity** (a vector) is the rate of change of position with respect to time: $\vec{v} = \frac{\mathrm{d}\vec{r}}{\mathrm{d}t} = (v_x, v_y, v_z) = (\frac{\mathrm{d}x}{\mathrm{d}t}, \frac{\mathrm{d}y}{\mathrm{d}t}, \frac{\mathrm{d}z}{\mathrm{d}t})$
- **speed** $v = |\vec{v}|$ is magnitude (scalar) of velocity (vector)
- In one dimension, speed is $v = |v_x|$, i.e. the absolute value of the x-component of velocity.
- We can talk about velocity at a given instant. Over a finite time interval, we can talk about the **average velocity** during the time from t_i to t_f.

$$\vec{v}_{\mathrm{av}} = \frac{\Delta\vec{r}}{t_f - t_i} \qquad v_{x,\mathrm{av}} = \frac{x_f - x_i}{t_f - t_i}$$

- The **average speed** during the finite time interval from t_i to t_f is the (distance traveled) divided by the (time interval)

$$v_{\mathrm{av}} = \frac{d_{\mathrm{traveled}}}{t_f - t_i}$$

Example 2.9 (modified)

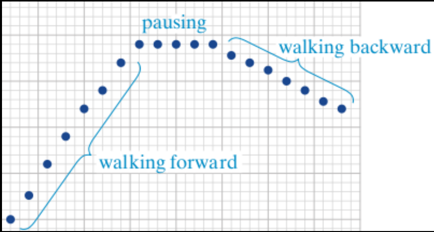
frame #	x (m)	t (s)
1	+1.0	0
2	+1.5	0.33
3	+2.2	0.67
4	+2.8	1.00
5	+3.4	1.33
6	+3.8	1.67
7	+4.4	2.00
8	+4.8	2.33
9	+4.8	2.67
10	+4.8	3.00
11	+4.8	3.33
12	+4.8	3.67
13	+4.6	4.00
14	+4.4	4.33
15	+4.2	4.67
16	+4.0	5.00
17	+3.8	5.33
18	+3.6	5.67
19	+3.4	6.00

Consider Eric's motion between frames 13 and 19 in textbook Figure 2.1. Let's use the values in Table 2.1 to answer to these questions:

(a) What is his average speed over this time interval?

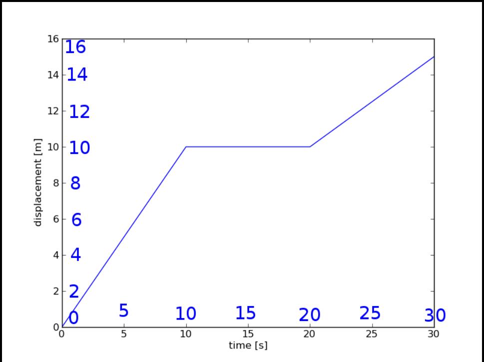
(b) What is the x component of his average velocity over this time interval?

(c) Write the average velocity (during this time interval) in terms of the unit vector $\hat{\imath}$.



Drawing position (or displacement) vs. time

Which statement best describes the motion depicted by this graph?



(A) I walk 1.0 m/s forward for 10 s. Then I rest 10 s. Then I walk 1.0 m/s backward for 10 s.

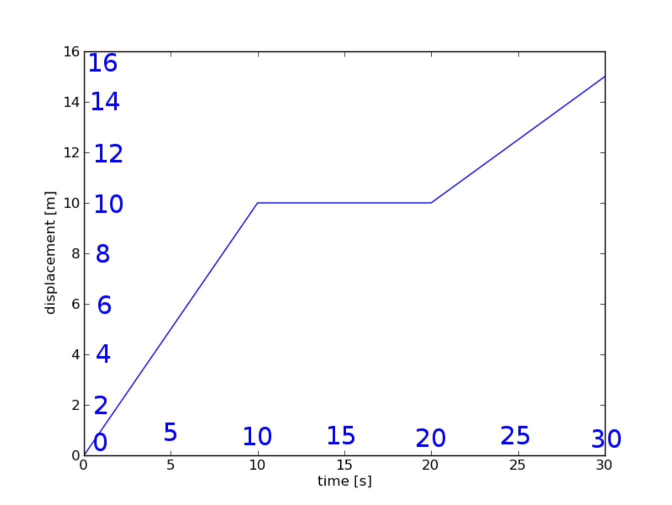
(B) I walk 0.5 m/s forward for 10 s. Then I rest 10 s. Then I walk 1.0 m/s forward for 10 s.

(C) I walk 0.5 m/s forward for 10 s. Then I rest 10 s. Then I walk 0.5 m/s forward for 10 s.

(D) I walk 1.0 m/s forward for 10 s. Then I rest 10 s. Then I walk 0.5 m/s forward for 10 s.

Average velocity

What is my average velocity $\vec{v}_{\mathrm{av}}$ during the 30 second interval shown on this graph? (Remember that $\hat{i}$ is the unit vector pointing forward along the x axis, i.e. pointing in the direction in which x increases.)



(A) $+1.0$ m/s $\hat{i}$

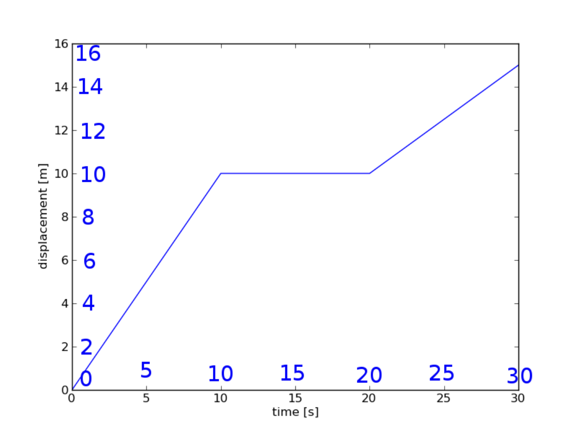
(B) $+0.75$ m/s $\hat{i}$

(C) $+0.5$ m/s $\hat{i}$

(D) -0.25 m/s $\hat{i}$

Instantaneous velocity

What is my instantaneous velocity $\vec{v}$ at time $t = 5$ s? What is $\vec{v}$ at time $t = 15$ s?



(A) $+1.0$ m/s $\hat{i}$ and 0 m/s $\hat{i}$, respectively

(B) $+0.5$ m/s $\hat{i}$ and $+1.0$ m/s $\hat{i}$, respectively

(C) $+1.0$ m/s $\hat{i}$ and $+0.5$ m/s $\hat{i}$, respectively

(D) $+0.5$ m/s $\hat{i}$ and $+0.5$ m/s $\hat{i}$, respectively

Slope of the x(t) curve

The slope of the curve in the x coordinate of position vs. time graph (graph of $x(t)$ vs. t) for an object's motion gives

(A) the object's speed

(B) the object's acceleration

(C) the object's average velocity

(D) the x component of the object's instantaneous velocity

(E) not covered in today's material

You walk 1.2 km (1200 m) due east from home to a restaurant in 20 min (1200 s), stay there for an hour (3600 s), and then walk back home, taking another 20 min. What is your **average speed** for the trip?

(A) $v_{\mathrm{av}} = 0.0\ \mathrm{m/s}$

(B) $v_{\mathrm{av}} = 0.4\ \mathrm{m/s}$

(C) $v_{\mathrm{av}} = 0.8\ \mathrm{m/s}$

(D) $v_{\mathrm{av}} = 1.0\ \mathrm{m/s}$

(E) $v_{\mathrm{av}} = 2.0\ \mathrm{m/s}$

You walk 1.2 km (1200 m) due east from home to a restaurant in 20 min (1200 s), stay there for an hour (3600 s), and then walk back home, taking another 20 min. What is your **average velocity** for the trip?

(A) $\vec{v}_{\mathrm{av}} = \vec{0}$

(B) $\vec{v}_{\mathrm{av}} = +0.4\ \mathrm{m/s}$ east

(C) $\vec{v}_{\mathrm{av}} = +0.8\ \mathrm{m/s}$ east

(D) $\vec{v}_{\mathrm{av}} = -0.4\ \mathrm{m/s}$ east

(E) $\vec{v}_{\mathrm{av}} = -0.8\ \mathrm{m/s}$ east

You drive an old car on a straight, level highway at 20 m/s for 20 km, and then the car stalls. You leave the car and, continuing in the direction in which you were driving, walk to a friend's house 4 km away, arriving 1000 s after you began walking. What is your average speed during the whole trip?

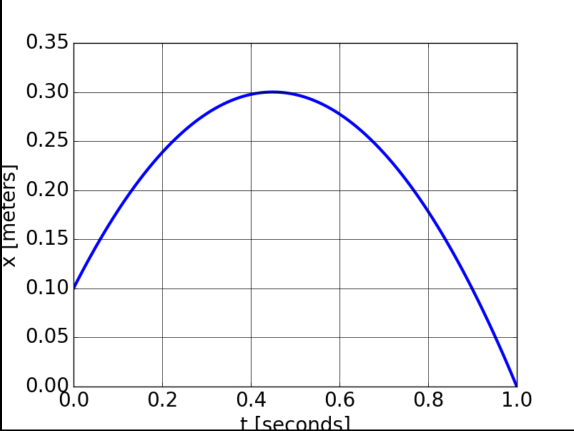
(A) $v_{\mathrm{av}} = 10\ \mathrm{m/s}$

(B) $v_{\mathrm{av}} = 12\ \mathrm{m/s}$

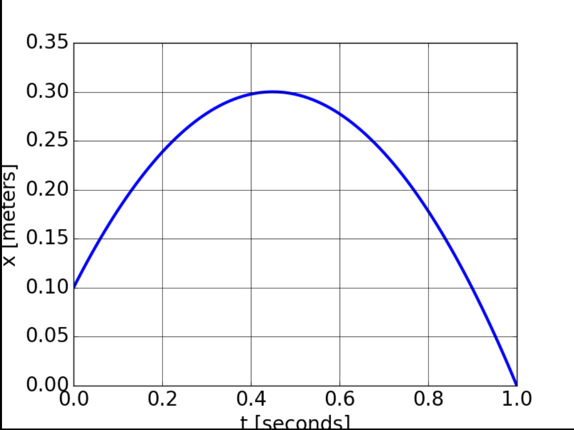
(C) $v_{\mathrm{av}} = 15\ \mathrm{m/s}$

(D) $v_{\mathrm{av}} = 20\ \mathrm{m/s}$

(E) $v_{\mathrm{av}} = 24\ \mathrm{m/s}$



- Where is the object moving forward?
- Where is the object moving backward?
- Where does the speed equal zero?
- Where is the speed largest?
- Where is v_x (the x component of velocity) largest?



For the motion represented in the figure above, what is the object's average velocity between $t = 0$ and $t = 1.0\,\mathrm{s}$?

What is its average speed during this same time interval?

Why is the average speed, for this motion, different from the magnitude of the average velocity?

Physics 8 — Wednesday, September 4, 2019

- Course www: `http://positron.hep.upenn.edu/physics8`
- Why are we talking about velocity and acceleration, when architectural structures generally do not move? Answer: to understand force and torque, we need first to discuss motion.

Unit vectors (yuck)

- We can define **unit vectors** in the x, y, and z directions: $\hat{i} = (1, 0, 0)$, $\hat{j} = (0, 1, 0)$, and $\hat{k} = (0, 0, 1)$.
- Then we can write $\vec{r} = (x, y, z) = x\hat{i} + y\hat{j} + z\hat{k}$.
- It's often convenient to define a coordinate system where the x-axis points east, the y-axis points north, and the z-axis points up, with the origin at some specified location (e.g. the center of the ground floor).
- Then if I'm standing 5 meters east of the origin, my position vector is $+5\,\mathrm{m}\ \hat{i}$, which we could also write as $(+5\,\mathrm{m}, 0, 0)$.
- If I'm 3 m west of the origin, then $\vec{r} = -3\,\mathrm{m}\ \hat{i} = (-3\,\mathrm{m}, 0, 0)$.
- If I'm 2 m north of the origin, then my position is $\vec{r} = +2\,\mathrm{m}\ \hat{j} = (0, +2\,\mathrm{m}, 0)$.
- Most students dislike Mazur's unit-vector notation, so I try to avoid using it. I will instead write, "The displacement is $+5$ meters eastward." I will usually use a word like "east" or "north" or "up" to avoid writing $\hat{i}$ or other unit vectors.

Vectors

- Vectors are very useful on a 2D map ((x, y) or geocode) or in a 3D CAD model (x, y, z).
- For the first 10 chapters of our textbook, all problems will be one-dimensional (we will use the x-axis only), which makes the use of vectors seem contrived at this stage.
- The reason for doing this is so that we can focus on the physics first before reviewing too much math.
- In one dimension, position is $\vec{r} = (x, 0, 0) = x\ \hat{i}$.
- The x component of vector $\vec{v}$ is v_x, and in one dimension $\vec{v} = (v_x, 0, 0) = v_x\ \hat{i}$.
- The x component of vector $\vec{r}$ is x. (Special case notation.)
- In 1D, magnitude of $\vec{r}$ is $|x|$, and magnitude of $\vec{v}$ is $|v_x|$.
- Vectors will seem more natural starting in Chapter 10, when we study motion in a two-dimensional plane.

- **position:** where is it located in space? $\vec{r} = (x, y, z)$
- **displacement:** where is it w.r.t. some earlier position?
- $\Delta\vec{r} \; = \; (\Delta x, \Delta y, \Delta z) \; = \; \Delta x\hat{\imath} + \Delta y\hat{\jmath} + \Delta z\hat{k}$
- position and displacement are both **vectors**: they have both a direction in space and a magnitude
- **distance** is a scalar (magnitude only, never negative)
- **unit vectors** $\hat{\imath} = (1, 0, 0)$, $\hat{\jmath} = (0, 1, 0)$, $\hat{k} = (0, 0, 1)$ are vectors pointing along x,y,z axes, with "unit" magnitude (length = 1). Until Chapter 10, we use only the x-axis. So $\hat{\imath}$ is the only unit vector introduced in Chapter 2.
- **average velocity** $\vec{v}_{\rm av} = \frac{\Delta\vec{r}}{\Delta t}$: (displacement) / (time interval)
  x-component of $\vec{v}_{\rm av}$ is $v_{x,\rm av} = \frac{\Delta x}{\Delta t}$
- **(instantaneous) velocity** $\vec{v} = \frac{{\rm d}\vec{r}}{{\rm d}t} = (\frac{{\rm d}x}{{\rm d}t}, \frac{{\rm d}y}{{\rm d}t}, \frac{{\rm d}z}{{\rm d}t})$
  x-component of $\vec{v}$ is $v_x = \frac{{\rm d}x}{{\rm d}t}$
- velocity is a vector (it has a direction in space),
  speed is a scalar (it has only a magnitude)
- For many people, the hardest part of this reading was getting used to the author's notation.

Let's start by asking how your neighbor's answer to the first reading question compares with your own:

- What is a vector, and what is it good for?
- By the way, what are two examples of vectors that are focal points of chapter 2? (See what your neighbor says.)
- Here's what one of you wrote:

"A vector quantity, unlike a scalar quantity, is one that not only has a magnitude but also a direction. An example of an important vector quantity is displacement — unlike distance, displacement takes into account the direction that something has travelled in (i.e. while someone may have run a 400 m distance on a track, their displacement would be 0 since they end up back where they started.)"

By the way: clear and complete answers make me very happy.

Potential sources of confusion from Chapter 2

- It takes a while to get used to the textbook's vector notation. Some people positively hate the book's notation!
  - But the book's notation is extremely self-consistent, even if the many subscripts and superscripts can be annoying.
  - And this book is excellent on the concepts.
- Also, it might take some practice to re-acclimate your brain to reading lots of equations, if it has been several years since your last math course. No worries.
- What is a unit vector? Yuck!
- Using only a single spatial dimension (until Chapter 10) makes the discussion of vectors seem contrived.
- Distinction between displacement & position vectors.
- Difference between average and instantaneous velocity.
- Anything to add to this list?

Now — onward to chapter 3 . . .

video segment break

- begin video preceding ws03
- Before third class meeting:
- first watch this video
- then skim through Mazur chapter 3, focusing mainly on the concepts half, and glossing over most equations

Defining acceleration

- Last week, we defined velocity as the rate of change of position with respect to time

$$v_x = \frac{\mathrm{d}x}{\mathrm{d}t}$$

(considering only the x component for now), and we learned to identify v_x visually as the slope on a graph of $x(t)$

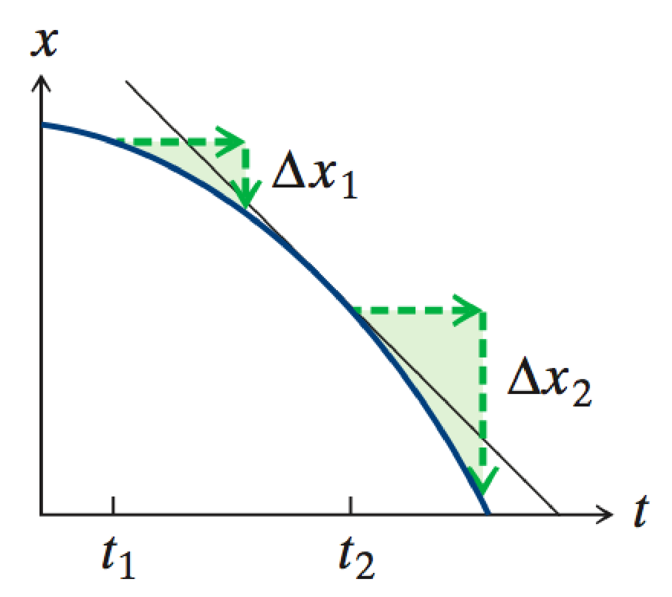
- Moving at constant velocity is not very interesting! So we need to be able to talk about changes in velocity.
- The rate of change of velocity with respect to time is called acceleration:

$$a_x = \frac{\mathrm{d}v_x}{\mathrm{d}t}$$

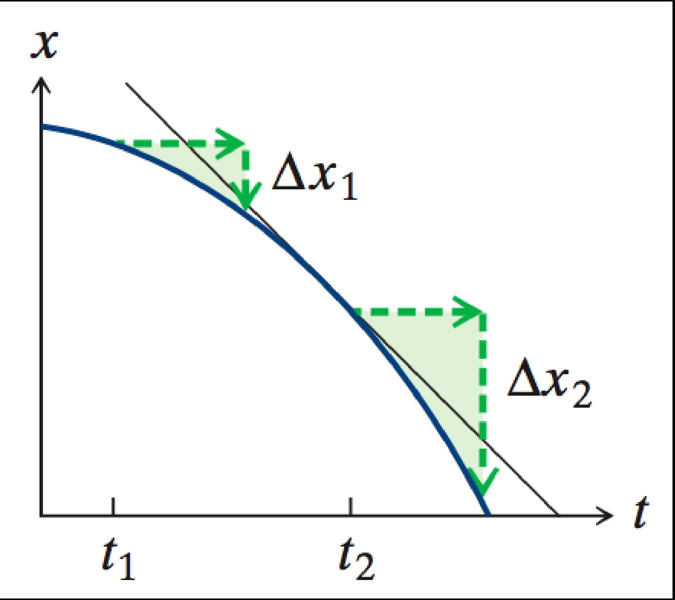
- While acceleration can also vary with time (!), there are many situations in which constant acceleration ($a_x = \mathrm{constant}$) gives a good description of the motion. We'll see soon what math lets us conclude, if we start with $a_x = \mathrm{constant}$.

At time t_2 in the position-vs-time graph below, the object is

(A) not moving
(B) moving at constant speed
(C) speeding up
(D) slowing down



At time t_2 in the position-vs-time graph below, is v_x (the x component of velocity) is



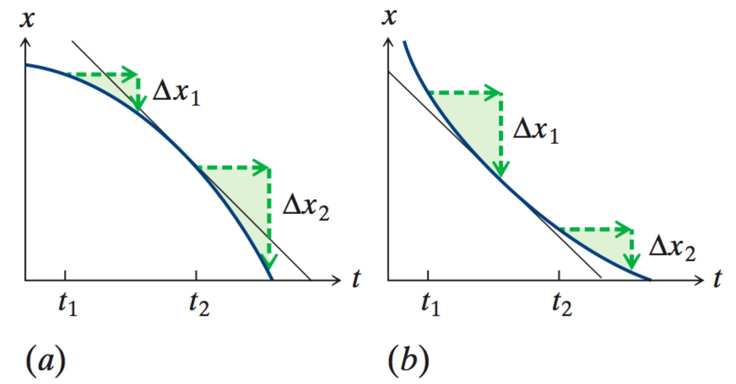
(A) zero

(B) not changing

(C) increasing

(D) decreasing

The x component of acceleration in these two graphs is



(A) positive in (a), negative in (b)

(B) negative in (a), positive in (b)

(C) negative in both (a) and (b)

(D) positive in both (a) and (b)

(E) zero in both (a) and (b)

Accelerating under gravity's influence

- One important situation in which constant acceleration ($a_x = \text{constant}$) gives a good description of the motion is "free fall" near Earth's surface.
- (Until Chapter 10, we will use only one axis in any given problem, and we will call that axis x. So for free-fall problems, for now, the x axis will be vertical, pointing upward.)
- *Free fall* is the motion of an object subject only to the influence of gravity.
  - Not being pushed or held by your hand or by the ground
  - When air resistance is small enough to neglect
- Close to Earth's surface, an object in free fall experiences a constant acceleration, of magnitude $|\vec{a}| = 9.8\ \mathrm{m/s^2}$ and pointing in the *downward* direction.
- If we define the x axis to point upward (as we often will, for free-fall problems before Ch10), then $a_x = -9.8\ \mathrm{m/s^2}$.
- Since we see the quantity $9.8\ \mathrm{m/s^2}$ so often, we give it a name: $g = 9.8\ \mathrm{m/s^2}$. Then $a_x = -g$.

(Checkpoint 3.7)

Let's pause here to go through Checkpoint 3.7 together.

- Does the speed of a falling object (A) increase or (B) decrease?
- If the positive x axis points up, does v_x (A) increase or (B) decrease as the object falls?
- is the x component of the acceleration (A) positive or (B) negative?

Discuss with your neighbor for a moment, and then we'll compare answers.

You and your neighbor might even want to graph $v_x(t)$, for a falling object, while you discuss.

Let's do what Galileo could only imagine doing!

- Let's see if different objects really do fall with the same acceleration

$$a_x = -g$$

if we are able to remove the effects of air resistance.

Equations we can derive from $a_x = \text{constant}$

- **You don't need to know how to do these derivations,** but if you like calculus, you might enjoy seeing where these often-used results come from.
- We defined $a_x = \frac{\mathrm{d}v_x}{\mathrm{d}t}$ and $v_x = \frac{\mathrm{d}x}{\mathrm{d}t}$, without worrying so far about whether or not a_x is changing with time.
- Integrating the first equation $\left(\frac{\mathrm{d}v_x}{\mathrm{d}t} = a_x\right)$ over time,

$$v_x(t) = v_{x,i} + \int_0^t a_x \, \mathrm{d}t$$

- If $a_x = \text{constant}$, then this integral becomes easy to do:

$$v_x(t) = v_{x,i} + a_x t$$

- We can also integrate the equation $\left(\frac{\mathrm{d}x}{\mathrm{d}t} = v_x\right)$ over time:

$$x(t) = x_i + \int_0^t v_x \, \mathrm{d}t$$

keeping in mind that v_x (unlike a_x) is changing with time

Equations we can derive from $a_x = \mathrm{constant}$

$$v_x(t) = v_{x,i} + a_x t$$

$$x(t) = x_i + \int_0^t v_x \, \mathrm{d}t$$

- Plugging our $v_x(t)$ result into the second integal:

$$x(t) = x_i + \int_0^t (v_{xi} + a_x t) \, \mathrm{d}t$$

$$x(t) = x_i + v_{x,i}\, t + \frac{1}{2}\, a_x\, t^2$$

Equations we can derive from $a_x = \text{constant}$

- That's all there is to it. Just writing down the assumption that a_x is constant allows us to integrate twice to get two results that you will use many times:

$$v_{x,f} = v_{x,i} + a_x t$$

$$x_f = x_i + v_{x,i} t + \frac{1}{2} a_x t^2$$

- If you plug one of these equations into the other, you can eliminate t to get one more very useful result

$$v_{x,f}^2 = v_{x,i}^2 + 2a_x\,(x_f - x_i)$$

- This last one is helpful e.g. to know how fast the dropped steel ball is traveling at the instant before it hits the ground.
- My point is that these equations are just the result of taking $a_x = \text{constant}$ and doing some math.

Inclined planes

- Falling to the ground at $a_x = -g$ happens so quickly that it can be difficult to see exactly what is happening.
- Maybe there is a way to "fall" in slow motion?
- Yes! We can slide down a hill.

$$|g| \;\rightarrow\; |g\sin\theta|$$

(We'll see in Chapter 10 why it's $\sin\theta$ here. Don't worry.)

- To get the $\pm$ sign right, you have to choose which direction to draw the x axis. Eric chooses the x axis to point *downhill*

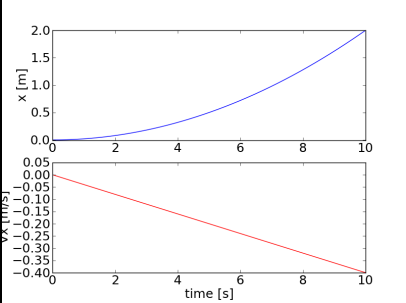
$$a_x = +g\sin\theta$$

- Let's look at **this contraption** and figure out which way it defines the x axis to point

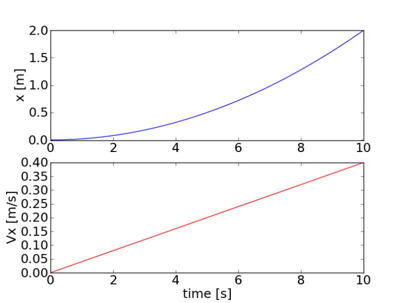
Inclined air track

- It looks as if the x axis points *downhill*, and the point on the top of the ramp is called $x = 0$.

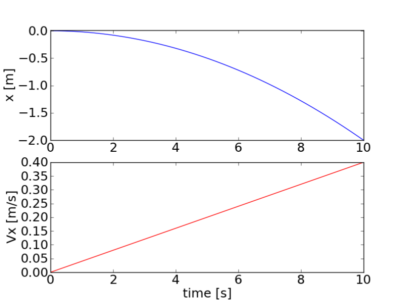
Which of the following shows the expected shapes of $x(t)$ [blue] and $v_x(t)$ [red] if I release the cart (at rest) from $x = 0$?



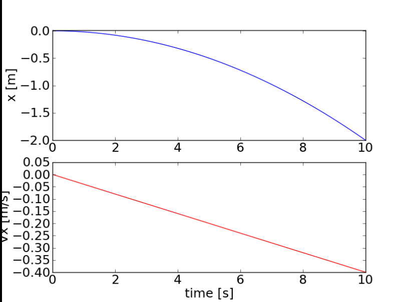
A



B

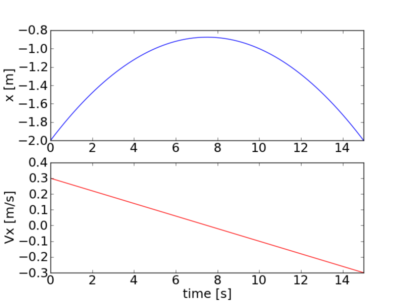


C

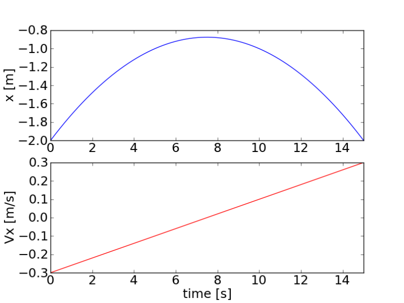


D

Which of the following shows the expected shapes of $x(t)$ [blue] and $v_x(t)$ [red] if I shove the cart upward starting from $x = +2$ m?



A



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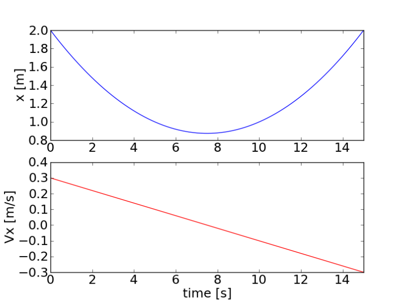

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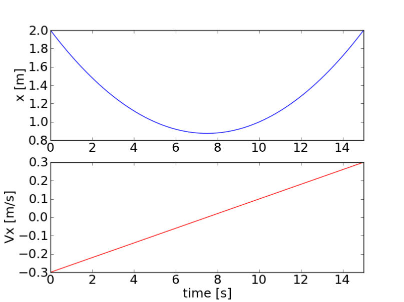


D

I shove the cart uphill and watch it travel up and back. We define the x axis to point *downhill*. At the top of its trajectory (where it turns around), v_x is

(A) positive

(B) negative

(C) zero

(D) infinite

(E) undefined

I shove the cart uphill and watch it travel up and back. We define the x axis to point *downhill*. At the top of its trajectory (where it turns around), a_x is

(A) positive

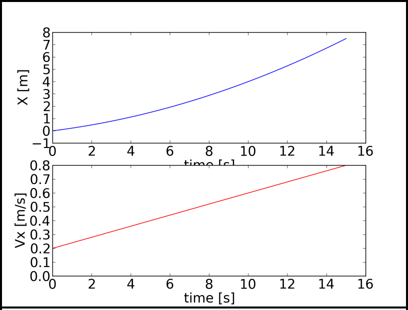
(B) negative

(C) zero

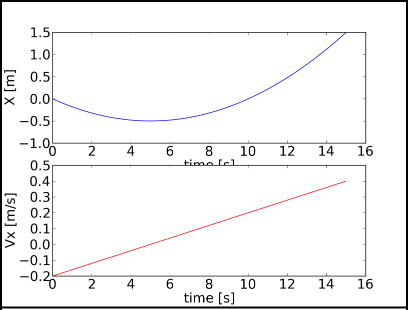
(D) infinite

(E) undefined

Which of the following shows the expected shapes of $x(t)$ [blue] and $v_x(t)$ [red] if I **shove** the cart gently downward from $x = 0\ \mathrm{m}$?



A



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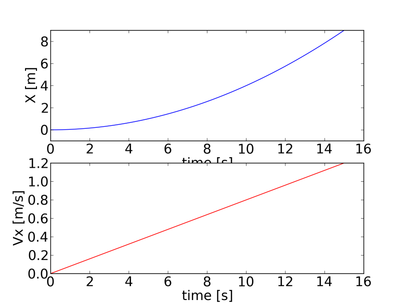
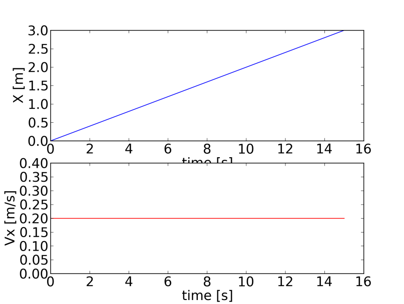


C



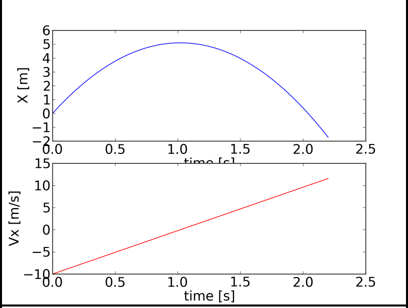
D

In a past year, someone asked an excellent question after class about the difference, in the previous slide, between scenario (A) and scenario (C). Let's ponder that.

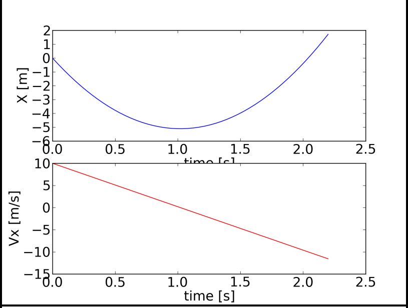
Physics 8 — Friday, September 6, 2019

- Course www: `http://positron.hep.upenn.edu/physics8`

What are the expected shapes of $x(t)$ [blue] and $v_x(t)$ [red] for a **basketball tossed upward**, when the x axis points *upward*?



A



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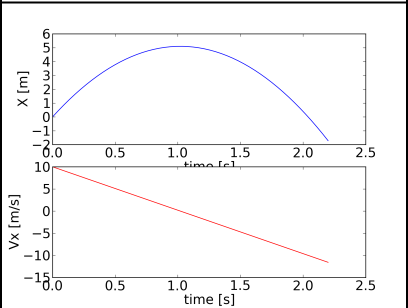

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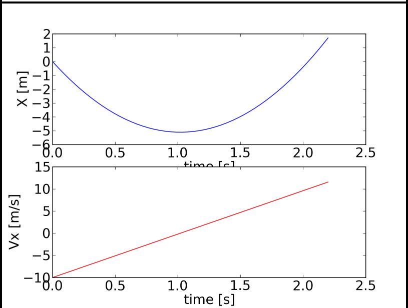


D

Basketball tossed upward

What are the values of v_x and a_x at the top of the basketball's trajectory (assuming that the x axis points upward)?

(A) $v_x < 0$, $a_x = -9.8\ \mathrm{m/s^2}$

(B) $v_x < 0$, $a_x = 0$

(C) $v_x = 0$, $a_x = -9.8\ \mathrm{m/s^2}$

(D) $v_x = 0$, $a_x = 0$

(E) $v_x = 0$, a_x is undefined

Ball thrown downward

If you stand up high and release an object with a downward shove, in the absence of air resistance, the motion (after release, but before hitting the ground) is best described by

(A) $v_x < 0$, $a_x = -9.8\ \mathrm{m/s^2}$

(B) $v_x < 0$, $a_x = 0$

(C) $v_x = 0$, $a_x = -9.8\ \mathrm{m/s^2}$

(D) $v_x = 0$, $a_x = 0$

(E) $v_x = 0$, a_x is undefined

(Where we've defined the x axis to point *upward* here.)

I'm going to drop the basketball from a few meters in the air, and I'll let it bounce twice before I catch it. Working with one or two people next to you, draw a graph of $v_x(t)$ (velocity) and a graph of $a_x(t)$ (acceleration), spanning the time from release to catch. Let the x axis point upward. Don't worry about labeling the axes with numerical values, but do be clear about positive vs. zero vs. negative values.

Put your name(s) on your sheet of paper (one, two, or three people per sheet — whatever you prefer) and turn it in at the end of class. I'll give you credit for showing up and making the effort, not so much for correctness.

There will be a couple of other things for you to work out together in class today, so you'll probably need a full sheet of paper.

Reminder

velocity is rate of change of position: $v_x = \frac{\mathrm{d}x}{\mathrm{d}t}$

acceleration is rate of change of velocity: $a_x = \frac{\mathrm{d}v_x}{\mathrm{d}t}$

If acceleration is **constant**, then: (write these on board)

$$v_{x,f} = v_{x,i} + a_x t$$

$$x_f = x_i + v_{x,i} t + \frac{1}{2} a_x t^2$$

$$v_{x,f}^2 = v_{x,i}^2 + 2a_x\,(x_f - x_i)$$

Important cases for which a_x is constant:

free fall: $a_x = -g$
(x axis points up)

inclined plane: $a_x = +g\sin\theta$
(x axis points downhill)

Q: If I stand $h = 20\,\mathrm{m}$ above the ground and release a steel ball from rest, how long does it take to reach the ground? (Hint: to avoid using a calculator, you can approximate $g \approx 10\ \mathrm{m/s^2}$.)

(A) 2.0 s

(B) 1.5 s

(C) 1.0 s

(D) 0.50 s

(E) 0.25 s

$$x_f = x_i + v_{x,i} t + \frac{1}{2} a_x t^2$$

$$0 = h + 0 - \frac{1}{2} g t^2$$

Q: If I stand 20 m above the ground and release a steel ball from rest, what is its velocity at the instant just before it reaches the ground? (Use $g = 10\ \mathrm{m/s^2}$ to simplify math.)

(A) 10 m/s, pointing downward

(B) 15 m/s, pointing downward

(C) 20 m/s, pointing downward

(D) 40 m/s, pointing downward

(E) 40 m/s, pointing upward

If you already solved for t in the previous question then:

$$v_{x,f} = v_{x,i} + a_x t$$

$$v_{x,f} = 0 - gt$$

Or if you don't already know t then:

$$v_{x,f}^2 = v_{x,i}^2 + 2a_x\,(x_f - x_i)$$

$$v_{x,f}^2 = 0^2 + 2(-g)\,(0 - h)$$

Work on this together with 1 or 2 nearby people!

A box is at the lower end of a frictionless ramp of length $L = 10\ \mathrm{m}$ that makes a nonzero angle $\theta = 30^\circ$ with the horizontal. A worker wants to give the box a shove so that it just reaches the top of the ramp. How fast must the box be going immediately after the shove (assumed to be instantaneous) for it to reach its goal? Remember $\sin 30^\circ = \frac{1}{2}$ and use $g \approx 10\ \mathrm{m/s^2}$ to keep the math simple.

(A) 1.0 m/s

(B) 5.0 m/s

(C) 7.0 m/s

(D) 10 m/s

(E) 20 m/s

Put your group's name(s) on the sheet of paper you work this out on, and turn it in at the end for "in class" credit.

Reminder (on board): results derived from $\boxed{a_x = \mathrm{constant}}$.

Work on this together with 1 or 2 neighbors!

A box is at the lower end of a frictionless ramp of length $L = 10\ \mathrm{m}$ that makes a nonzero angle $\theta = 30^\circ$ with the horizontal. A worker wants to give the box a shove so that it just reaches the top of the ramp. She shoves the box, as we worked out on the previous page: the box's initial speed is 10 m/s. (Again, use $g \approx 10\ \mathrm{m/s^2}$ to keep the math simple.)

What is the box's speed when it is halfway up the ramp?

(A) 1.0 m/s

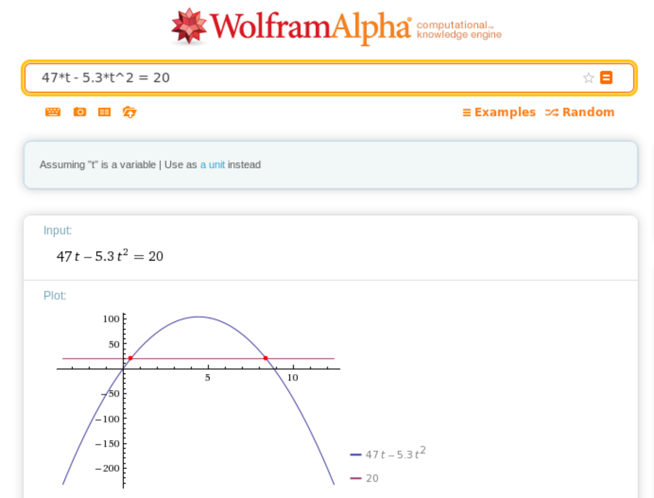
(B) 5.0 m/s

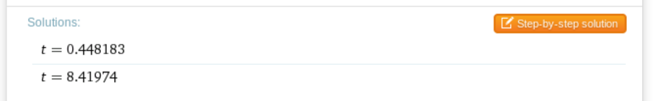
(C) 7.1 m/s

(D) 10.0 m/s

(E) 20.0 m/s

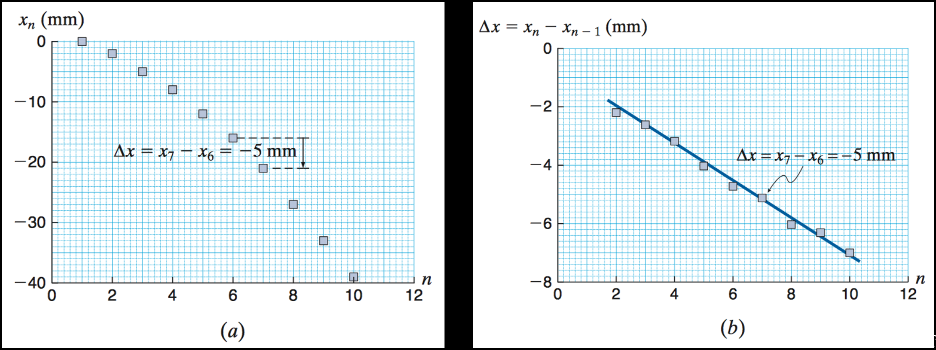
Another trick: Wolfram Alpha knows the quadratic formula





Potential sources of confusion from **today's** reading (Chapter 3)

- Inclined planes are new to many people.
- How do you draw a motion diagram?
- Don't follow Eric's reasoning about what is happening (v_x, a_x) at the very top of the motion for a ball tossed upward.
- Some of the mathy parts at the end are hard to follow.
- For checkpoint 3.6, you have to stare at Figure 3.6b for a while before you see that, since the points are all equal steps in time, the quantity being graphed is proportional to v_x, the x component of velocity.



video segment break

- begin video preceding ws04
- This time, actually **read** Mazur chapter 4, then come back to watch this video. Try to do the checkpoints, but you can gloss over most of the equations.

Physics 8 — Monday, September 9, 2019

- Course www: `http://positron.hep.upenn.edu/physics8`
- Question: momentum is what times what?

Chapter 4: momentum

- An object's momentum is $\vec{p} = m\vec{v}$ $p_x = mv_x$
- m is for "mass" a.k.a. "inertia." Mass plays two roles in physics: how strongly an object is attracted by gravity, and how difficult it is to change an object's velocity. We say "inertia" for now to focus on this latter aspect of mass. Inertia equals mass.
- Momentum is *conserved*: it can be transferred between interacting objects, but it cannot be created or destroyed.
- If the objects within a system have no interactions with the outside world ("isolated system"), then the momentum of that system is constant (cannot change).
- Imagine how it feels to throw a very heavy ball.
- Now imagine that you are standing on a sheet of ice!
- The difference is the *impulse* you get from the interaction between your shoes and the non-slippery floor.

For two carts colliding on a frictionless track, I can define "the system" to include just the two carts. Then $\boxed{\Delta\vec{p}_{\text{system}} = \vec{0}}$ because the system is isolated (i.e. interactions with the outside are negligible).

Here are 7 different ways of saying the exact same thing:

$$\Delta\vec{p}_1 + \Delta\vec{p}_2 = \vec{0} \quad \text{(isolated system)}$$

$$\vec{p}_{1i} + \vec{p}_{2i} = \vec{p}_{1f} + \vec{p}_{2f}$$

$$p_{1x,i} + p_{2x,i} = p_{1x,f} + p_{2x,f}$$

$$m_1\ \Delta v_{1x} \ + \ m_2\ \Delta v_{2x} \ = \ 0$$

$$\boxed{m_1 v_{1x,i} + m_2 v_{2x,i} = m_1 v_{1x,f} + m_2 v_{2x,f}}$$

$$m_1\ \Delta v_{1x} \ + \ m_2\ \Delta v_{2x} \ = \ 0$$

$$\frac{\Delta v_{1x}}{\Delta v_{2x}} = -\frac{m_2}{m_1}$$

The boxed equation is most useful for problem solving. The last equation is most useful for visual observation of collisions.

(For an isolated system of two objects)

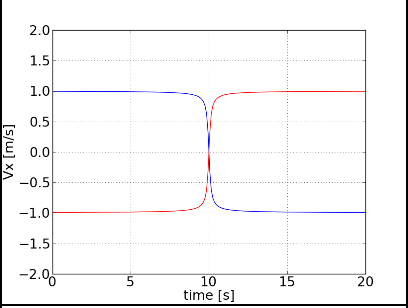
$$m_1 v_{1x,i} + m_2 v_{2x,i} = m_1 v_{1x,f} + m_2 v_{2x,f}$$

$$\frac{\Delta v_{1x}}{\Delta v_{2x}} = -\frac{m_2}{m_1}$$

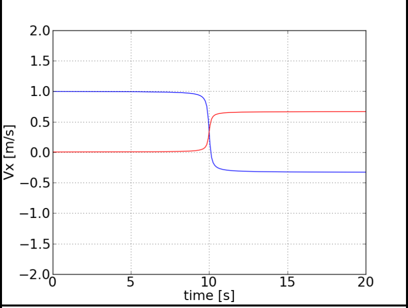
Let's watch the collision between a cart of mass m and a cart of mass $3m$ that you considered after last Tuesday night's reading.

Then let's watch the case where $m_1 = m_2$.

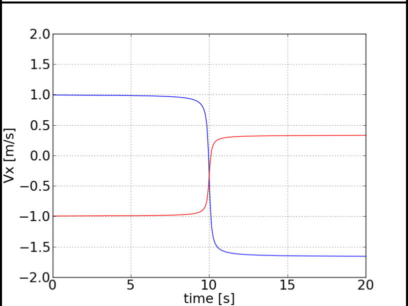
What are the expected shapes of $v_{1,x}(t)$ [blue] and $v_{2,x}(t)$ [red] when $m_2 = m_1$, and initially cart 1 is moving to the right and cart 2 is stationary?



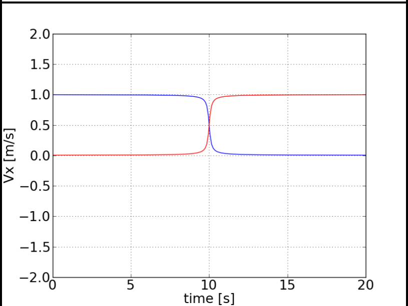
A



B

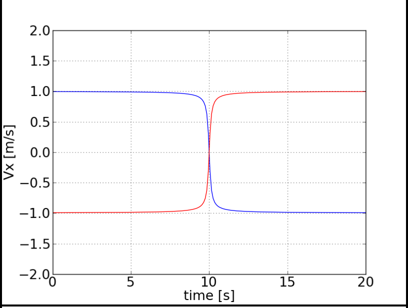


C

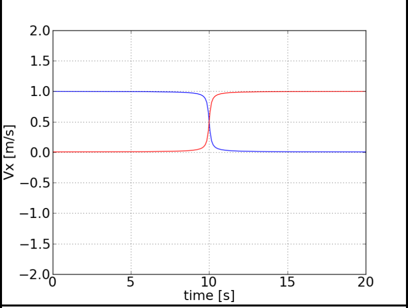


D

What are the expected shapes of $v_{1,x}(t)$ [blue] and $v_{2,x}(t)$ [red] when $m_2 = m_1$, and initially cart 1 is moving to the right and cart 2 is moving to the left?



A



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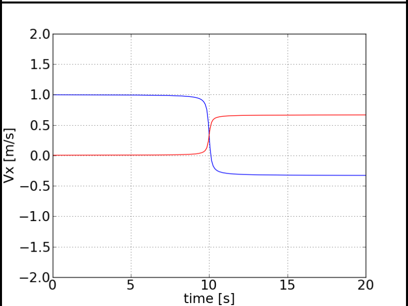
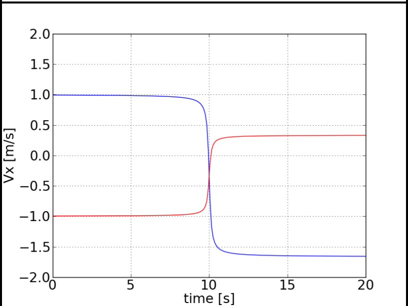
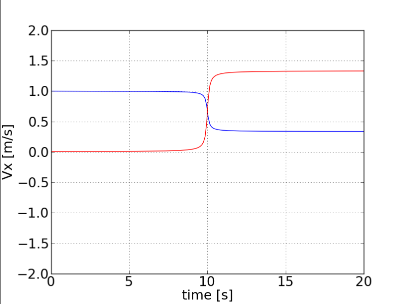


C

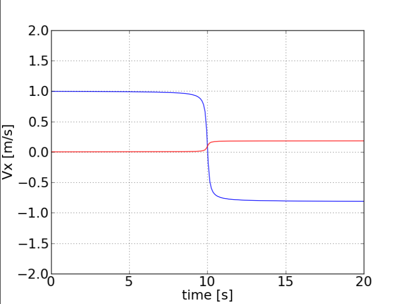


D

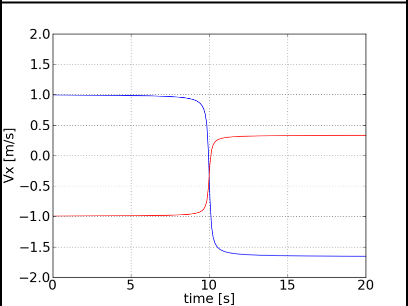
What are the expected shapes of $v_{1,x}(t)$ [blue] and $v_{2,x}(t)$ [red] when $m_2 = 2m_1$, and initially cart 1 is moving to the right and cart 2 is stationary?



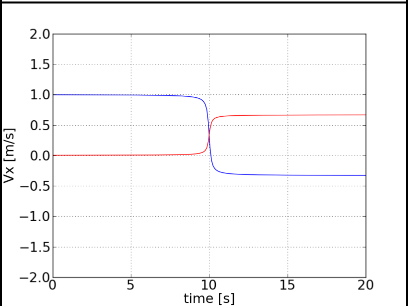
A



B



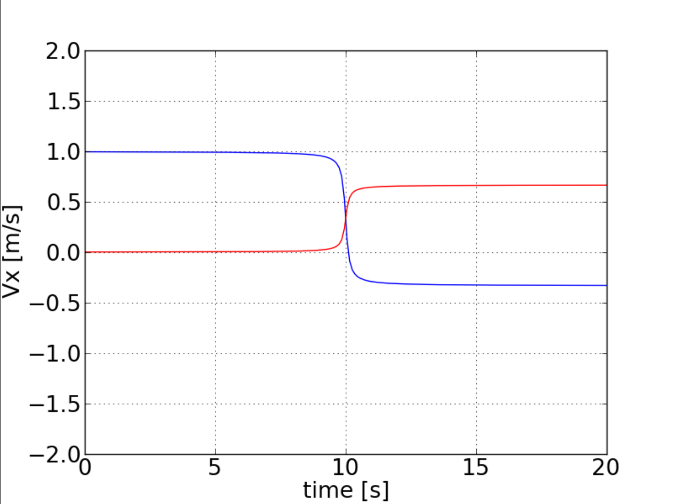
C



D

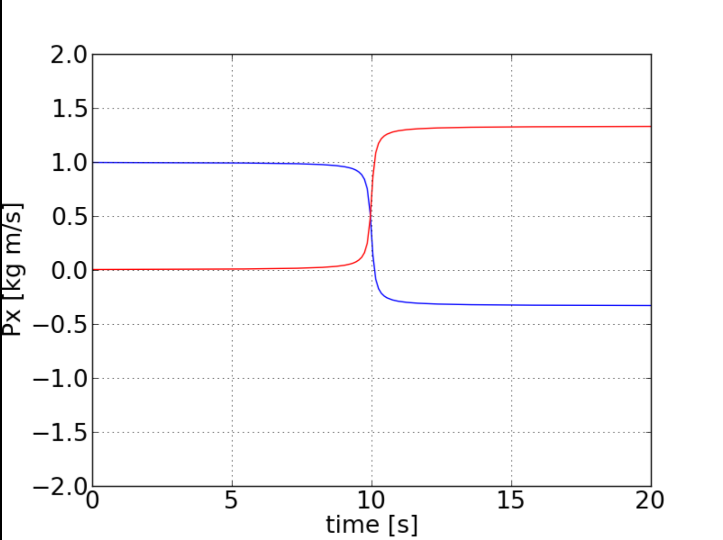
By the way: how would this graph look if we were to graph **momentum** instead of velocity for each cart? (This graph shows velocities. Graph on next page will show momenta.)

(…when $m_2 = 2m_1$, and initially cart 1 is moving to the right and cart 2 is stationary?)

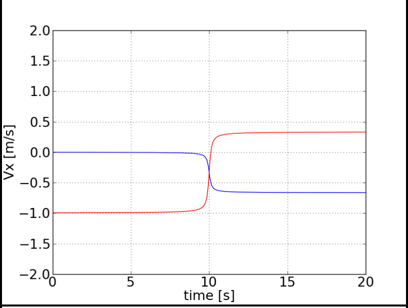


Let's look at momentum p_x instead of velocity v_x:

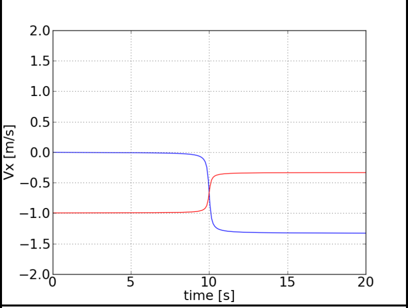
(…when $m_2 = 2m_1$, and initially cart 1 is moving to the right and cart 2 is stationary?)



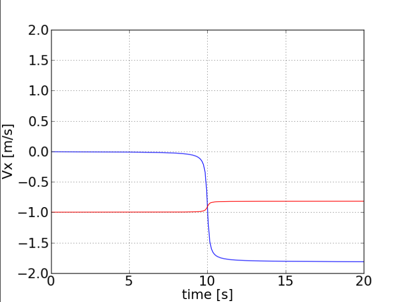
What are the expected shapes of $v_{1,x}(t)$ [blue] and $v_{2,x}(t)$ [red] when $m_2 = 2m_1$, and initially cart 1 is stationary and cart 2 is moving to the left?



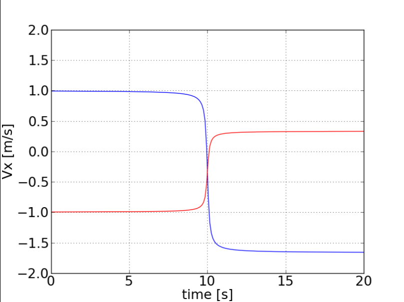
A



B

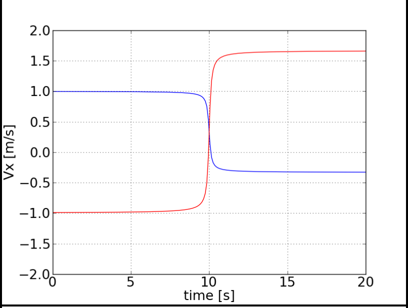


C

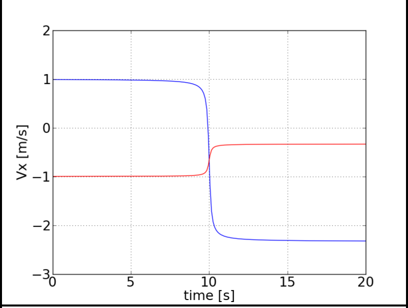


D

What are the expected shapes of $v_{1,x}(t)$ [blue] and $v_{2,x}(t)$ [red] when $m_2 = 2m_1$, and initially cart 1 is moving to the right and cart 2 is moving to the left?



A



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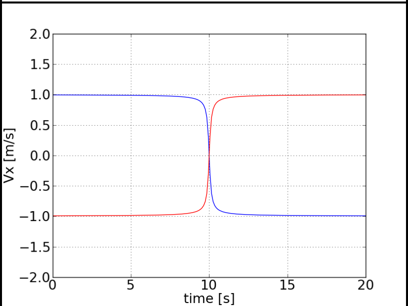

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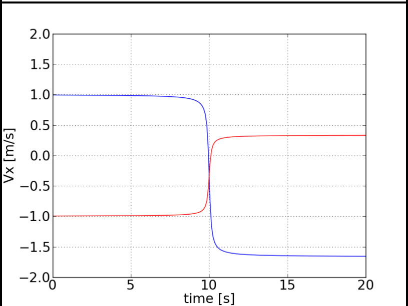


D

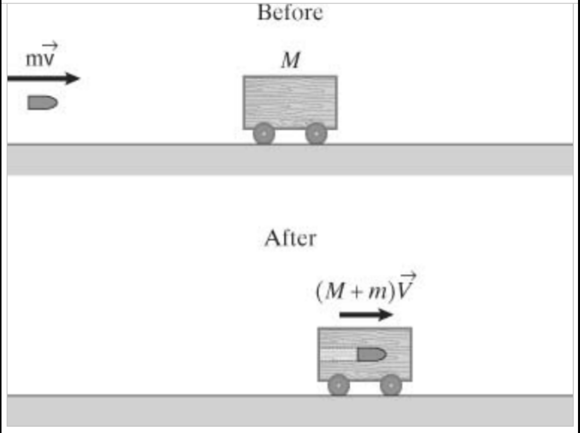
Which has more momentum —
a 0.50 kg baseball pitched at 40 m/s or
a 0.010 kg bullet fired at 400 m/s?

(A) Magnitude of baseball's momentum is larger.

(B) Magnitude of bullet's momentum is larger.

(C) The two momenta are equal in magnitude.

The speed of a bullet can be measured by firing it at a wooden cart initially at rest and measuring the speed of the cart with the bullet embedded in it. The figure shows a 0.0100 kg bullet fired at a 4.00 kg cart. After the collision, the cart rolls at 2.00 m/s. What is the bullet's speed before it strikes the cart? (Once you write down the right expression, the math works out pretty easily without a calculator.)



(A) 4.00 m/s
(B) 798 m/s
(C) 800 m/s
(D) 802 m/s

An old exam problem started like this . . .

You have been hired to check the technical correctness of an upcoming made-for-TV murder mystery. The mystery takes place in the space shuttle. In one scene, an astronaut's safety line is sabotaged while she is on a space walk, so she is no longer connected to the space shuttle. She checks and finds that her thruster pack has also been damaged and no longer works. She is 200 meters from the shuttle and moving with it. That is, she is not moving with respect to the shuttle. There she is — drifting in space — with only 4 minutes of air remaining. To get back to the shuttle, she decides to unstrap her 10 kg tool kit and . . .

What do you think the rest of the problem says she does with her 10 kg tool kit?

(Segue: low-tech carts rolling on track.)

Physics 8 — Wednesday, September 11, 2019

- Course www: `http://positron.hep.upenn.edu/physics8`
- Next week, you will read Ch7 (interactions) [for Monday] and Ch8 (force) [for Wednesday]. Ch6 and Ch7 take some time to read, but they don't add many new equations ("quantitative results") to learn how to work with. Ch5 (energy) and Ch8 (force) will get more class time than Ch6 and Ch7. So there is really one **substantive** chapter this week and one next week.
- We are going pretty quickly through the early chapters of the textbook. We will slow down for the more difficult topics in Ch10,11,12. The faster pace now lets us make time for the fun applications to structures later. You'll be glad we did.

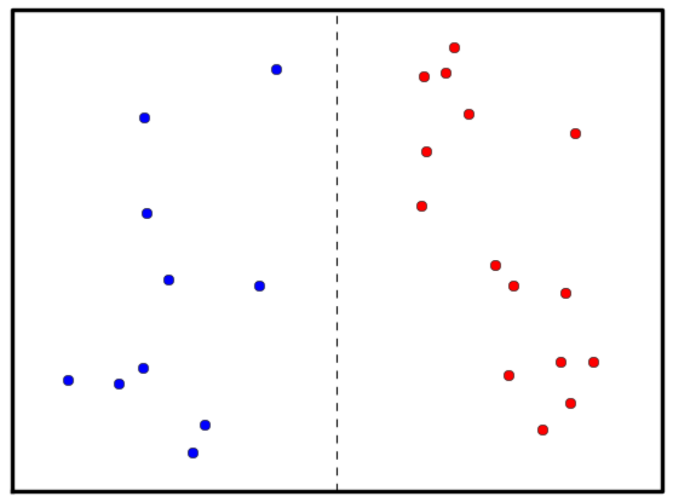
Q about chapter 4: "extensive" quantities

- A quantity Q describing a system is **extensive** if when you divide up the system into two parts,

$$Q(\mathrm{part1}) + Q(\mathrm{part2}) = Q(\mathrm{combined})$$

- Typical examples are volume, money, mass, number of atoms
- Some counterexamples (*not* extensive) are humidity, density, color, temperature.
- Some (just a few) extensive quantities are **conserved**, meaning they can be transferred but can never be created or destroyed. **Momentum** and **energy** are examples of conserved quantities in physics.
- All conserved quantities are extensive, but only a few extensive quantities are conserved.

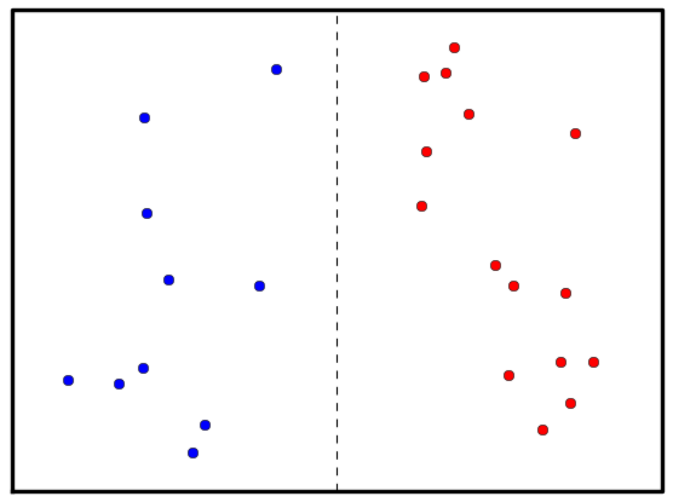
Is **number of dots** an extensive quantity?



(A) Yes.

(B) No.

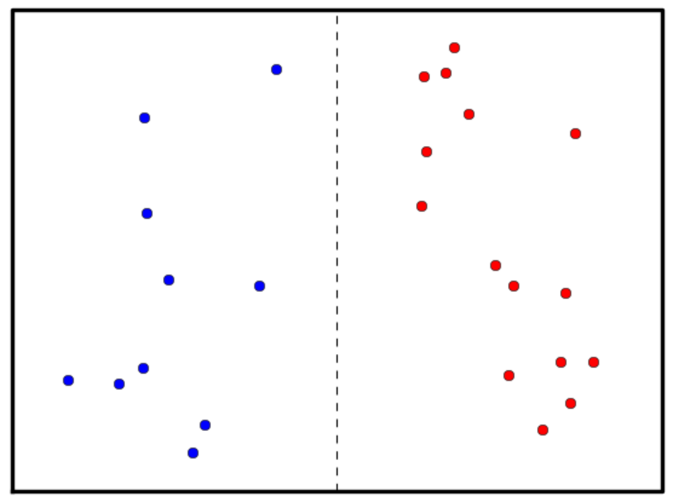
Is **dot diameter** an extensive quantity?



(A) No.

(B) Yes.

Is **total area covered by dots** an extensive quantity?



(A) No.

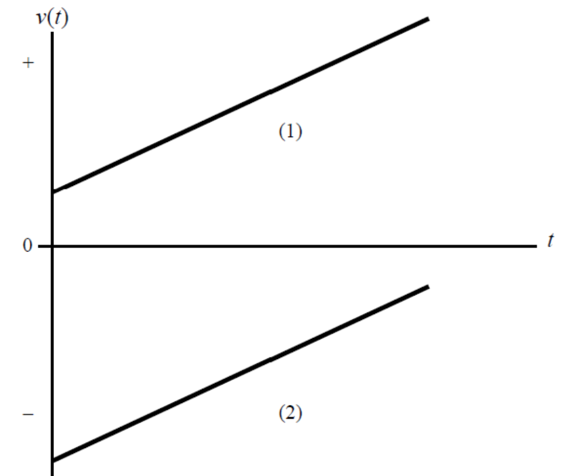
(B) Yes.

(C) Yes, as long as the dots can't overlap.

This may help you with HW2 #11(d)

(The issue is what non-negligible friction would look like on a velocity-vs-time graph.)

The velocity-vs-time graph below shows the motion of two different objects moving across a horizontal surface. Could the change in velocity with time be attributed to friction in each case?



(a) Yes for the top curve, no for the bottom curve.
(b) No for the top curve, yes for the bottom curve.
(c) Yes for both curves.
(d) No for both curves.
(e) I have no idea how friction would affect a velocity-vs-time graph!

Here once again are the key results from Chapter 4 (momentum):

Momentum $\boxed{\vec{p} = m\vec{v}}$. Constant for *isolated* system: no external pushes or pulls (next week we'll say "forces"). Conservation of momentum in isolated two-body collision implies

$$m_1 v_{1x,i} + m_2 v_{2x,i} = m_1 v_{1x,f} + m_2 v_{2x,f}$$

which then implies (for isolated system, two-body collision)

$$\frac{\Delta v_{1x}}{\Delta v_{2x}} = -\frac{m_2}{m_1}$$

If system is not isolated, then we *cannot* write $\vec{p}_f - \vec{p}_i = 0$. Instead, we give the momentum imbalance caused by the external influence a name ("impulse") and a symbol ($\vec{J}$). Then we can write $\vec{p}_f - \vec{p}_i = \vec{J}$. You will rarely use $\vec{J}$, other than to consider whether or not it is nonzero.

Do you remember the key results from Ch 3 (acceleration)?

video segment break

- begin video preceding ws05
- This time, first **read** Mazur chapter 5, then come back to watch this video. Try do do the checkpoints, but you can gloss over most of the equations.

Checkpoint 5.13 typo (in PDF: printed book is good)

5.13 Yes; cart 1 gets twice as much energy as cart 2: $K_{1f} = \frac{1}{2} m_1 v_{1f}^2 = \frac{1}{2}(0.25 \text{ kg})(2.0 \text{ m/s})^2 = 0.50 \text{ J}$, $K_{2f} = \frac{1}{2} m_2 v_{2f}^2 = \frac{1}{2}(0.50 \text{ kg})(1.0 \text{ m/s})^2 = 0.25 \text{ J}$. The reason is that the system's final momentum needs to be zero, and so v_{1f} must be $2v_{2f}$. Because $m_2 = 2m_1$, you have $K_{2f} = \frac{1}{2} m_2 v_{2f}^2 = \frac{1}{2}(2m_1)(\frac{1}{2}v_{1f})^2 = \frac{1}{4} m_1 v_{1f}^2 = \frac{1}{2} K_{1f}$.

He means "no" here.

Most of this answer is fine, but when he writes, "Yes" at the beginning, he really means to write, "No." (Even Harvard professors make mistakes once in a while!)

Chapter 5: Energy

What is the expression for the kinetic energy of an object of mass m that is moving at speed v ?

(Assume the object is not rotating — we'll deal with that later.)

Kinetic energy

$$K = \frac{1}{2}mv^2$$

- is the energy of *motion*.
- is unchanged (in total) in an *elastic* collision.

e.g.

$$\frac{1}{2}m_1 v_{1i}^2 + \frac{1}{2}m_2 v_{2i}^2 = \frac{1}{2}m_1 v_{1f}^2 + \frac{1}{2}m_2 v_{2f}^2$$

but it's much easier in practice to write (equivalently)

$$|v_{12,i}| = |v_{12,f}|$$

i.e. relative *speed* is the same before and after an elastic collision

$$(v_{1x,f} - v_{2x,f}) = -(v_{1x,i} - v_{2x,i}) \qquad \text{[Eqn. 5.4]}$$

What are 4 types of collision? What distinguishes them?

Types of collisions

- Elastic collision: objects recoil with same *relative speed* as before they collided. Kinetic energy $K_i = K_f$.

$$(v_{1x,f} - v_{2x,f}) = -(v_{1x,i} - v_{2x,i}) \qquad \text{[Eqn. 5.4]}$$

- Totally inelastic collision: objects stick together.

$$(v_{1x,f} - v_{2x,f}) = 0$$

- Inelastic collision: objects recoil, but with a reduction in relative speed

$$(v_{1x,f} - v_{2x,f}) = -e(v_{1x,i} - v_{2x,i}) \qquad \text{with } 0 < e < 1$$

- Explosive separation: imagine T.I.C. movie played in reverse.

$$(v_{1x,i} - v_{2x,i}) = 0$$

$$(v_{1x,f} - v_{2x,f}) \neq 0$$

Q (tricky): what value of e describes an explosive separation?!

If I play in reverse a movie of an elastic collision, what sort of collision would I appear to see?

(a) elastic
(b) inelastic
(c) totally inelastic
(d) explosive separation
(e) it depends!

When we collide (on a low-friction track) two carts whose masses and initial velocities are known, conservation of momentum allows us to write

$$m_1 v_{1x,i} + m_2 v_{2x,i} = m_1 v_{1x,f} + m_2 v_{2x,f}$$

We have one equation, but two unknowns. Knowing something about energy gives us a second equation. Relative speed = key.

- elastic: $(v_{1x,f} - v_{2x,f}) = -(v_{1x,i} - v_{2x,i})$
- totally inelastic: $(v_{1x,f} - v_{2x,f}) = 0$
- if e is given: $(v_{1x,f} - v_{2x,f}) = -e(v_{1x,i} - v_{2x,i})$
- if change in internal energy is given:

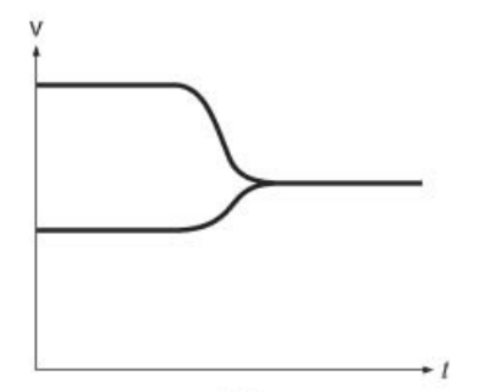
$$\frac{1}{2}m_1 v_{1i}^2 + \frac{1}{2}m_2 v_{2i}^2 = \frac{1}{2}m_1 v_{1f}^2 + \frac{1}{2}m_2 v_{2f}^2 + \Delta E_{\text{internal}}$$

(or equivalently)

$$K_{1i} + K_{2i} + E_{i,\text{internal}} = K_{1f} + K_{2f} + E_{f,\text{internal}}$$

(We'll work some HW-like examples on Friday or Monday.)

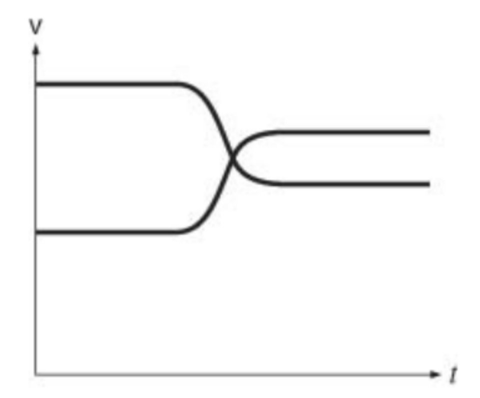
What sort of collision is illustrated by this velocity-vs-time graph?



(A) elastic
(B) inelastic
(C) totally inelastic
(D) explosive separation
(E) can't tell from given information

(By the way, can you infer the ratio of masses?)

What sort of collision is illustrated by this velocity-vs-time graph?



(A) elastic
(B) inelastic
(C) totally inelastic
(D) explosive separation
(E) can't tell from given information

(By the way, can you infer the ratio of masses?)

Suppose you find an isolated system in which two objects about to collide have equal and opposite momenta. If the collision is totally inelastic, what can you say about the motion after the collision?

(Discuss with your neighbor, and then I'll call on a few people to see what you think. If some of us disagree on the answer, it's not a problem: we will all learn by discussing.)

Imagine making two springy devices, each made up of a dozen or so metal blocks loosely connected by springs, and then colliding the two head-on. Do you expect the collision to be elastic, inelastic, or totally inelastic? (Think about what happens to the kinetic energy.)

(A) elastic

(B) inelastic

(C) totally inelastic

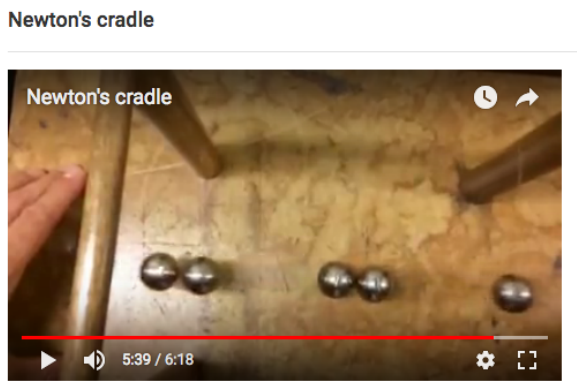
`http://youtu.be/SJIKCmg2Uzg`

Physics 8 — Friday, September 13, 2019

"Newton's cradle:" what do you expect to happen if I pull back two of the spheres and release them?

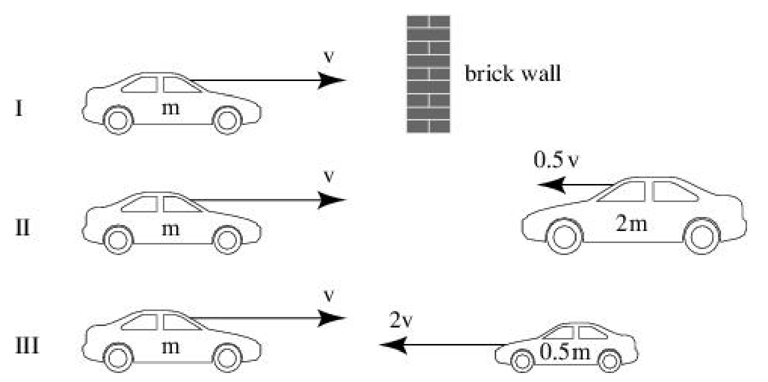
What do you expect to happen if I put a piece of play dough between two of the spheres?

Newton's cradle (slow motion video from my smartphone)



https://youtu.be/rrrs81pl_DU

If all three collisions in the figure shown here are totally inelastic, which bring(s) the car on the left to a halt?



(A) I

(B) II

(C) II,III

(D) all three

(E) III

Which of these systems are isolated?

(1) While slipping on ice, a car collides totally inelastically with another car. System: both cars (ignore friction)
(2) Same situation as in (a). System: slipping car
(3) A single car slips on a patch of ice. System: car
(4) A car brakes to a stop on a road. System: car
(5) A ball drops to Earth. System: ball
(6) A billiard ball collides elastically with another billiard ball on a pool table. System: both balls (ignore friction)

(A) (1) only
(B) (6) only
(C) (1) + (2) + (3) + (4) + (5)+ (6)
(D) (1) + (2) + (3) + (4) + (6)
(E) (1) + (3) + (6)

We've now spent a week watching two carts collide on low-friction tracks. Conservation of momentum lets us write one equation:

$$m_1 v_{1x,i} + m_2 v_{2x,i} = m_1 v_{1x,f} + m_2 v_{2x,f}$$

Often we know both initial velocities, but we don't know either of the two unknown final velocities. So we have two unknowns. Energy adds a second equation, which usually involves **relative speed** $|v_{1x} - v_{2x}|$ of the two carts.

- elastic: $(v_{1x,f} - v_{2x,f}) = -(v_{1x,i} - v_{2x,i})$
- totally inelastic: $(v_{1x,f} - v_{2x,f}) = 0$
- if e is given: $(v_{1x,f} - v_{2x,f}) = -e(v_{1x,i} - v_{2x,i})$
- if change in internal energy is given:

$$K_{1i} + K_{2i} + E_{i,\text{internal}} = K_{1f} + K_{2f} + E_{f,\text{internal}}$$

Let's try using these results.

Write this up with your neighbor(s) and turn it in at the end of class. If you miss class today or if you forget to hand it in on your way out, you can scan & email it to me later if you wish. Remember that in-class work like this is re-scaled so that 80% gets full credit at the end of the term, so missing a couple is OK.

Two carts, of inertias (masses) $m_1 = 1.0\ \mathrm{kg}$ and $m_2 = 1.0\ \mathrm{kg}$, collide head-on on a low-friction track. Before the collision, which is elastic, cart 1 is moving to the right at 1.0 m/s and cart 2 is at rest. What are the two carts' final velocities?

```
In[88]:= ClearAll["Global`*"];
         m1 = 1.0; m2 = 1.0; v1xi = +1.0; v2xi = 0.0;
         Reduce[{
           m1 v1xi + m2 v2xi == m1 v1xf + m2 v2xf,
           (v1xf - v2xf) == -(v1xi - v2xi)
          }]

Out[90]= v2xf == 1. && v1xf == 0.
```

(Keep writing with your neighbor(s).)

Two carts, of inertias $m_1 = 1.0\ \mathrm{kg}$ and $\boxed{m_2 = 9.0\ \mathrm{kg}}$, collide head-on on a low-friction track. Before the collision, which is elastic, cart 1 is moving to the right at 1.0 m/s and cart 2 is at rest. What are the two carts' final velocities?

In[91]:=
```
ClearAll["Global`*"];
m1 = 1.0; m2 = 9.0; v1xi = +1.0; v2xi = 0.0;
Reduce[{
  m1 v1xi + m2 v2xi == m1 v1xf + m2 v2xf,
  (v1xf - v2xf) == -(v1xi - v2xi)
 }]
```

Out[93]=
```
v2xf == 0.2 && v1xf == -0.8
```

Digression: notice what happens if I change the 1:9 ratio of masses into a 1:14 ratio, as in HW2 problem 12 (which you only needed to sketch, not solve with equations).

```
ClearAll["Global`*"];
m1 = 1; m2 = 14; v1xi = +1; v2xi = 0;
Reduce[{
  m1 v1xi + m2 v2xi == m1 v1xf + m2 v2xf,
  (v1xf - v2xf) == -(v1xi - v2xi)
 }]
```

$$v2xf == \frac{2}{15} \,\&\&\, v1xf == -\frac{13}{15}$$

```
N[%]
```

v2xf == 0.133333 && v1xf == -0.866667

(Keep writing with your neighbor(s).)

Two carts, of inertias $m_1 = 1.0\ \mathrm{kg}$ and $m_2 = 9.0\ \mathrm{kg}$, collide head-on on a low-friction track. Before the collision, which is **totally inelastic**, cart 1 is moving to the right at 1.0 m/s and cart 2 is at rest. What are the two carts' final velocities?

```
In[94]:= ClearAll["Global`*"];
         m1 = 1.0; m2 = 9.0; v1xi = +1.0; v2xi = 0.0;
         Reduce[{
            m1 v1xi + m2 v2xi == m1 v1xf + m2 v2xf,
            (v1xf - v2xf) == 0
           }]

Out[96]= v2xf == 0.1 && v1xf == 0.1
```

(Keep writing with your neighbor(s).)

Two carts, of inertias $m_1 = 1.0\ \mathrm{kg}$ and $m_2 = 1.0\ \mathrm{kg}$, collide head-on on a low-friction track. Before the collision, cart 1 is moving to the right at 2.0 m/s and cart 2 is moving to the left at 2.0 m/s. After the collision, cart 1 is moving to the left at 1.0 m/s and cart 2 is moving to the right at 1.0 m/s.

Let "the system" be cart 1 + cart 2. With the given values, is the system's total momentum the same before and after the collision?

What is the coefficient of restitution, e, for this collision?

Initial and final momentum are both zero, as you can verify. The relative speed of the two objects is reduced by a factor $e = 0.5$.

Physics 8 — Monday, September 16, 2019

- You read Ch7 (interactions) for today and you'll read Ch8 (force) for Wednesday. [Then we can finally start using Newton's three laws, as we will for the rest of the semester!]

A system consists of two 1.00 kg carts attached to each other by a compressed spring. Initially, the system is at rest on a low-friction track. When the spring is released, internal energy that was initially stored in the spring is converted into kinetic energy of the carts. The change in the spring's internal energy during the separation is 4.00 joules. What are the two carts' final velocities?

A system consists of two 1.00 kg carts attached to each other by a compressed spring. Initially, the system is at rest on a low-friction track. When the spring is released, internal energy that was initially stored in the spring is converted into kinetic energy of the carts. The change in the spring's internal energy during the separation is 4.00 joules. What are the two carts' final velocities?

```
ClearAll["Global`*"];
m1 = 1.00; m2 = 1.00; v1xi = 0.0; v2xi = 0.0;
Eispring = 4.00; Efspring = 0.00;
Reduce[{
  m1 v1xi + m2 v2xi == m1 v1xf + m2 v2xf,
  1/2 m1 v1xi^2 + 1/2 m2 v1xi^2 + Eispring == 1/2 m1 v1xf^2 + 1/2 m2 v2xf^2 + Efspring,
  v2xf > 0
}]

v2xf == 2. && v1xf == -2.
```

HW3 problem 10

A system consists of a 2.00 kg cart and a 1.00 kg cart attached to each other by a compressed spring. Initially, the system is at rest on a low-friction track. When the spring is released, an explosive separation occurs at the expense of the internal energy of the compressed spring. If the decrease in the spring's internal energy during the separation is 10.0 J, what is the speed of each cart right after the separation?

Since the two-cart system is isolated, what equation can we write down?

Since the spring's internal energy is converted into the carts' kinetic energies, we can account for the initial and final energies of the cart + spring + cart system and can see that this system is closed. (No energy goes in or out of the system.) What second equation can we write down?

(This one can be done without writing down much at all.)

Two carts, of inertias $m_1 = 1.0\ \mathrm{kg}$ and $m_2 = 1.0\ \mathrm{kg}$, collide head-on on a low-friction track. Before the collision, cart 1 is moving to the right at 2.0 m/s and cart 2 is moving to the left at 2.0 m/s. After the collision, cart 1 is moving to the left at 1.0 m/s and cart 2 is moving to the right at 1.0 m/s.

Let "the system" be cart 1 + cart 2. With the given values, is the system's total momentum the same before and after the collision?

What is the coefficient of restitution, e, for this collision?

Initial and final momentum are both zero, as you can verify. The relative speed of the two objects is reduced by a factor $e = 0.5$.

A battery-powered car, with bald tires, sits on a sheet of ice. Friction between the bald tires and the ice is negligible. The driver steps on the accelerator, but the wheels just spin (frictionlessly) on the ice without moving the car. Is the car an isolated system (considering only the coordinate along the car's axis) — i.e. does nothing outside the system push/pull on anything inside the system? Is it a closed system (i.e. negligible energy is transferred in/out of the system)?

(A) Closed but not isolated.

(B) Isolated but not closed.

(C) Both closed and isolated.

(D) Isolated: yes. Closed: very nearly so, yes.

(E) Neither closed nor isolated.

A battery-powered Aston Martin car, with James-Bond-like spiked tires, sits on a sheet of ice. Agent 007 (or maybe it is really Austin Powers?) steps on the pedal, and the car accelerates forward. Is the car an isolated system (considering only the coordinate along the car's axis), i.e. nothing outside the system pushes/pulls on anything inside the system? Is it a closed system (i.e. negligible energy is transferred in/out of the system)?

(A) Closed but not isolated.

(B) Isolated: no. Closed: very nearly so, yes.

(C) Isolated but not closed.

(D) Both closed and isolated.

(E) Neither closed nor isolated.

A battery-powered Aston Martin car, with James-Bond-like spiked tires, sits on a sheet of ice. Agent 007 steps on the accelerator, and the car accelerates forward. All the while, a high-tech solar panel on the car's roof rapidly charges the car's battery. Is the car an isolated system (considering only the coordinate along the car's axis), i.e. nothing outside the system pushes/pulls on anything inside the system? Is it a closed system (i.e. negligible energy is transferred in/out of the system)?

(A) Closed but not isolated.

(B) Isolated but not closed.

(C) Both closed and isolated.

(D) Neither closed nor isolated.

A battery-powered Aston Martin, with James-Bond-like spiked tires, sits atop an iceberg that floats in the North Sea. Agent 007 steps on the accelerator, and the car accelerates forward. (What happens to the iceberg?) All the while, a high-tech solar panel on the car's roof rapidly charges the car's battery. Ignore any friction (or viscosity, drag, etc.) between the water and the iceberg. Which statement is true?

(A) "Car alone" system is isolated but not closed.

(B) "Car + iceberg" system is isolated but not closed.

(C) "Iceberg alone" system is isolated but not closed.

(D) "Car alone" system is isolated and closed.

(E) "Car + iceberg" system is isolated and closed.

(F) "Iceberg alone" system is isolated and closed.

(G) None of the above.

- An **isolated** system has no mechanism for momentum to get in/out of the system from/to outside of the system. This means nothing outside of the system can push/pull on anything inside of the system. (Later this week, we'll say: "no external forces act on the system.")
- This will make more sense when we discuss *forces*, next time!
- A hugely important idea in physics is that if the parts of a system interact only with each other (do not push/pull on anything outside of the system), then the total momentum of that system does not change.
- A **closed** system has no mechanism for energy to get in/out of the system. Examples so far are contrived, but soon we will learn to calculate energy stored in springs, energy stored in Earth's gravitational field, etc. The concept of a closed system is much more useful once we learn how to account for the many ways energy can be stored.
- Accounting for movement of energy in/out of a system will make more sense when we discuss *work*, just after forces.

I put two carts on a low-friction track, with a compressed spring between them. I release the spring by remote control, which sets the carts moving apart. What system is isolated?

(A) One cart.

(B) Cart + spring + other cart.

(C) One cart plus the spring.

(D) None of the above.

I put two carts on a low-friction track, with a compressed spring between them. I release the spring by remote control, which sets the carts moving apart. Is the cart + spring + other cart system closed?

(A) Yes, for all practical purposes, because the system's total energy $K_1 + K_2 + E_{\mathrm{spring}}$ is the same before and after releasing the spring, and other tiny transfers of energy (escaping sound, etc.) are negligible by comparison.

(B) No.

I put two carts on a low-friction track, with a compressed spring between them. I release the spring by remote control, which sets the carts moving apart. Is the spring alone a closed system?

(A) Yes.

(B) No, because it transferred energy to the carts, which are outside of what you're now calling "the system."

I put two carts on a low-friction track, with a lighted firecracker between them. The firecracker explodes, which sets the carts moving apart. Is the cart + firecracker + other cart system closed?

(A) Yes, by analogy with the cart + spring + cart system.

(B) Yes, for some other reason.

(C) No, because realistically, some of the firecracker's energy will escape in the form of heat, flying debris, etc. So really energy conservation only provides an upper limit on $K_1 + K_2$ after the explosion, because accounting for where the energy goes is more difficult here than for a simple spring.

(D) No, for some other reason.

(E) I still don't understand what "closed" means.

A variation on HW2 #8

A rock dropped from the top of a building travels 21.5 m in the last second before it hits the ground. Assume that air resistance is negligible. (Homework asked, "How tall is the building?") Which of the following statements is true? (Let x-axis point upward.)

(A) The rock's average velocity $v_{x,\mathrm{av}}$ during the last 1.0 s of its fall is $-21.5\ \mathrm{m/s}$.

(B) The rock's instantaneous velocity v_x one second before it hits the ground is $-21.5\ \mathrm{m/s}$.

(C) The rock's instantaneous velocity v_x at the instant just before it hits the ground is $-21.5\ \mathrm{m/s}$.

(D) Statements (A), (B), (C) are all true.

(E) Statements (A), (B), (C) are all false.

A rock dropped from the top of a building travels 21.5 m in the last second before it hits the ground. Assume that air resistance is negligible. (Homework asked, "How tall is the building?") At the instant just before hitting the ground, the rock's **speed** is

(A) 21.5 $\mathrm{m/s}$

(B) $-$21.5 $\mathrm{m/s}$

(C) Somewhat faster than 21.5 $\mathrm{m/s}$

(D) Somewhat slower than 21.5 $\mathrm{m/s}$

(E) We don't have enough information to decide.

A rock dropped from the top of a building travels 21.5 m in the last second before it hits the ground. Assume that air resistance is negligible. (Homework asked, "How tall is the building?") Let the building height be h. Let the total time the rock falls be t. Which is a true statement about the problem?

(A) $h - \frac{1}{2}gt^2 = 0$

(B) $0 - gt = -21.5\ \mathrm{m/s}$

(C) $h - \frac{1}{2}g[t - 1.0\ \mathrm{s}]^2 = 21.5\ \mathrm{m}$

(D) $0 - g[t - 1.0\ \mathrm{s}] = -21.5\ \mathrm{m/s}$

(E) (A) and (B) are both true.

(F) (A) and (C) are both true.

(G) (A), (B), (C), and (D) are all true.

(H) (A), (B), (C), and (D) are all false.

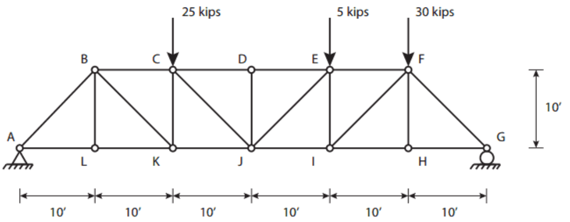
video segment break

- begin video preceding ws06
- There are very few results from chapter 6 that you will need to remember, so this time watch the video lecture first, then **skim** Mazur chapter 6. Having first watched the video will, I hope, help to you know where to focus your attention as you skim through the chapter.
- The main reason for chapter 6 is to build some intuition that will help us, once we reach chapter 8, to better understand forces.
- Your patience will soon be rewarded: once we reach chapter 8, we'll be using Newton's three laws. We'll then continue to use them throughout the rest of the course.

- Ch6 and Ch7 take some time to read, but they don't add many new equations ("quantitative results") to learn how to work with. Ch5 (energy) and Ch8 (force) get more class time than Ch6 and Ch7. So there is really one **substantive** chapter this week and one next week.
- We are going quickly through the early chapters & will slow down for the more difficult topics in Ch10,11,12. The faster pace now lets us make time for "structures" applications later.

Where are we (going)?

3) Find the forces in members BK and CJ. State whether they are in tension or compression.



To analyze structures, you need a thorough understanding of **forces**, **torques** (a.k.a. “moments” of forces), and **vectors**.

Where are we (going)?

(Richard Wesley: "A course should tell a story.")

- To analyze structures, you need a thorough understanding of **forces**, **torques** (a.k.a. "moments" of forces), and **vectors**.
- To understand forces well, you need a solid grasp of
  - How forces affect motion.
  - How different forces relate to one another.
  - How objects interact with one another via forces.
- In ch2–3, we studied the key concepts of motion: position, velocity, acceleration, etc.
- In ch4–5, we studied two key conservation laws (momentum, energy) and some of the restrictions they place on how colliding objects can interact with one another.
- Finally in ch8 we'll discuss forces! We're preparing your mind for forces in ch6–7 by learning a few more of the restrictions imposed, as a consequence of momentum & energy conservation, on how objects can interact with one another.

If "inertial reference frames" baffled you:

- Imagine yourself trying to pour a cup of coffee while standing up on an airplane that is cruising smoothly at constant velocity. No problem.
- Now imagine trying to pour coffee while the airplane is taking off, landing, turning sharply, or experiencing turbulence. Your eye and hand are working from the perspective of a non-inertial reference frame — a set of coordinate axes that is accelerating w.r.t. "the fixed stars." The usual rules of physics don't work. To use the usual rules of physics, you have to analyze the situation from the perspective of an inertial frame.
- If you want more detail on frames of reference, watch this 30-minute educational video from 1960. Email me a few sentences detailing what you learned for extra credit.

Physics: Frames of Reference 1960 PSSC Physical Science Study Committee; Reference & Relativity

https://youtu.be/bJMYoj4hHqU

Chapter 6 included a few key ideas, some of which were obscured by the notation and equations.

Law of inertia — this is big deal! (a.k.a. Newton's law #1.)

- In an inertial reference frame, an isolated object at rest remains at rest, and an isolated object in motion keeps moving at a constant velocity.
- You can't "feel" the difference between being at rest in Earth's frame vs. being at rest in some other inertial reference frame.
- Imagine that you're sitting on an airplane, pouring a cup of coffee, juggling, or maybe just tossing a single ball into the air and catching it. If the airplane is cruising at constant velocity, is all of this activity feasible?
- What if you try the same thing while the airplane is rapidly screeching to a halt on the runway immediately after landing?
- (Illustrate with "ball popper.")

The **law of inertia** states that in an inertial reference frame, any isolated object that is at rest remains at rest, and any isolated object in motion keeps moving at a constant velocity.

Imagine that you are in a jet airplane that has just landed and is in the midst of screeching to a stop on the runway. (You are wearing your seatbelt!) Is the frame of reference in which the airplane is at rest an inertial frame? Will a marble, initially sitting at rest on the floor of the airplane, as observed from the frame in which the airplane is at rest (i.e. as observed by a passenger, with the window shades down), remain at rest as the airplane screeches to a stop?

(A) Yes (inertial frame) and Yes (marble)

(B) Yes (inertial frame) and No (marble)

(C) No (inertial frame) and Yes (marble)

(D) No (inertial frame) and No (marble)

Suppose I'm a passenger on a train that is speeding toward NYC at 40 m/s (heading "north"). In search of coffee, I walk toward the back of the train at 2 m/s, just as the train whizzes past Princeton Junction. From the perspective of a passenger watching me from the train platform, my velocity is

(A) 2 m/s northward
(B) 2 m/s southward
(C) 42 m/s northward
(D) 40 m/s northward
(E) 38 m/s northward

$$\vec{v}_{\text{Earth,me}} = \vec{v}_{\text{Earth,Train}} + \vec{v}_{\text{Train,me}}$$

"(My velocity w.r.t. Earth) =
(Train's velocity w.r.t. Earth) + (my velocity w.r.t. Train)"

I'm driving east at 50 kph. A little kid looks out the window of a westbound car that is going 40 kph. From the kid's point of view, what is my velocity?

(A) 10 kph east
(B) 40 kph east
(C) 50 kph east
(D) 90 kph east
(E) 10 kph west
(F) 40 kph west
(G) 50 kph west
(H) 90 kph west

I'm driving east at 50 kph. A truck driving east at 60 kph overtakes me. As I look out my window, how fast does the truck appear to be moving?

(a) 10 kph
(b) 50 kph
(c) 60 kph
(d) 110 kph

We stopped here on Monday. We'll finish up this stuff from chapters 6 & 7 and then we'll start talking about forces some time during Wednesday's class.

More chapter 6 key ideas

- Center of mass: basically a weighted-average of positons.

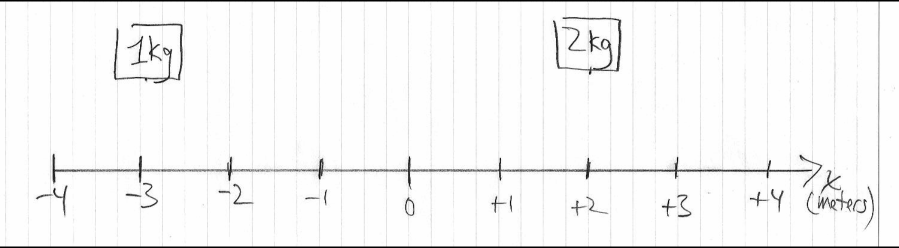
$$\vec{r}_{cm} = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2 + \cdots}{m_1 + m_2 + \cdots}$$

$$x_{cm} = \frac{m_1 x_1 + m_2 x_2 + \cdots}{m_1 + m_2 + \cdots}$$

- CoM of an object lies along axis of symmetry (if there is one).
- When analyzing the motion of a complicated object (composed of many pieces), it is often useful to consider separately the motion of its CoM and the motion of the various internal parts w.r.t. the CoM.
- Illustrate by tossing complicated object in the air.

$$x_{cm} = \frac{m_1 x_1 + m_2 x_2 + \cdots}{m_1 + m_2 + \cdots}$$

At what value of x is the CoM of this pair of masses?



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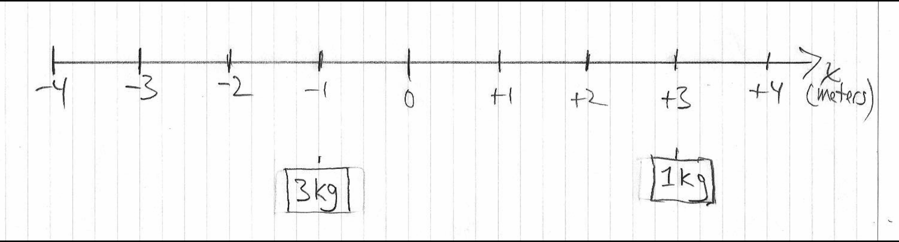

More chapter 6 key ideas

- "Center-of-mass velocity" is the velocity of the CoM of a system of objects:

$$\vec{v}_{cm} = \frac{m_1 \vec{v}_1 + m_2 \vec{v}_2 + \cdots}{m_1 + m_2 + \cdots}$$

$$v_{x,cm} = \frac{m_1 v_{x1} + m_2 v_{x2} + \cdots}{m_1 + m_2 + \cdots}$$

- An isolated system's CoM velocity cannot change!
- You can see this by noticing that the numerator in $\vec{v}_{cm}$ is the system's total momentum, which you know is constant for an isolated system.
- If you observe this system from a camera that is moving at $\vec{v}_{cm}$, the system's CoM will appear to be at rest. This camera's frame-of-reference is called the "ZM frame," because the system's momentum is zero as seen from that frame (i.e. as seen by that moving camera).

A friend and I take our little track and our two little colliding carts, and we set them up (probably in the dining car) on board a moving train (train moving north at constant velocity 30 $\mathrm{m/s}$), with our little track aligned with the train axis.

I push a 1 kg cart north toward my friend at 1 m/s. She pushes a 1 kg cart south toward me at 1 m/s.

As seen by a camera mounted on the ceiling of the train, what is the velocity (north) of the center-of-mass of the two-cart system?

(A) $+30\,\mathrm{m/s}$ (B) $-30\,\mathrm{m/s}$ (C) $+1\,\mathrm{m/s}$ (D) $-1\,\mathrm{m/s}$ (E) $0\,\mathrm{m/s}$

As seen by a camera mounted on the train platform at Princeton Junction (looking in the window as we go by), what is the (northward) velocity of the center-of-mass of the two-cart system?

Is one of these two cameras watching from the "zero-momentum" frame of the two-cart system?

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(B) The center-of-mass velocity (as seen from the ZM frame) is zero.
(C) When observing from **Earth's** frame of reference, a system's center-of-mass velocity will be the same as the velocity (w.r.t. Earth) of the ZM reference frame.
(D) (A) and (B)
(E) (A) and (C)
(F) (B) and (C)
(G) (A), (B), and (C)

- If you find it tedious to do algebra by hand, you could consider learning to use Mathematica, which is free (via site license) for all SAS and Wharton students. I have some excellent self-study Mathematica materials you could go through for extra credit. Email if you're interested.

```
m1 = 1.0; m2 = 9.0; v1xi = +1.0; v2xi = 0.0;
Reduce[{
  m1 v1xi + m2 v2xi == m1 v1xf + m2 v2xf,
  (v1xf - v2xf) == -(v1xi - v2xi)
 }]

Out[3]= v2xf == 0.2 && v1xf == -0.8
```

Physics 8 — Wednesday, September 18, 2019

More chapter 6 key ideas

- Center of mass: basically a weighted-average of positons.

$$\vec{r}_{cm} = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2 + \cdots}{m_1 + m_2 + \cdots}$$

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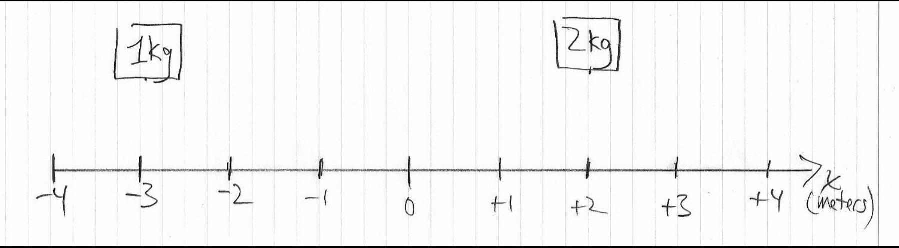

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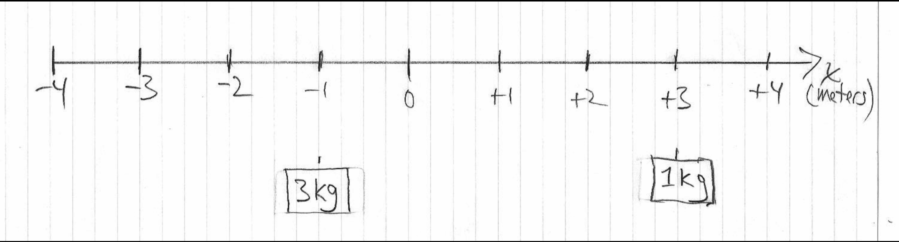

More chapter 6 key ideas

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(D) (A) and (B)
(E) (A) and (C)
(F) (B) and (C)
(G) (A), (B), and (C)

More chapter 6 key ideas

- An isolated system's CoM velocity cannot change!
- A somewhat obscure consequence of this fact is that even in a totally inelastic collision, it is not necessarily possible to convert 100% of the initial kinetic energy into heating up, mangling, etc. the colliding objects.
- Momentum conservation requires that the CoM velocity cannot change, so if the CoM is moving initially, it has to keep moving after the collision.
- Textbook: "convertible" vs. "translational" parts of a system's kinetic energy.
- That idea is worth remembering, **but the math is not.**
- (Can illustrate using colliding carts.)

You really only need these first two equations from Ch6. The third one is in the "obscure" category. Don't worry about it.

(Chapter 6: relative motion)

Center of mass:

$$x_{CM} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3 + \cdots}{m_1 + m_2 + m_3 + \cdots}$$

Center of mass velocity (equals velocity of ZM frame):

$$v_{ZM,x} = \frac{m_1 v_{1x} + m_2 v_{2x} + m_3 v_{3x} + \cdots}{m_1 + m_2 + m_3 + \cdots}$$

Convertible kinetic energy: $K_{\text{conv}} = K - \frac{1}{2} m v_{CM}^2$

Zero-Momentum (ZM) frame for two-object collisions

- Very useful (but difficult to visualize) tool: ZM frame.
- Elastic collision analyzed in ZM ("$*$") frame:

$$v^*_{1i,x} = v_{1i,x} - v_{ZM,x}, \quad v^*_{2i,x} = v_{2i,x} - v_{ZM,x}$$

$$\boxed{v^*_{1f,x} = -v^*_{1i,x}, \quad v^*_{2f,x} = -v^*_{2i,x}}$$

$$v_{1f,x} = v^*_{1f,x} + v_{ZM,x}, \quad v_{2f,x} = v^*_{2f,x} + v_{ZM,x}$$

- Inelastic collision analyzed in ZM frame (restitution coeff. e):

$$\boxed{v^*_{1f,x} = -ev^*_{1i,x}, \quad v^*_{2f,x} = -ev^*_{2i,x}}$$

- Step 1: shift velocities into ZM frame, by subtracting $v_{ZM,x}$
- Step 2: write down (very simple!!) answer in ZM frame
- Step 3: shift velocities back into Earth frame, by adding $v_{ZM,x}$

You can try this on some XC problems. Otherwise, skip it.

There are actually three pretty neat situations that you can analyze quite easily using the "ZM frame" trick:

- When a stationary golf ball is hit by a much more massive golf club, the golf ball's outgoing speed is $2\times$ the incoming speed of the (end of the) club.
- It's easier to hit a home run off of a fastball than a slow pitch.
- When you drop a basketball with a tennis ball resting atop the basketball, the result is quite remarkable.

I was planning to skip these as "obscure," and leave them as topics for extra-credit problems. But if there is overwhelming demand, we could work them out one day in class?

(A) Leave it for XC (B) Do it in class

video segment break

- begin video preceding ws07
- Watch this (pretty short) video first, then **skim** Mazur chapter 7, as chapter 7 contains only two new equations worth knowing.
- Again, chapter 7 is here mainly to prepare your mind, and your intuition, for the force concept. **Force** will be introduced in chapter 8 and then used throughout the rest of the course.

Here are the only two equations worth knowing from Chapter 7.
By contrast, Chapter 8 will have quite a few worth knowing!

(Chapter 7: interactions)

For two objects that form an isolated system (i.e. interacting only with one another), the ratio of accelerations is

$$\frac{a_{1x}}{a_{2x}} = -\frac{m_2}{m_1}$$

When an object near Earth's surface moves a distance Δx in the direction away from Earth's center (i.e. upward), the change in gravitational potential energy of the Earth+object system is

$$\Delta U = mg\Delta x$$

- **Hugely important:** when two objects interact only with one another:

$$\Delta p_{1x} = -\Delta p_{2x}$$

$$\Delta v_{1x}/\Delta v_{2x} = -m_2/m_1$$

$$\boxed{\frac{a_{1x}}{a_{2x}} = -\frac{m_2}{m_1}}$$

- When the medicine ball and I push apart from one another, we both accelerate: in opposite directions, and in inverse proportion to our masses.

- Lifting an object up a height Δx in Earth's gravity changes its gravitational **potential energy** by

$$\Delta U^G = mg\Delta x$$

- I usually remember $U = mgh$ where h is height
- Basketball: back & forth between $\frac{1}{2}mv^2$ and mgh until mechanical energy is dissipated into thermal energy

Problem: I release a 1 kg ball from rest, from an initial height $x_i = +5.0\ \mathrm{m}$ above the ground. (Use $g \approx 10\ \mathrm{m/s^2}$.)

(A) What is the ball's initial G.P.E. ?
(Let's define $x = 0$ to be $U^G = 0$.)

(B) What is the ball's initial K.E. ?

(C) What is the ball's G.P.E. immediately before it reaches the ground?

(D) What is the ball's K.E. immediately before it reaches the ground?

(E) What is the ball's speed immediately before it reaches the ground?

(F) If the ball bounces elastically off of the floor, what height will it reach after bouncing?

(G) If instead the ball bounces off of the floor with a restitution coefficient $e = 0.9$, what height will it reach after bouncing?

Physics 8 — Friday, September 20, 2019

- Wolfram Mathematica is free (site license) for SAS and Wharton students. I have some very helpful self-study Mathematica materials you can do for XC. Email if interested.

Problem: Suppose your friend's mass is about 50 kg, and she climbs up 30 flights of stairs (that's about 100 m) to check out a great rooftop view of the city's architecture.

(A) By how many joules did climbing the stairs change her G.P.E.?

(B) Where did this gravitational potential energy come from? I mean what source energy was converted into this G.P.E.?

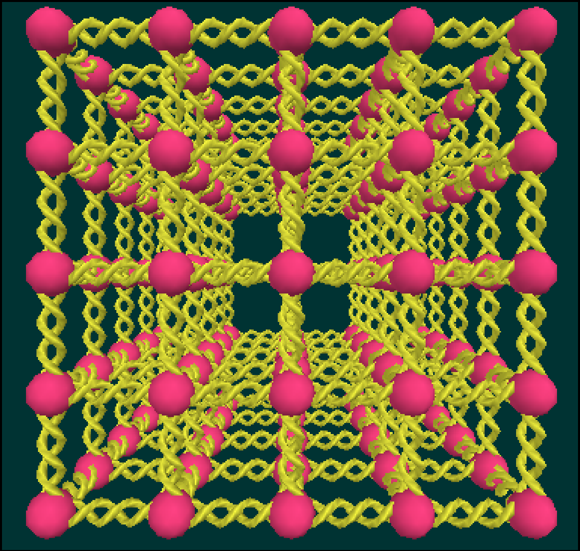
(C) How many food Calories did she burn (assuming, unrealistically, that one's muscles are 100% efficient at converting food into mechanical work)? [Realistically, your metabolism/muscles are very roughly about 20% efficient at turning stored food into mechanical work.]

(Begin digression.)

Dissipative / incoherent / irreversible

A simple ball / spring model of the atoms in a solid.

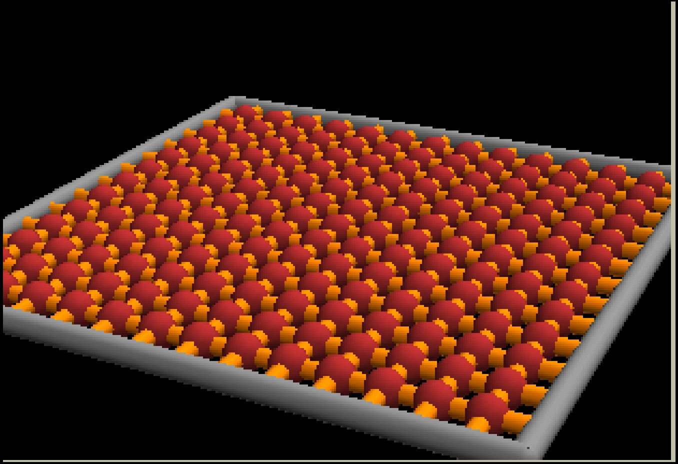
This is sometimes a useful picture to keep in your head.



Dissipative / incoherent / irreversible

2D version for simplicity

illustrate "reversible" and "irreversible" deformation with e.g. marbles and egg crate



Dissipative / incoherent / irreversible

I showed you once before my low-tech animation of two objects in a totally inelastic collision. Collision dissipates coherent motion (kinetic energy) into incoherent vibration of atoms (thermal energy)

`https://youtu.be/SJIKCmg2Uzg`

Here's a high-speed movie of a (mostly) reversible process a golf ball bouncing off of a wall at 150 mph.

https://www.youtube.com/watch?v=AkB81u5IM3I

(End digression.)

video segment break

- begin video preceding ws08
- This time, carefully **read** Mazur chapter 8, then watch this video.

Chapter 8: Force

- Forces **always** come in pairs: when A and B interact,

$$\vec{F}_{\rm A\ on\ B} = -\vec{F}_{\rm B\ on\ A}$$

- "Interaction pairs" have equal magnitude, opposite direction. **Always.** That's called Newton's third law. Difficult idea!

- The acceleration of object A is given by vector sum of all of the forces acting **ON** object A, divided by m_A. (Law #2.)

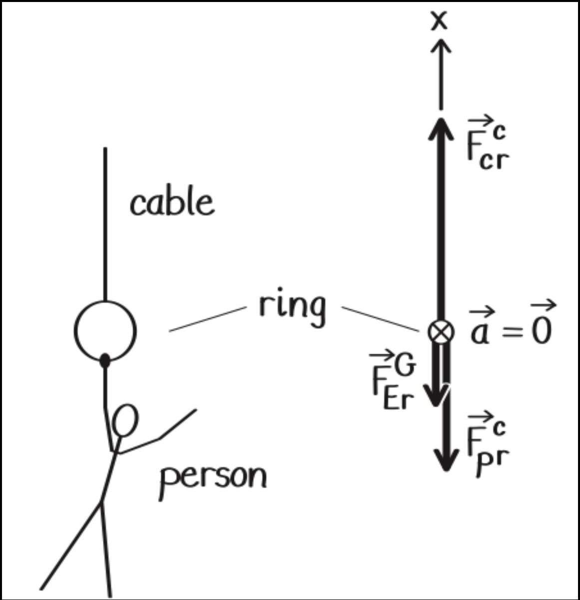
$$\vec{a}_A = \frac{1}{m_A} \sum \vec{F}_{\rm on\ A}$$

- In an inertial frame of reference, object A moves at constant velocity (or stays at rest) if and only if the vector sum $(\sum \vec{F}_{\rm on\ A})$ equals zero. (Law #1.) #1 seems redundant?!

You push with a steady force of 25 N on a 50 kg desk fitted with (ultra-low-friction) casters on its four legs. How long does it take you (starting from rest) to get the desk across a room that is 25 m wide?

(A) 0.71 s
(B) 1.0 s
(C) 1.4 s
(D) 5.0 s
(E) 7.1 s
(F) 10 s
(G) 14 s

Free-body diagram: A sort of visual accounting procedure for adding up the forces acting **ON** a given object. FBD for ring:



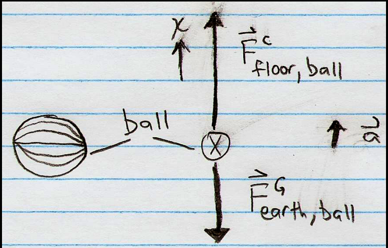
Chapter 8 ("force") reading Q #1

"Think about the familiar example of a basketball dropped from eye level and allowed to bounce a few times. Describe the forces acting on the basketball at its lowest point, as it is in contact with the floor and is changing direction from downward to upward motion."

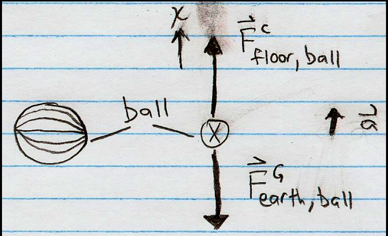
- Working with 1-2 nearby people, draw a free-body diagram of the ball at its lowest point (while it is most squished). Include all forces acting ON the ball. Indicate the direction of each force with its vector arrow. Indicate the relative magnitudes of the forces by the lengths of the arrows. Indicate the direction of the ball's acceleration with an arrow (or a dot).
- When you finish that, draw a second free-body diagram for the ball — this time while the ball is in the air. Will the diagram be different while the ball is rising vs. falling?
- Discuss! I may call on people to describe their diagrams.

(My diagram appears on the next slide.)

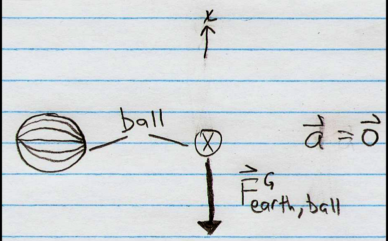
Which free-body diagram best represents the forces acting on the basketball at the *bottom* of its motion?



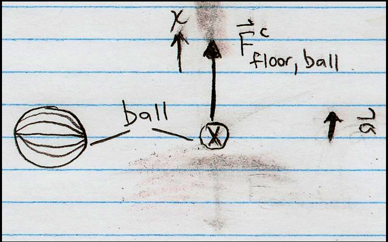
A



B

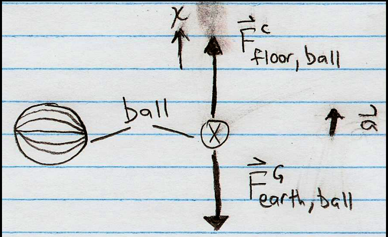


C

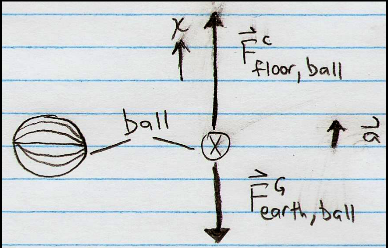


D

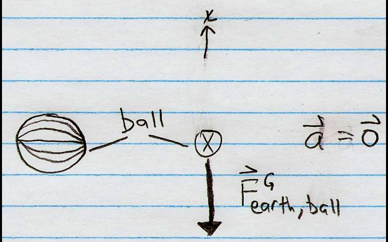
Which free-body diagram best represents the forces acting on the basketball at the *top* of its motion?



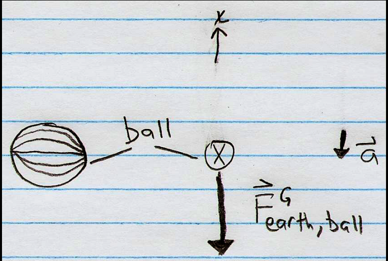
A



B



C



D

If I were to draw a free-body diagram for the basketball when it is halfway back down to the ground, that new diagram would be

(A) the same as

(B) slightly different from

(C) very different from

the drawing for the basketball when it is at the top of its motion? (Neglect air resistance.)

Equal and opposite forces?

Consider a car at rest on a road. We can conclude that the downward gravitational pull of Earth on the car and the upward contact force of the road on the car are equal and opposite because

(A) the two forces form an interaction pair.

(B) the net force on the car is zero.

(C) neither: the two forces are not equal and opposite

(D) both (A) and (B)

We stopped after this — will resume here.

Physics 8 — Monday, September 23, 2019

Chapter 8 ("force") reading Q #2

"Explain briefly in your own words what it means for the interaction between two objects to involve 'equal and opposite' forces. Can you illustrate this with an everyday example?"

- For instance, if I push against some object O that moves, deforms, or collapses in response to my push, is the force exerted by O on me still equal in magnitude and opposite in direction to the force exerted by me on O?
- If every force is paired with an equal and opposite force, why is it ever possible for any object to be accelerated? Don't they all just cancel each other out?
- (I think the next example may help.)

Have you ever spotted the Tropicana juice train?!

vocab: powered “locomotive” pulls the unpowered “cars”

Equal and opposite forces?

An engine ("locomotive") (the first vehicle of the train) pulls a series of train cars. Which is the correct analysis of the situation?

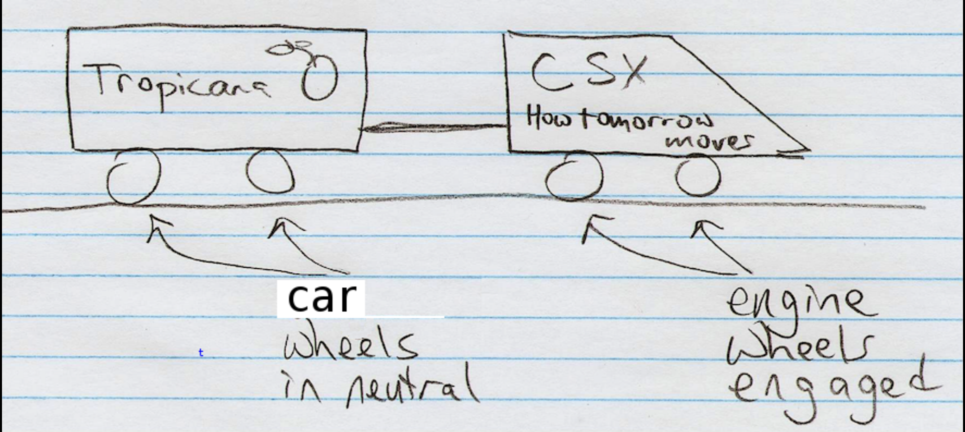
(A) The train moves forward because the locomotive pulls forward slightly harder on the cars than the cars pull backward on the locomotive.

(B) Because action always equals reaction, the locomotive cannot pull the cars — the cars pull backward just as hard as the locomotive pulls forward, so there is no motion.

(C) The locomotive gets the cars to move by giving them a tug during which the force on the cars is momentarily greater than the force exerted by the cars on the locomotive.

(D) The locomotive's force on the cars is as strong as the force of the cars on the locomotive, but the frictional force by the track on the locomotive is forward and large while the backward frictional force by the track on the cars is small.

(E) The locomotive can pull the cars forward only if its inertia (i.e. mass) is larger than that of the cars.



Let's see the effect of including or not including the frictional force of the tracks pushing forward on the wheels of the engine.

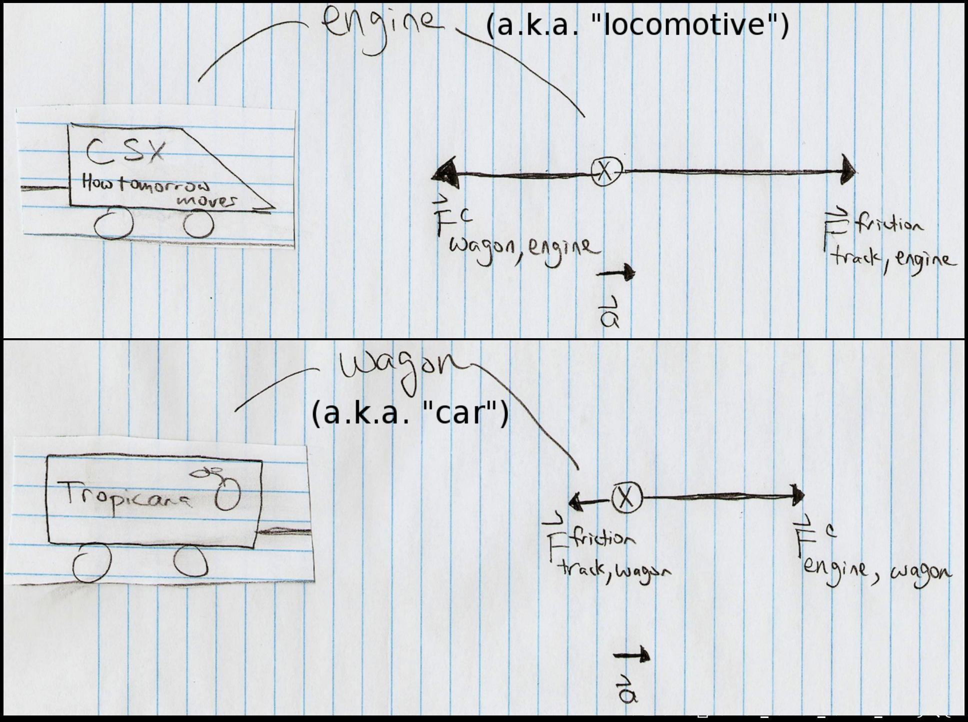
I'll pretend to be the engine!

Only *external* forces can accelerate a system's CoM

Let's define "system" to be locomotive+car. Remember that forces internal to system cannot accelerate system's CoM.

To change the velocity of the CoM, we need a force that is *external* to the system.

(By the way, when you look at the two free-body diagrams on the next page, tell me if you see an "interaction pair" of forces somewhere!)



engine
(a.k.a. "locomotive")
CSX
How tomorrow moves
$\vec{F}^{c}_{wagon, engine}$
$\vec{F}^{friction}_{track, engine}$
$\vec{a}$
wagon
(a.k.a. "car")
Tropicana
$\vec{F}^{friction}_{track, wagon}$
$\vec{F}^{c}_{engine, wagon}$
$\vec{a}$

$$\vec{a}_{\mathrm{CoM}} = \frac{\sum \vec{F}^{\mathrm{external}}}{m_{\mathrm{total}}}$$

It's useful to remember that even if the several pieces of a system are behaving in a complicated way, you can find the acceleration of the CoM of the system by considering only the **external** forces that act **on** the system.

Once again, a careful choice of "system" boundary often makes the analysis much easier. We'll see more examples of this soon. (This topic also arises in HW4 #9 and #10, so we'll try to practice it today or Wednesday.)

Hooke's law

- When you pull on a spring, it stretches
- When you stretch a spring, it pulls back on you
- When you compress a spring, it pushes back on you
- For an ideal spring, the pull is proportional to the stretch
- Force **by** spring, **on** load is

$$F_x = -k\ (x - x_0)$$

- The constant of proportionality is the "spring constant" k, which varies from spring to spring. When we talk later about properties of building materials, we'll see where k comes from.
- The minus sign indicates that if I move my end of the spring to the right of its relaxed position, the force exerted by the the spring on my finger points left.

Let's look at some examples of springs.

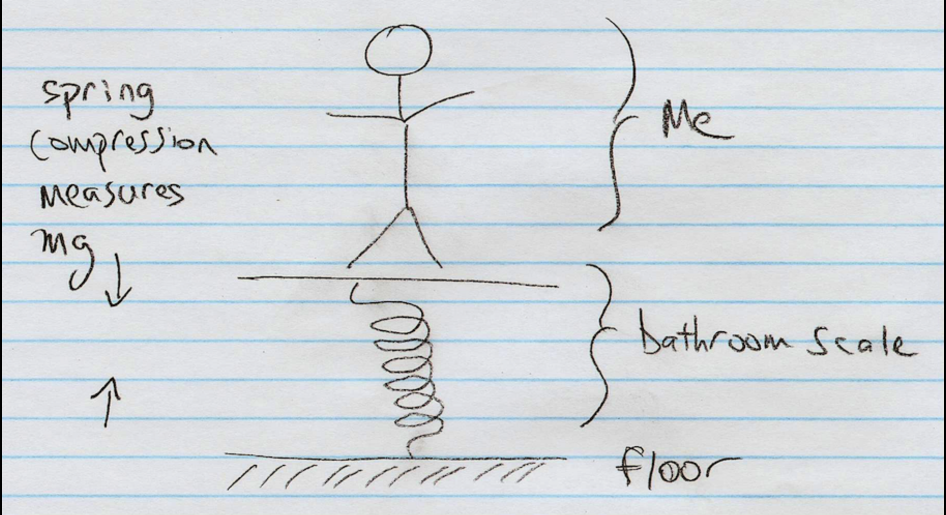
A spring hanging from the ceiling is 1.0 m long when there is no object attached to its free end. When a 4.0 kg brick is attached to the free end, the spring is 2.0 m long. (For easier math, use $g = 10\,\mathrm{m/s^2} = 10\,\mathrm{N/kg}$.) What is the spring constant of the spring?

[Hint: draw a FBD for the brick, to figure out what magnitude force the spring must be exerting on the brick. The magnitude of the force exerted by the spring is the spring constant (k) times how far the spring is stretched w.r.t. its relaxed length.]

(A) 5.0 N/m
(B) 10 N/m
(C) 20 N/m
(D) 30 N/m
(E) 40 N/m

Measuring your weight ($F = mg$) with a spring scale

Most bathroom scales work something like this:



Now suppose I take my bathroom scale on an elevator . . .

Bathroom scale on an accelerating elevator

A bathroom scale typically uses the compression of a spring to infer the gravitational force ($F = mg$) exerted by Earth on you, which we call your *weight*.

Suppose I am standing on such a scale while riding an elevator. With the elevator parked at the bottom floor, the scale reads 700 N. I push the button for the top floor. The door closes. The elevator begins moving upward. At the moment when I can feel (e.g. in my stomach) that the elevator has begun moving upward, the scale reads

(A) a value smaller than 700 N.

(B) the same value: 700 N.

(C) a value larger than 700 N.

You might want to try drawing a free-body diagram for your body, showing the downward force of gravity, the upward force of the scale pushing on your feet, **and your body's acceleration**.

Tension vs. compression

- When a force tries to squish a spring, that is called *compression*, or a compressive force
- When a force tries to elongate a spring, that is called *tension*, or a tensile force
- We'll spend a lot of time next month talking about compression and tension in columns, beams, etc.
- For now, remember that tension is the force trying to pull apart a spring, rope, etc., and compression is the force trying to squeeze a post, a basketball, a mechanical linkage, etc.

Physics 8 — Wednesday, September 25, 2019

- Thanks to a 2019 student, here's a neat video showing that the CoM of a dropped slinky falls at acceleration g, even though the top and bottom of the slinky do not move in unison:
  https://www.youtube.com/watch?v=eCMmmEEyOO0&t=43
  super-sized version (harder to see than original version):
  https://www.youtube.com/watch?v=JsytnJ_pSf8&t=88

Since Monday, we are finally talking about forces

- The **force** concept quantifies interaction between two objects.
- Forces always come in "**interaction pairs**." The force exerted by object "A" on object "B" is equal in magnitude and opposite in direction to the force exerted by B on A:

$$\vec{F}_{AB} = -\vec{F}_{BA}$$

- The acceleration of object "A" is given by the vector sum of the forces acting **on** A, divided by the mass of A:

$$\vec{a}_A = \frac{\sum \vec{F}_{\mathrm{(on\ A)}}}{m_A}$$

- The vector sum of the forces acting **on** an object equals the rate of change of the object's momentum:

$$\sum \vec{F}_{\mathrm{(on\ A)}} = \frac{\mathrm{d}\vec{p}_A}{\mathrm{d}t}$$

- An object whose momentum is not changing is in translational **equilibrium**. We'll see later that this will be a big deal for the members of a structure! To achieve this, we will want all forces acting **on** each member to sum vectorially to zero.
- The unit of force is the **newton**. $1\ \mathrm{N} = 1\ \mathrm{kg}\cdot\mathrm{m/s}^2$.
- **Free-body diagrams** depict all of the forces acting **on** a given object. They are used all the time in analyzing structures!

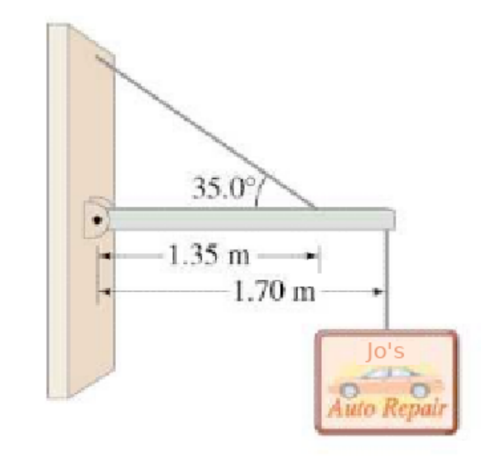
- The force exerted by a compressed or stretched spring is proportional to the displacement of the end of the spring w.r.t. its relaxed value x_0. k is "spring constant."

$$F_x^{\mathrm{spring}} = -k(x - x_0)$$

- When a rope is held taut, it exerts a force called the **tension** on each of its ends. Same magnitude T on each end.

Tension in cables (repeated from Monday)

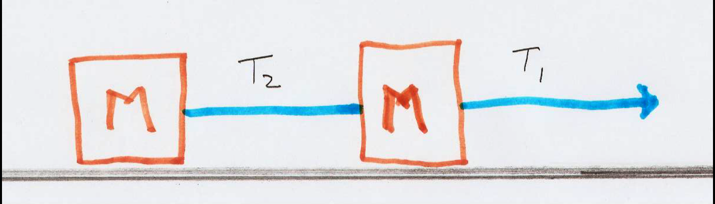
- A large category of physics problems (and even architectural structures, e.g. a suspension bridge) involves two objects connected by a rope, a cable, a chain, etc.
- These things (cables, chains, ropes) can pull but can't push. There are two cables in this figure:



Tension in cables

- Usually the cables in physics problems are considered light enough that you don't worry about their inertia (we pretend $m = 0$), and stiff enough that you don't worry about their stretching when you pull on them (we pretend $k = \infty$).
- The cable's job is just to transmit a force from one end to the other. We call that force the cable's *tension*, T.
- A cable always pulls on both ends with same magnitude (T), though in opposite directions. [Formally: we neglect the cable's mass, and the cable's acceleration must be finite.]
- (We stopped here on Monday.)
- E.g. hang basketball from ceiling. Cable transmits mg to ceiling. Gravity pulls ball down. Tension pulls ball up. Forces on ball add (vectorially) to zero.
- Let's try an example.

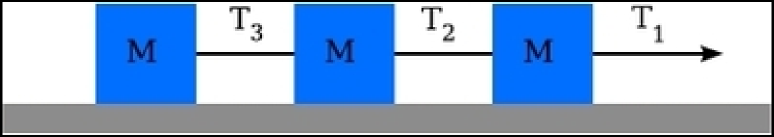
Two blocks of equal mass are pulled to the right by a constant force, which is applied by pulling at the arrow-tip on the right. The blue lines represent two identical sections of rope (which can be considered massless). Both cables are taut, and friction (if any) is the same for both blocks. What is the ratio of T_1 to T_2?



(A) zero: $T_1 = 0$ and $T_2 \neq 0$.
(B) $T_1 = \frac{1}{2} T_2$
(C) $T_1 = T_2$
(D) $T_1 = 2T_2$
(E) infinite: $T_2 = 0$ and $T_1 \neq 0$.

It's worth drawing an FBD first for the two-mass system, then for the left mass, then for the right mass.

Three blocks of equal mass are pulled to the right by a constant force. The blocks are connected by identical sections of rope (which can be considered massless). All cables are taut, and friction (if any) is the same for all blocks. What is the ratio of T_1 to T_2?



(A) $T_1 = \frac{1}{3} T_2$

(B) $T_1 = \frac{2}{3} T_2$

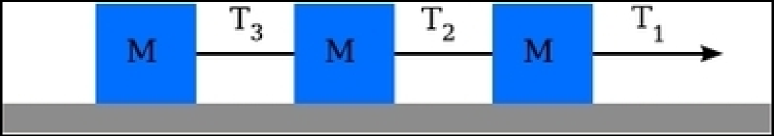
(C) $T_1 = T_2$

(D) $T_1 = \frac{3}{2} T_2$

(E) $T_1 = 2 T_2$

(F) $T_1 = 3 T_2$

Three blocks of equal mass are pulled to the right by a constant force. The blocks are connected by identical sections of rope (which can be considered massless). All cables are taut, and friction (if any) is the same for all blocks. What is the ratio of T_1 to T_3?



(A) $T_1 = \frac{1}{3} T_3$
(B) $T_1 = \frac{2}{3} T_3$
(C) $T_1 = T_3$
(D) $T_1 = \frac{3}{2} T_3$
(E) $T_1 = 2T_3$
(F) $T_1 = 3T_3$

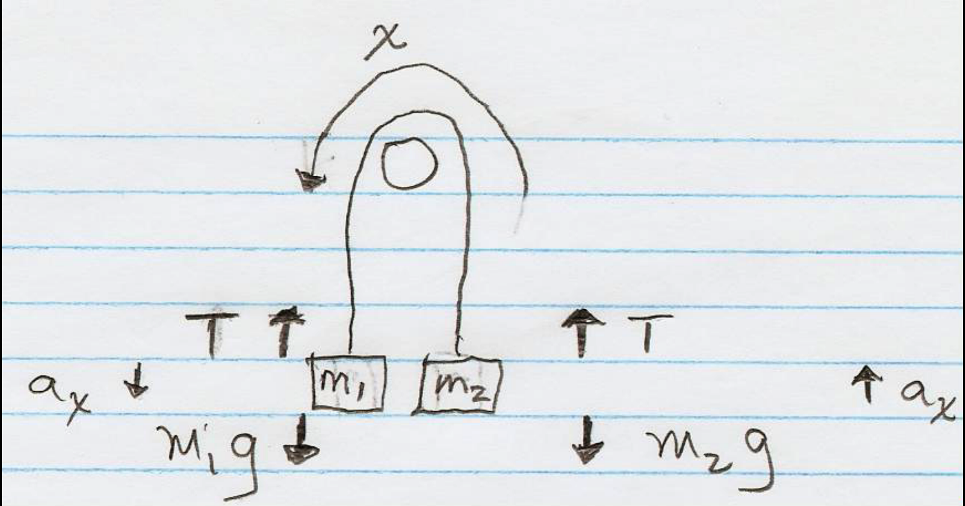
Atwood machine — discuss with your neighbors

A contraption something like this appears in HW4 (but with a spring added, to keep things interesting).

- Why aren't the two masses accelerating?
- What is the tension in the cable when the two masses are equal (both 5.0 kg) and stationary, as they are now?
- If I make one mass equal 5.0 kg and the other mass equal 5.1 kg, what will happen? Can you predict what the acceleration will be?
- If I make one mass equal 5.0 kg and the other mass equal 6.0 kg, will the acceleration be larger or smaller than in the previous case?
- Try drawing a free-body diagram for each of the two masses
- By how much do I change the gravitational potential energy of the machine+Earth system when I raise the 6 kg mass 1 m?

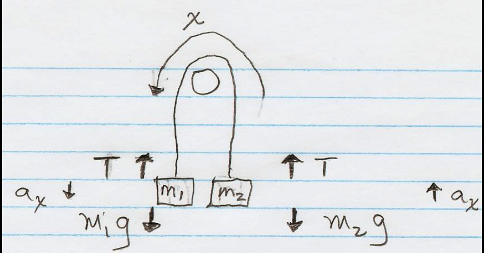
- Two more comments:
- This machine was originally invented as a mechanism for measuring g and for studying motion with constant acceleration.
- The same concept is used by the “counterweight” in an elevator for a building.

Atwood machine: take $m_1 > m_2$



Pause here: how can we solve for a_x? Try it before we go on.

Atwood machine: write masses' equations of motion



$$m_1 g - T = m_1 a_x$$

$$T - m_2 g = m_2 a_x$$

Solve second equation for T; plug T into first equation; solve for a_x:

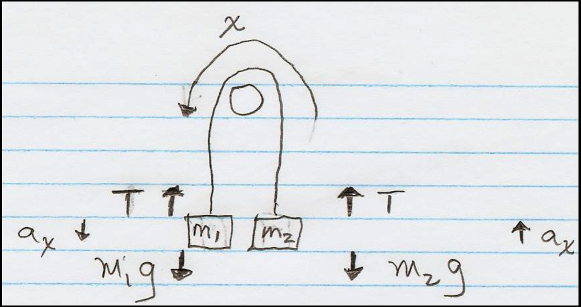
$$T = m_2 a_x + m_2 g \quad \Rightarrow \quad m_1 g - (m_2 a_x + m_2 g) = m_1 a_x \quad \Rightarrow$$

$$(m_1 - m_2) g = (m_1 + m_2) a_x \quad \Rightarrow \quad \boxed{a_x = \frac{m_1 - m_2}{m_1 + m_2}\, g}$$

For $m_2 = 0$, $\quad a_x = g \quad$ (just like picking up m_1 and dropping it)

For $m_1 \approx m_2$, $a_x \ll g$: small difference divided by large sum.

$$a_x = \frac{m_1 - m_2}{m_1 + m_2}\, g$$



For example, $m_1 = 4.03\ \mathrm{kg}$, $m_2 = 3.73\ \mathrm{kg}$:

$$a_x = \frac{m_1 - m_2}{m_1 + m_2}\, g = \left(\frac{0.30\ \mathrm{kg}}{7.76\ \mathrm{kg}}\right)\left(9.8\ \mathrm{m/s^2}\right) = 0.38\ \mathrm{m/s^2}$$

How long does it take m_1 to fall 2 meters?

$$x = \frac{a_x t^2}{2} \quad \Rightarrow \quad t = \sqrt{\frac{2x}{a_x}} = \sqrt{\frac{(2)(2\ \mathrm{m})}{(0.38\ \mathrm{m/s^2})}} \approx 3.2\ \mathrm{s}$$

You can also solve for T if you like (eliminate a_x), to find the tension while the two masses are free to accelerate (no interaction with my hand or the floor).

Start from masses' equations of motion:

$$m_1 g - T = m_1 a_x, \qquad T - m_2 g = m_2 a_x$$

Eliminate a_x:

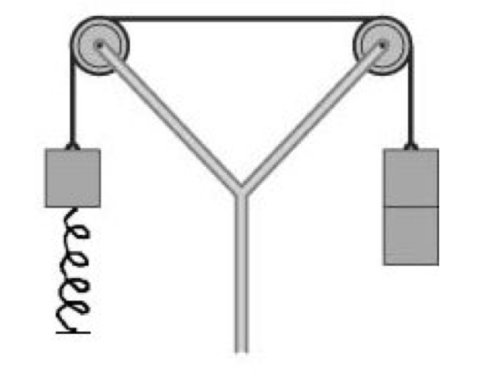
$$\frac{m_1 g - T}{m_1} = \frac{T - m_2 g}{m_2} \quad \Rightarrow \quad m_1 m_2 g - m_2 T = m_1 T - m_1 m_2 g$$

$$\Rightarrow 2 m_1 m_2 g = (m_1 + m_2) T \quad \Rightarrow \quad T = \frac{2 m_1 m_2}{m_1 + m_2} g$$

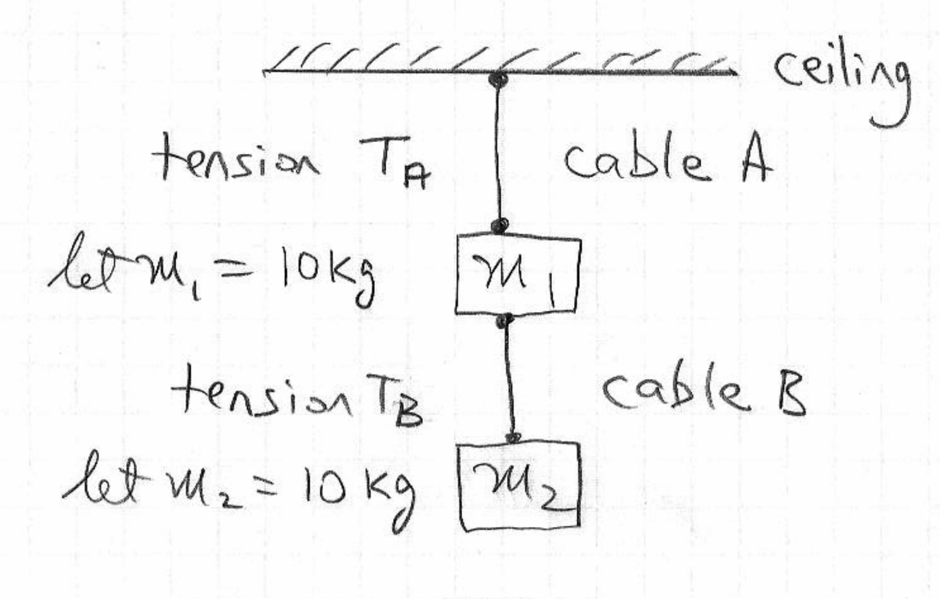
consider extreme cases: $m_2 = m_1$ vs. $m_2 \ll m_1$.

HW4 / problem 7: tricky!

7*. A modified Atwood machine is shown below. Each of the three blocks has the same inertia m. One end of the vertical spring, which has spring constant k, is attached to the single block, and the other end of the spring is fixed to the floor. The positions of the blocks are adjusted until the spring is at its **relaxed** length. The blocks are then released from rest. What is the acceleration of the two blocks on the right after they have fallen a distance D?

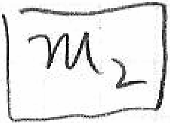


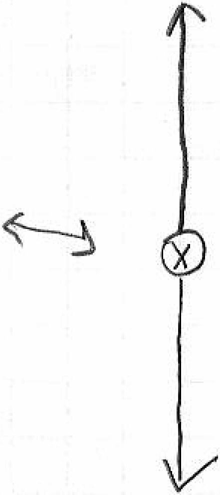
Physics 8 — Friday, September 27, 2019



Work with your neighbor to draw a FBD for mass 2. Then draw a FBD for mass 1. Assume that $\vec{a} = \vec{0}$ for both masses.

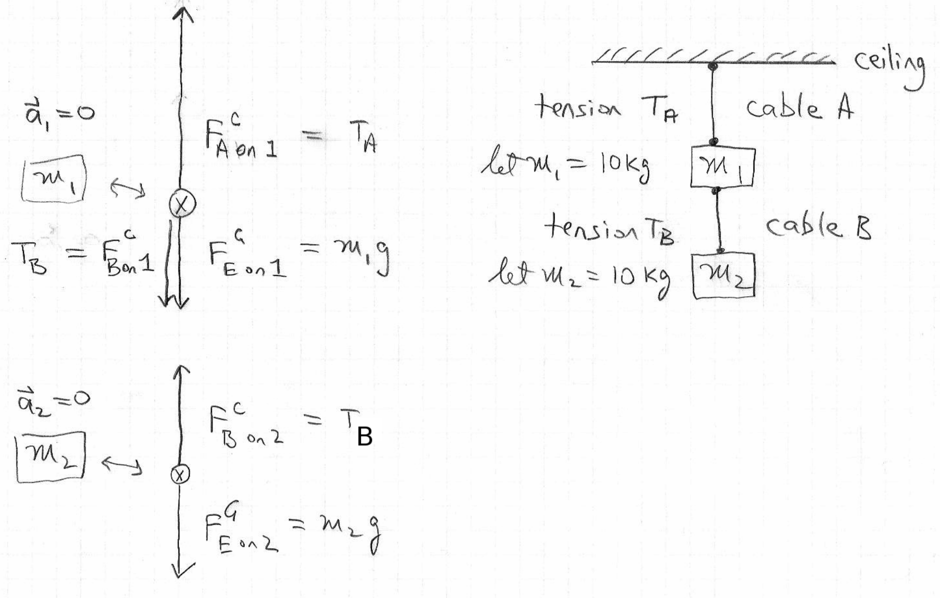
$\vec{a}_2 = 0$



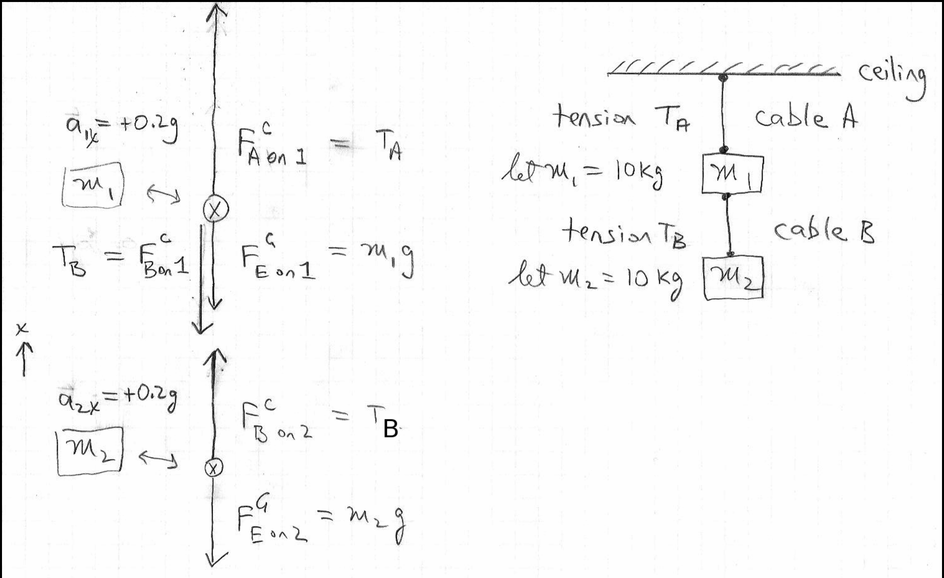


$F^{C}_{B\,on\,2} = T_B$

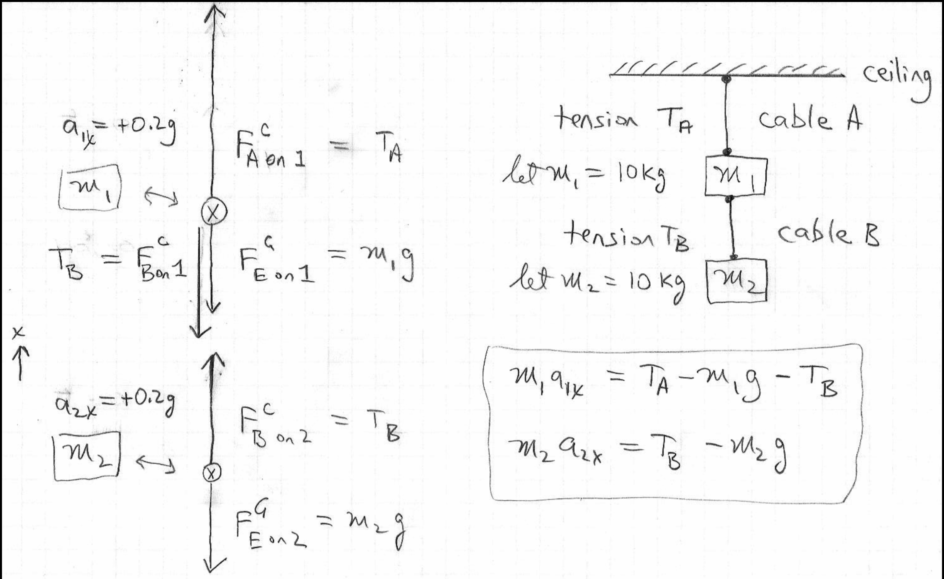
$F^{G}_{E\,on\,2} = m_2 g$



Next: How would these two diagrams change if we imagine that the ceiling is actually the ceiling of an elevator that is **accelerating upward** at $a_x = +1.96\,\mathrm{m/s^2}$ (that's $0.2g$ — you can round off).



How do you use these two FBDs to write Newton's 2nd law for each of the two masses?



Note: because the length of an (**idealized**) taut cable doesn't change as its tension increases, $a_{1x} = a_{2x}$. Distance between blocks only changes if the cable goes slack (no longer in tension).

In the 17th century, Otto von Güricke, a physicist in Magdeburg, fitted two hollow bronze hemispheres together and removed the air from the resulting sphere with a pump. Two eight-horse teams could not pull the halves apart even though the hemispheres fell apart when air was readmitted. Suppose von Güricke had tied both teams of horses to one side and bolted the other side to a giant tree trunk. In this case, the tension on the hemispheres would be

(A) twice

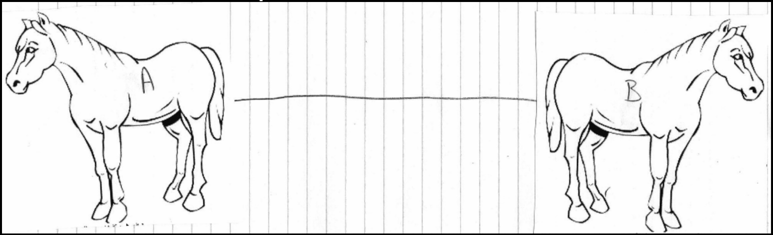
(B) exactly the same as

(C) half

what it was before.

(To avoid confusion, you can replace the phrase "the hemispheres" with the phrase "the cable" if you like. The original experiment was a demonstraton of air pressure, but we are interested in tension.)

Suppose a horse can pull 1000 N



$$\vec{F}_{A \text{ on } B} = -\vec{F}_{B \text{ on } A}$$

$$|\vec{F}_{A \text{ on } B}| = |\vec{F}_{B \text{ on } A}| = 1000 \text{ N}$$

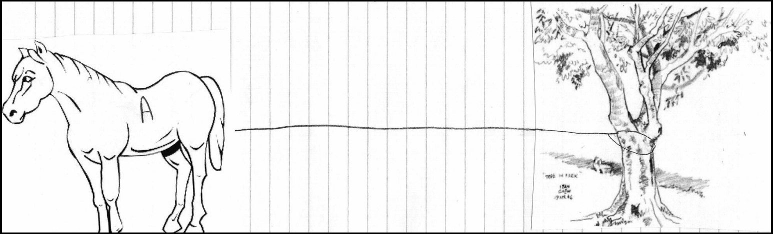
$$T = 1000 \text{ N}$$

$$\vec{a}_A = \vec{0}$$

$$\vec{a}_B = \vec{0}$$

The acceleration of each horse is zero. What are the two horizontal forces acting on horse A? What are the two horizontal forces acting on horse B?

Suppose tree stays put, no matter how hard horse pulls



$$\vec{F}_{A \text{ on tree}} = -\vec{F}_{\text{tree on } A}$$

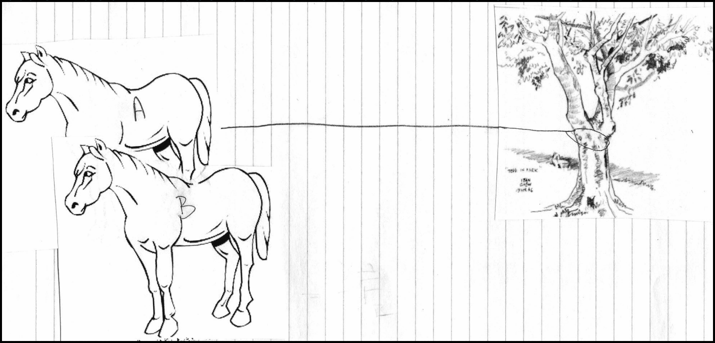
$$|\vec{F}_{A \text{ on tree}}| = |\vec{F}_{\text{tree on } A}| = 1000 \text{ N}$$

$$T = 1000 \text{ N}$$

$$\vec{a_A} = \vec{0}$$

What are the two horizontal forces acting on horse A?

Suppose tree stays put, no matter how hard horses pull. Somehow we attach both horses to the left end of the same cable.



$$\vec{F}_{A+B \text{ on tree}} = -\vec{F}_{\text{tree on } A+B}$$

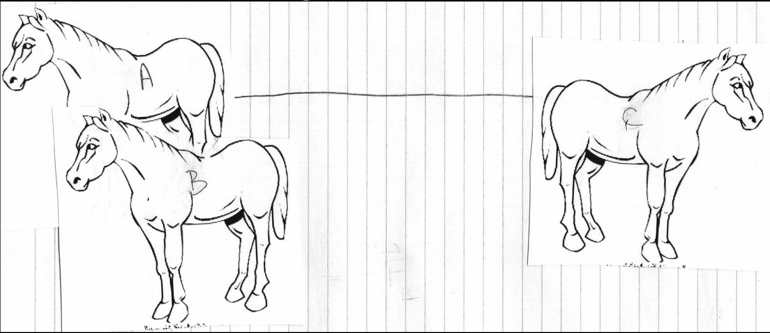
$$|\vec{F}_{A+B \text{ on tree}}| = |\vec{F}_{\text{tree on } A+B}| = 2000 \text{ N}$$

$$T = 2000 \text{ N}$$

$$\vec{a}_{\text{horses}A+B} = \vec{0}$$

What are the external forces acting on the two-horse system (system = horse A + horse B)?

Horse C loses his footing when he pulls > 1000 N



$$|\vec{F}_{A+B \text{ on C}}| = |\vec{F}_{\text{C on } A+B}| = 2000 \text{ N}$$

$$T = 2000 \text{ N}$$

Force of ground on C is 1000 N to the right. Tension pulls on C 2000 N to the left. C accelerates to the left.

$$|\vec{a_C}| = (2000 \text{ N} - 1000 \text{ N})/m_C$$

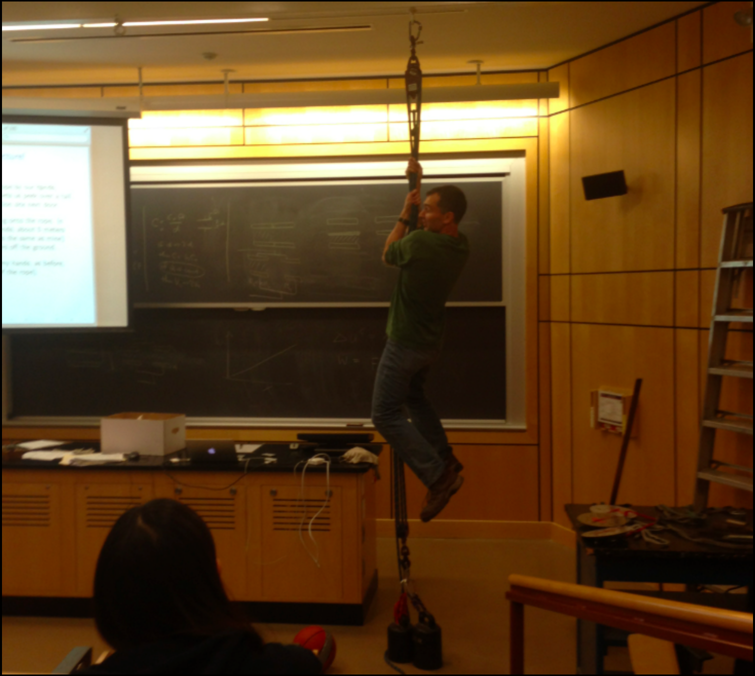
Today, while we happen to have this rope attached to the ceiling, I want to re-visit something (related to forces) that I demonstrated on the first day of class. Believe it or not, this relates pretty directly to architecture.

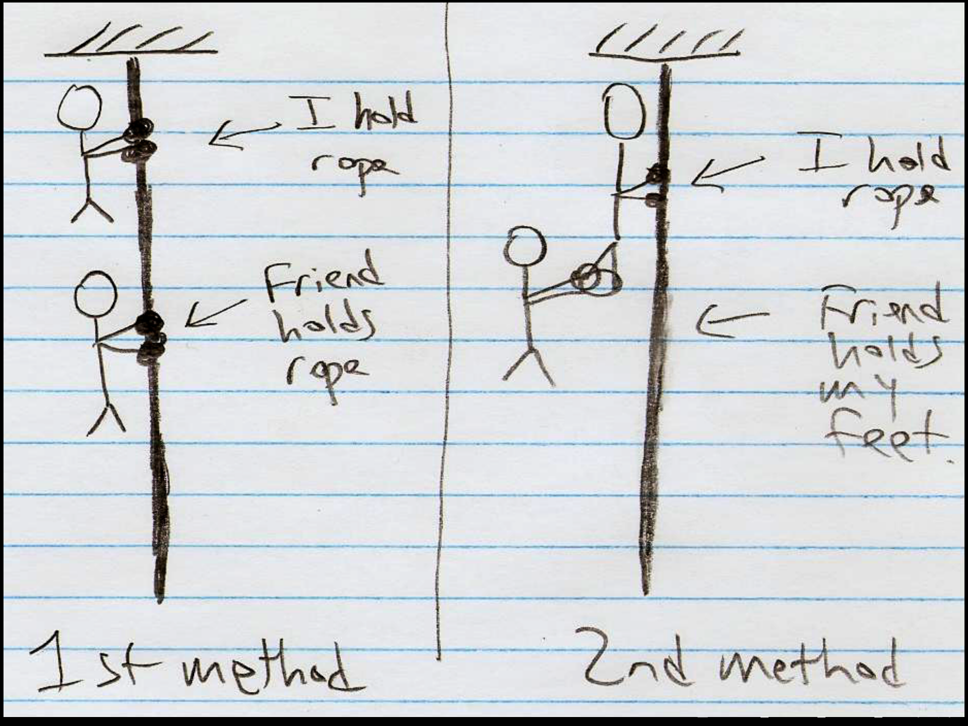
My friend and I both want to hang on to a rope by our hands, perhaps because being up above the ground lets us peek over a tall fence and see into an amazing new construction site next door.

We consider two different methods of hanging onto the rope. In the first method, I hold the rope with my hands, about 5 meters off the ground, and my friend (whose mass is the same as mine) holds the rope with his hands, about 3 meters off the ground.

In the second method, I told the rope with my hands, as before, and my friend holds onto my feet (instead of the rope).

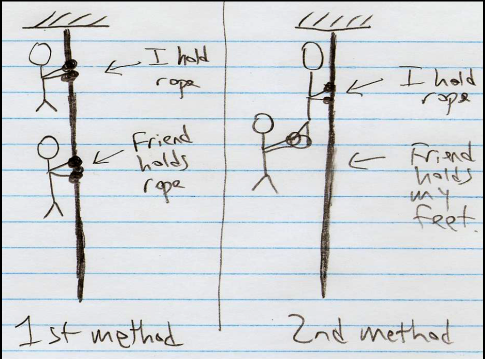
Let's draw a picture, to make it more clear.





I hold rope
Friend holds rope
1st method
I hold rope
Friend holds my feet.
2nd method

The downward force exerted by *my* hands on the rope is . . .



(A) The same for both methods: equal to mg (m = my mass)

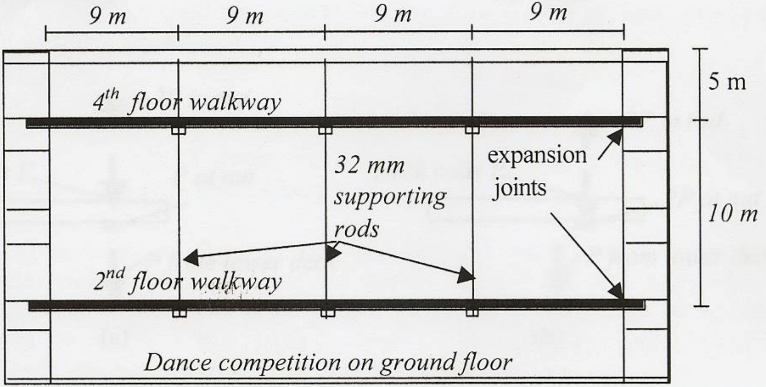
(B) The same for both methods: equal to $2mg$

(C) Twice as much for 1st method ($2mg$ vs. mg)

(D) Twice as much for 2nd method ($2mg$ vs. mg)

Kansas City Hyatt Regency skywalk collapse

On 7^{th} July *1981*, a dance was being held in the lobby of the Hyatt Regency Hotel, Kansas City. As spectators gathered on suspended walkways above the dance floor, the support gave way and the upper walkway fell on the lower walkway, and the two fell onto the crowded dance floor, killing *114* people and injuring over *200*.



For more like this, read *To Engineer is Human* by Henry Petroski.

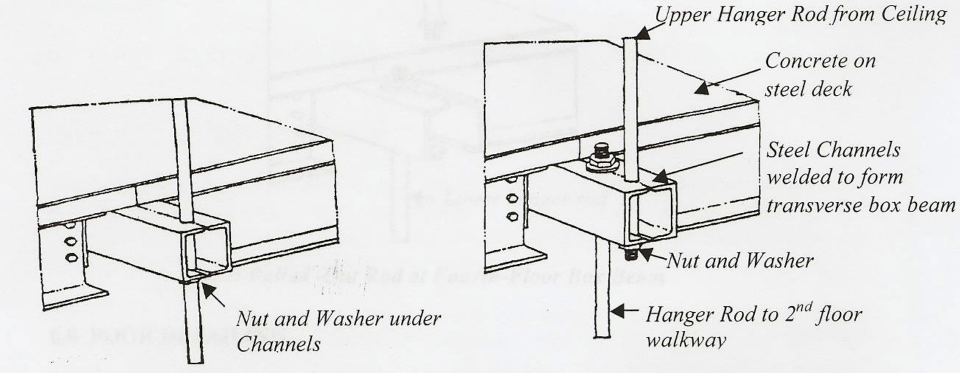


Fig. 5(a): Hyatt Regency Hanger Details As-Designed

Fig. 5(b): Hyatt Regency Hanger Details As-Built

As designed, each of the two skywalks hangs onto the rope with its own hands. As built, the lower skywalk's hands are effectively hanging onto the upper skywalk's feet! So the upper skywalk's grip on the rope feels 2× larger force than in original design. Oops!

A real-world use for free-body diagrams! But these diagrams aren't careful to single out one object, to indicate clearly what that object is, and to draw only the forces acting ON that object. (Alas.)

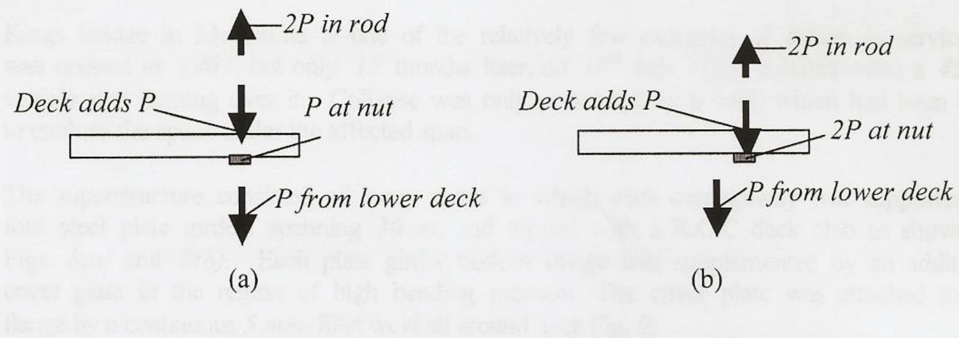


Fig. 6: Free-Body Diagram (a) As Designed (b) As Built

The author uses the symbol P for a "point" force (or point load, or a "concentrated load"), as is the custom in engineering and architecture. When you see "P" here, pretend it says "F" or "mg" instead.

Upper skywalk loses its grip on the "rope"

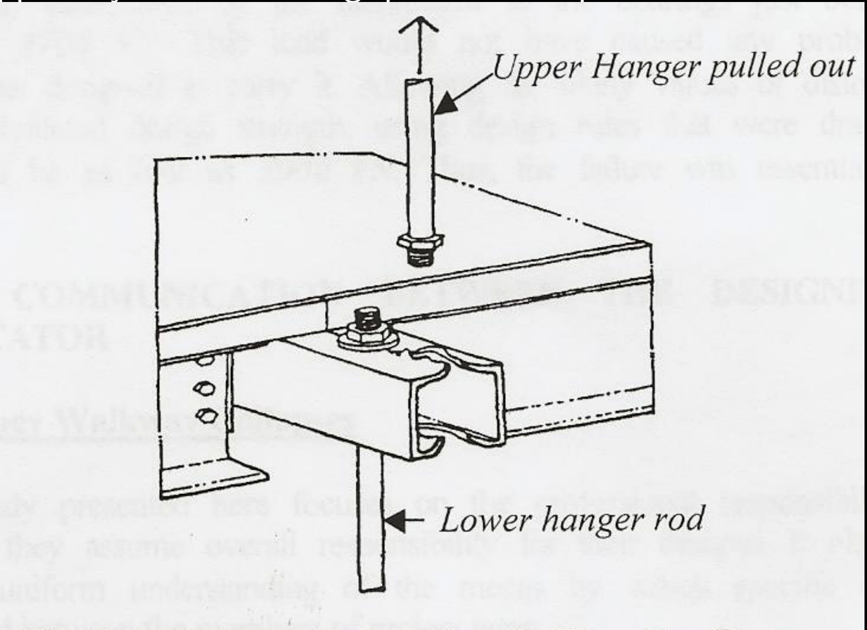


Fig. 7: Pulled -Out Rod at Fourth-Floor Box Beam

HW4 / problem 9: slightly modified (skip?)

9*. A tugboat pulls two barges (connected in series, like a train, with taut ropes as couplings) down a river. The barge connected to the tugboat, carrying coal, has inertia m_1. The other barge, carrying pig iron, has inertia m_2. The frictional force exerted by the water on the coal barge is F^f_{w1}, and that exerted by the water on the pig-iron barge is F^f_{w2}. The common acceleration of all three boats is a_x. Even though the ropes are huge, the gravitational force exerted on them is negligible, as are the ropes' inertias. How can you solve for the tension in each rope?

HW4 / problem 10 (modified): (skip?)

10*. A red cart of mass m_{red} is connected to a green cart of mass m_{green} by a **relaxed** spring of spring constant k. The green cart is resting against a blue cart of mass m_{blue}. All are on a low-friction track. You push the red cart to the right, in the direction of the green cart, with a constant force $F^c_{\mathrm{you,green}}$. (a) What is the acceleration of the center-of-mass of the three-cart system? (b) What is the acceleration of each cart **the instant you begin to push**? (c) What is the acceleration of each cart the instant when the spring is compressed a distance D with respect to its relaxed length?

(skip?)

Estimate the spring constant of your car springs. (Experiment: sit on one fender.)

(What do you think?)

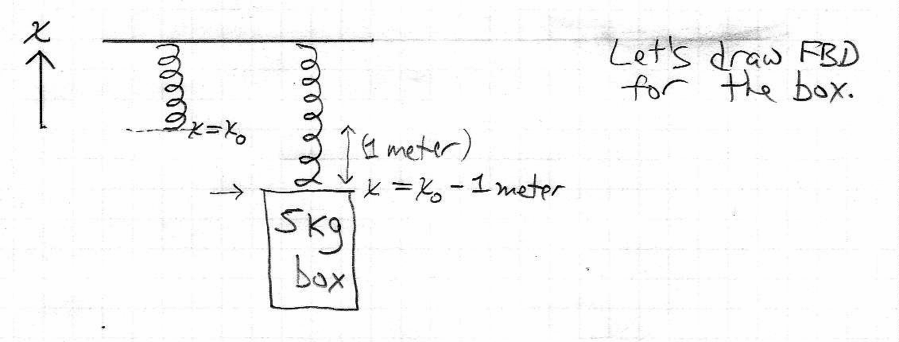
(skip)

When a 5.0 kg box is suspended from a spring, the spring stretches to 1.0 m beyond its equilibrium length. In an elevator accelerating upward at 0.98 $\mathrm{m/s^2}$ (that's "0.1 g"), how far will the spring stretch with the same box attached?

(A) 0.50 m
(B) 0.90 m
(C) 1.0 m
(D) 1.1 m
(E) 1.2 m
(F) 1.9 m
(G) 2.0 m

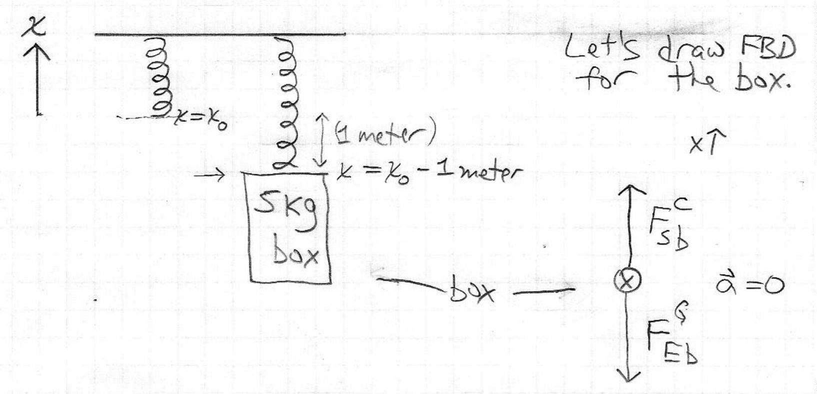
(By the way: When a tall building sways back and forth in the wind, the uncomfortable acceleration experienced by the occupants is often measured as a fraction of "g.")

(skip)



Let's start by drawing a FBD for the box when the elevator is **not** accelerating.

(skip)



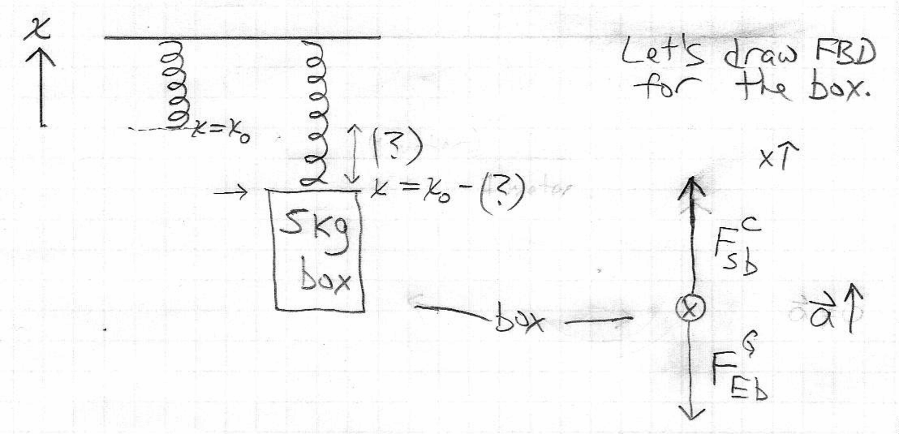
$$F^c_{sb,x} + F^G_{Eb,x} = ma_x = 0$$

$$F^c_{sb,x} = -k\,(x - x_0) = -k(-1\text{ meter}) \qquad F^G_{Eb,x} = -mg$$

$$+k(1\text{ meter}) - mg = ma_x = 0$$

Next, what happens if elevator is accelerating upward at 1 $\mathrm{m/s^2}$?

(skip)



$$F^c_{sb,x} + F^G_{Eb,x} = ma_x = +1\ \mathrm{m/s^2}$$

$$F^c_{sb,x} = -k\,(x - x_0) \qquad F^G_{Eb,x} = -mg$$

$$-k(x - x_0) - mg = ma_x = +0.1mg$$

combine with $+k(1\ \mathrm{meter}) - mg = 0$ from last page

(skip)

$$-k(x - x_0) - mg = ma_x = +0.1g \quad \Rightarrow \quad \boxed{-k(x - x_0) = +1.1mg}$$

combine with $+k(1 \text{ meter}) - mg = 0 \quad \Rightarrow \quad \boxed{+k(1 \text{ meter}) = mg}$

Divide two boxed equations: get $\quad x - x0 = -1.1 \text{ meters}$

So the spring is now stretching 1.1 meters beyond its relaxed length (vs. 1.0 meters when $a_x = 0$).

The upward force exerted by the spring on the box is $\quad m(g + a_x)$.

(skip)

When a 5.0 kg box is suspended from a spring, the spring stretches to 1.0 m beyond its equilibrium length. In an elevator accelerating upward at 0.98 $\mathrm{m/s^2}$, how far will the spring stretch with the same box attached?

(A) 0.50 m
(B) 0.90 m
(C) 1.0 m
(D) 1.1 m
(E) 1.2 m
(F) 1.9 m
(G) 2.0 m

video segment break

- begin video preceding ws09
- Read/skim Mazur chapter 9, then watch this video.

Ch9: work. Two definitions of work

- Work equals the change in energy of a system due to external forces. If the energy of a system increases, the (arithmetic sum of) work done by external forces on the system is positive; if the energy of a system decreases, the (sum of) work done by external forces on the system is negative.

$$\Delta E_{\mathrm{system}} = W_{\mathrm{done\ ON\ system}}$$

- The work done by an external force $\vec{F}$ on a system (in one dimension) is $\boxed{W = \int F_x(x)\,\mathrm{d}x}$ or just $\boxed{W = F_x\,\Delta x}$ for a constant force. When the force and the "displacement of the point of application of the force" point in the same (opposite) direction, the work done by $\vec{F}$ is positive (negative).

Let's initially focus on the second, more familiar, definition.

Chapter 9: first reading question

1. If you graph the work, $W(x)$, done by a force on an object as a function of the object's position, x, what graphical feature represents the force, $F(x)$, exerted on the object?

(A) The force is the area under the work curve.
(B) The force is the slope of the work curve.
(C) The vertical axis, i.e. the height of the work curve.
(D) The second derivative.

Chapter 9: first reading question

1. If you graph the work done by a force on an object as a function of the object's position, what graphical feature represents the force exerted on the object?

Since work equals the integral of force w.r.t. displacement, $W = \int F_x \mathrm{d}x$ or $W = F_x \Delta x$, the force is equal to the work per unit displacement. On a graph of W vs. x, the slope, $\mathrm{d}W/\mathrm{d}x$, is equal to the force.

Suppose you want to ride your mountain bike up a steep hill. Two paths lead from the base to the top, one twice as long as the other. (Your bicycle has only one gear.) Compared to the average force you would exert if you took the short path, the average force you exert along the longer path is

(A) one-fourth as large.

(B) one-third as large.

(C) one-half as large.

(D) the same.

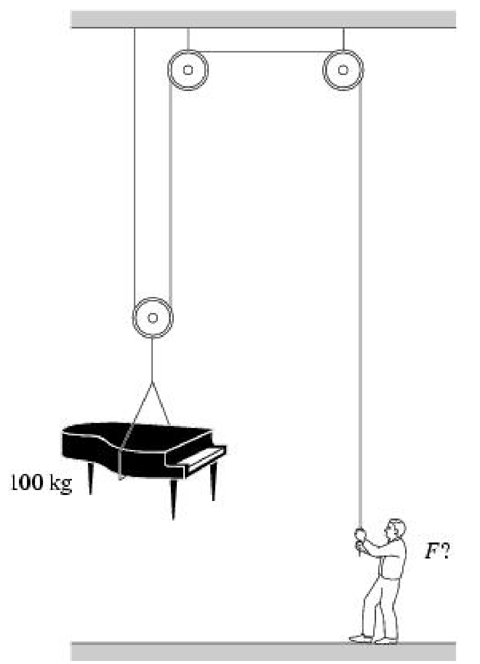
(E) twice as large.

(F) undetermined — it depends on the time taken

(Imagine how hard you have to press down on the pedals, on average, to make the bike go up one path vs. the other. As a kid, did you ever zig-zag up a really steep hill on your one-speed bike, or if your multi-speed bike's lowest gear was still not low enough?)

Imagine me towing Alfie up a steep hill behind my bicycle . . . .

https://youtu.be/Yigqi7zGCfQ
https://youtu.be/ewvet0I1YiM



A piano mover raises a 100 kg piano at a constant speed using the pulley system shown here. With how much force is she pulling on the rope? (Ignore friction and assume $g \approx 10\ \mathrm{m/s^2}$.)

(A) 2000 N

(B) 1500 N

(C) 1000 N

(D) 750 N

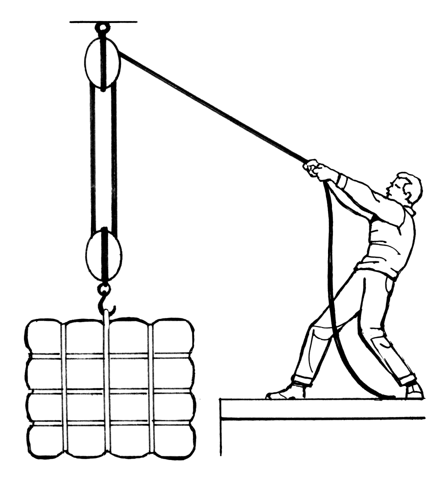
(E) 500 N

(F) 200 N

(G) 50 N

(H) impossible to determine.

Block and tackle: “mechanical advantage”



This graphic shows a 2:1 mechanical advantage. The block & tackle in the classroom shows a 4:1 advantage. How would you get a HUGE mechanical advantage, like 1000:1 ? (Phys 009 topic.)

A spring-loaded toy dart gun is used to shoot a dart straight up in the air, and the dart reaches a maximum height of 8 m. The same dart is shot straight up a second time from the same gun, but this time the spring is compressed only half as far before firing. How far up does the dart go this time (neglecting friction)?

(A) 1 m

(B) 2 m

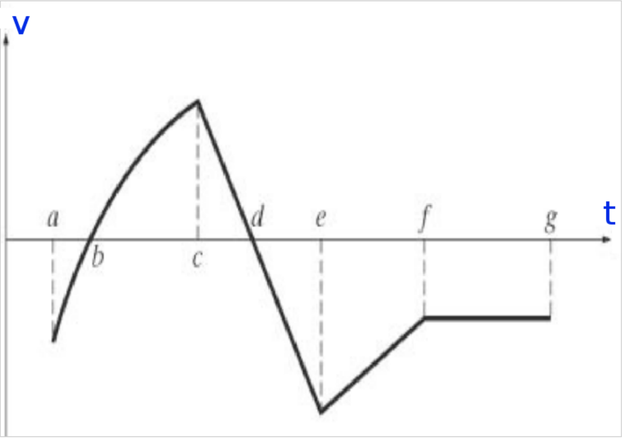
(C) 4 m

(D) 8 m

(E) 16 m

(F) 32 m

2. The velocity of an object as a function of time is shown in the figure below. Over what intervals is the work done on the object (a) positive, (b) negative, (c) zero? (Hint: make a table showing sign of acceleration (hence sign of net force), sign of displacement, and sign of their product, for each segment.)



(Consider work done by whatever external force is causing the object's velocity to change.)

Stretching a certain spring 0.10 m from its equilibrium length requires 10 J of work. How much more work does it take to stretch this spring an additional 0.10 m from its equilibrium length?

(A) No additional work

(B) An additional 10 J

(C) An additional 20 J

(D) An additional 30 J

(E) An additional 40 J

A block initially at rest is allowed to slide down a frictionless ramp and attains a speed v at the bottom. To achieve a speed $2v$ at the bottom, how many times as high must a new ramp be?

(A) 1

(B) 1.414

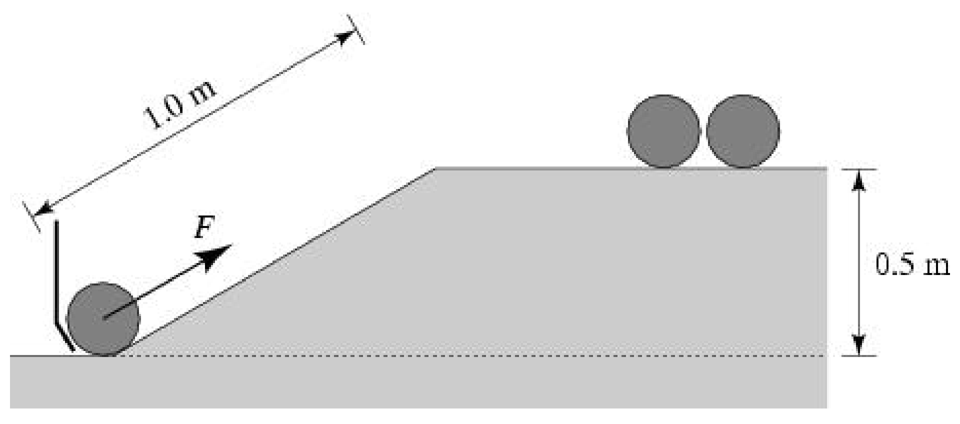
(C) 2

(D) 3

(E) 4

(F) 5

(G) 6



At the bowling alley, the ball-feeder mechanism must exert a force to push the bowling balls up a 1.0 m long ramp. The ramp leads the balls to a chute 0.5 m above the base of the ramp. About how much force must be exerted on a 5.0 kg bowling ball?

(A) 200 N
(B) 100 N
(C) 50 N
(D) 25 N
(E) 5.0 N
(F) impossible to determine.

Suppose you drop a 1 kg rock from a height of 5 m above the ground. When it hits, how much force does the rock exert on the ground? (Take $g \approx 10 \text{ m/s}^2$.)

(A) 0.2 N

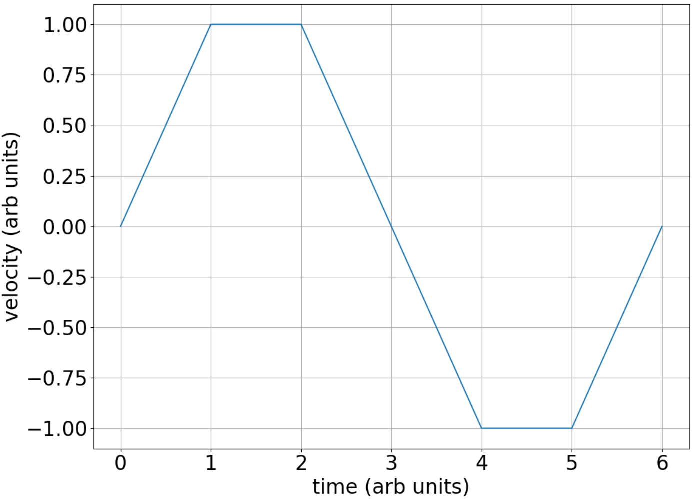
(B) 5 N

(C) 50 N

(D) 100 N

(E) impossible to determine without knowing over what distance the rock slows when it impacts the ground.

The velocity of an object as a function of time is shown. Over what time intervals is the work done on the object (a) positve, (b) negative, (c) zero? Hint: make a table showing sign of acceleration (hence sign of net force), sign of displacement, and sign of their product, for each segment. (Consider work done by whatever external force is causing the object's velocity to change.)



From a bridge at initial height h above the water, I release from rest an object of mass m which is attached to a "bungee cord" (a spring) of relaxed length ℓ_0 spring constant k. Which equation correctly expresses, assuming that no mechanical energy is dissipated into heat, the speed v_f of the object when it reaches the water surface? (One end of the bungee cord is tied to the bridge. The cord is initially slack does not begin to stretch until the object has fallen a distance equal to the cord's relaxed length.)

(A) $mg = kh$

(B) $mg = k\,(h - \ell_0)$

(C) $mgh + \frac{1}{2}mv_f^2 = \frac{1}{2}kh^2$

(D) $mgh + \frac{1}{2}mv_f^2 = \frac{1}{2}k\,(h - \ell_0)^2$

(E) $mgh = \frac{1}{2}mv_f^2 + \frac{1}{2}kh^2$

(F) $mgh = \frac{1}{2}mv_f^2 + \frac{1}{2}k\,(h - \ell_0)^2$

A motor lifts an object of mass m at constant upward **velocity** $v_y = \mathrm{d}y/\mathrm{d}t$. How much power (work per unit time) does the motor supply?

(A) $\text{power} = mgv_y$

(B) $\text{power} = mgy$

(C) $\text{power} = \frac{1}{2}mv_y^2$

(D) $\text{power} = \frac{1}{2}mv_y^2 + mgy$

(E) $\text{power} = \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{1}{2}mv_y^2 + mgy\right)$

(F) (A) and (E) are both correct.

(G) (B) and (E) are both correct.

A motor lifts an object of mass m at constant upward **acceleration** $a_y = \mathrm{d}v_y/\mathrm{d}t$. How much power (work per unit time) does the motor supply?

(A) $\text{power} = mgv_y$

(B) $\text{power} = m(a_y + g)v_y$

(C) $\text{power} = \frac{1}{2}mv_y^2$

(D) $\text{power} = \frac{1}{2}mv_y^2 + mgy$

(E) $\text{power} = \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{1}{2}mv_y^2 + mgy\right)$

(F) (A) and (E) are both correct.

(G) (B) and (E) are both correct.

An object is said to be in *stable equilibrium* if a displacement in either direction requires positive work to be done on the object by an external force. Let's suppose that there is some potential energy associated with every position of the object, i.e. there is a potential energy curve $U(x)$, where x is the object's position. How do you expect $U(x)$ to change as you move the object away (in either direction) from its position of stable equilibrium?

(A) When displacing the object away from its equilibrium position, the positive work done (on the object plus its environment) by the external force causes a positive change in the potential energy function $U(x)$. So $U(x)$ must have a local minimum at the object's stable equilibrium position.

(B) $U(x)$ must have a local maximum at the object's stable equilibrium position.

(C) The derivative $\mathrm{d}U(x)/\mathrm{d}x$ must have a local minimum at the object's stable equilibrium position.

(D) The derivative $\mathrm{d}U(x)/\mathrm{d}x$ must be zero at the object's stable equilibrium position.

(E) Both (A) and (D) are true.

Chapter 9 reading question

2. When you stand up from a seated position, you push down with your legs. So then do you do negative work when you stand up?

"In this situation, we have 2 systems. Firstly, in the system of just the person, the action of standing up will result in a loss of internal or chemical energy, thereby resulting in a loss of system energy and hence positive work (BY the system) [which implies negative work done ON the system, by Earth's gravitational force]. For the system of the person and Earth, the action of standing up increases the [system's] potential energy at the expense of [the person's] internal [food] energy. In this situation, there is no change in system energy and therefore no work is done."

Reading question 2 had no really simple answer

When you stand up from a seated position, you push down with your legs. Does this mean you do negative work when you stand up?

Suppose "system" = me + Earth + floor + chair

- $\Delta K = 0$
- $\Delta U = mg\ (\Delta x)_{\text{my c.o.m.}} > 0$
- $\Delta E_{\text{thermal}} = 0$ (debatable but irrelevant)
- $\Delta E_{\text{source}} = -mg\ (\Delta x)_{\text{my c.o.m.}} < 0$

- $W = 0$

There are no external forces. Everything of interest is inside the system boundary.

Let's try choosing a different "system."

When you stand up from a seated position, you push down with your legs. Does this mean you do negative work when you stand up?

Suppose "system" = me + floor + chair

- $\Delta K = 0$
- $\Delta U = 0$ (U^G undefined if Earth not in system)
- $\Delta E_{\text{thermal}} = 0$ (debatable but irrelevant)
- $\Delta E_{\text{source}} = -mg\,(\Delta x)_{\text{my c.o.m.}} < 0$
- $W = -mg\,(\Delta x)_{\text{my c.o.m.}} < 0$

External gravitational force, exerted by Earth on me, does negative work on me. Point of application of this external force is my body's center of mass. Force points downward, but displacement is upward. $W < 0$. System's total energy decreases.

Let's try answering a slightly different question.

When **a friend** *stands me up from a chair (e.g. my knees are weak today), does my friend do positive or negative work?*

Suppose "system" = me + Earth + floor + chair

- $\Delta K = 0$
- $\Delta U = mg\ (\Delta x)_{\text{my c.o.m.}} > 0$
- $\Delta E_{\text{thermal}} = 0$ (debatable but irrelevant)
- $\Delta E_{\text{source}} = 0$
- $W = mg\ (\Delta x)_{\text{my c.o.m.}} > 0$

My friend applies an upward force beneath my arms. The point of application of force is displaced upward.

Let's include my friend as part of "the system."

When a friend stands me up from a chair (e.g. my knees are weak today), does my friend do positive or negative work?

Suppose "system" $=$ me $+$ Earth $+$ floor $+$ chair $+$ friend

- $\Delta K = 0$
- $\Delta U = mg\ (\Delta x)_{\text{my c.o.m.}} > 0$
- $\Delta E_{\text{thermal}} = 0$ (debatable but irrelevant)
- $\Delta E_{\text{source}} = -mg\ (\Delta x)_{\text{my c.o.m.}} < 0$
- $W = 0$

There is no external force. Everything is within the system.

Back to the original reading question

When you stand up from a seated position, you push down with your legs. Does this mean you do negative work when you stand up?

I think the work done **ON the system BY my legs** is either positive (if my legs are considered "external" to the me+Earth+floor system and are supplying the energy to lift me) or zero (if my legs are part of the system).

Remember the one way we got a negative answer: In the case in which Earth was not part of the system, we found that the external force of Earth's gravity did negative work on me. But I was pushing Earth downward, away from me. I lost energy. So even in this case (where the work done **on me** was negative), the work done **by me** was positive.

Key point: what you call "work" depends on how you define "the system."

A few key ideas from Chapters 8 (force) and 9 (work)

Impulse (i.e. momentum change) delivered by external force:

$$\text{force} = \frac{\mathrm{d}(\text{momentum})}{\mathrm{d}t} \quad \Leftrightarrow \quad \vec{J} = \int \vec{F}_{\text{external}}\,\mathrm{d}t$$

External force exerted ON system:

$$\text{force} = \frac{\mathrm{d}(\text{work})}{\mathrm{d}x} \quad \Leftrightarrow \quad W = \int F_x\,\mathrm{d}x$$

Force exerted BY spring, gravity, etc.:

$$\text{force} = -\frac{\mathrm{d}(\textit{potential energy})}{\mathrm{d}x}$$

ΔE_{system} = flow of energy into system = work done ON system:

$$\text{work} = \Delta(\text{energy}) = \Delta K + \Delta U + \Delta E_{\text{source}} + \Delta E_{\text{thermal}}$$

Notice that work : energy :: impulse : momentum

Some equation sheet entries for Chapters 8+9

http://positron.hep.upenn.edu/physics8/files/equations.pdf

Work (external, nondissipative, 1D):

$$W = \int F_x(x)\ dx$$

which for a constant force is

$$W = F_x\ \Delta x$$

Power is rate of change of energy:

$$P = \frac{\mathrm{d}E}{\mathrm{d}t}$$

Constant external force, 1D:

$$P = F_x v_x$$

G.P.E. near earth's surface:

$$U_{\mathrm{gravity}} = mgh$$

Force of gravity near earth's surface (force is $-\frac{\mathrm{d}U_{\mathrm{gravity}}}{\mathrm{d}x}$):

$$F_x = -mg$$

Potential energy of a spring:

$$U_{\mathrm{spring}} = \frac{1}{2}k(x - x_0)^2$$

Hooke's Law (force is $-\frac{\mathrm{d}U_{\mathrm{spring}}}{\mathrm{d}x}$):

$$F_{\mathrm{by\ spring\ ON\ load}} = -k(x - x_0)$$

video segment break

- begin video preceding ws10

Physics 8 — Monday, September 30, 2019

- You probably noticed by now that I try my best to motivate you to spend time each week reading, working checkpoints, and solving problems. I give you a significant amount of work to do. By **doing** all of the work, you will learn a lot, and you will do very well in the course. That's the bargain we offer.

Things to understand before studying architectural structures:

- forces ✓ (but we will continue to use, all term!)
- vectors — (now)
- torques — (chapter 12)

- The next few chapters (10,11,12) are the most difficult material in the course. We will slow down for them. After that, the fun begins: we can apply our knowledge of forces (ch8), vectors (ch10), and torque (ch12) to structures.

A Chapter 10 reading question:

Can an object be accelerated without changing its kinetic energy?

Answer: Yes. You can change an object's direction without changing its speed. So its velocity can change without changing its kinetic energy.

Over a finite time interval, this is easy to arrange.

Over an infinitessimal time interval, if the acceleration vector is perpendicular to the velocity vector, then direction changes, but speed does not. This will be important in Chapter 11!

Let's start with the familiar "ball-popper" cart

New (ch10): use two coordinate axes. In most cases, make y-axis point upward (vertical), and x-axis point to the right (horizontal).

Vertical equation of motion ($a_y = -g$ is constant):

$$y = y_i + v_{i,y} t - \frac{1}{2} g t^2$$

$$v_y = v_{i,y} - g t$$

Horizontal equation of motion ($v_x = v_{i,x}$ is constant):

$$x = x_i + v_{i,x} t$$

If you let $x_i = 0$ (simpler) and solve horizontal eqn. for t, you get

$$t = \frac{x}{v_{i,x}}$$

Now plug this into the equation for y ...

$$y = y_i + v_{i,y} t - \frac{1}{2} g t^2$$

Now plug $t = \frac{x}{v_{i,x}}$ into the equation for y:

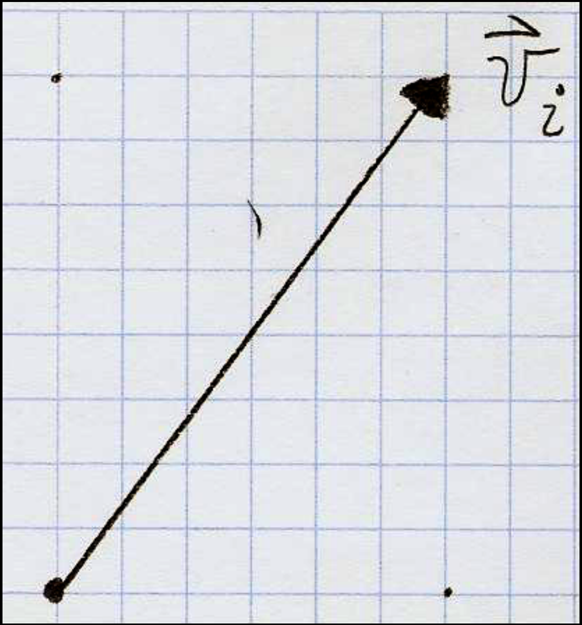
$$y = y_i + v_{i,y} \left(\frac{x}{v_{i,x}} \right) - \frac{1}{2} g \left(\frac{x}{v_{i,x}} \right)^2$$

Separate out the constants to see that $y(x)$ is a parabola:

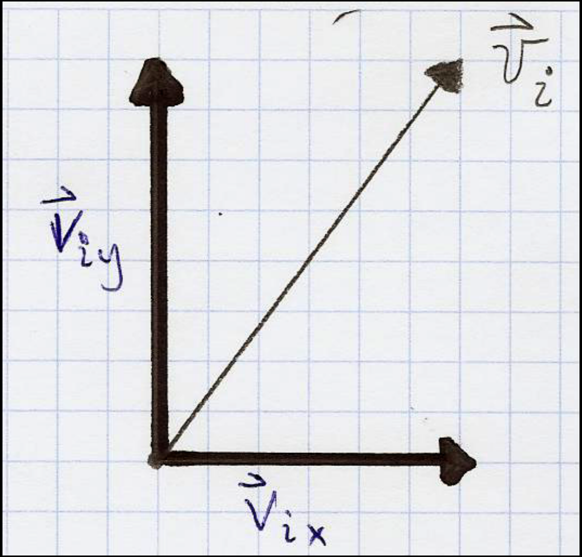
$$y = y_i + \left(\frac{v_{i,y}}{v_{i,x}} \right) x - \left(\frac{g}{2 v_{i,x}^2} \right) x^2$$

(You can "see" this either by drawing a graph or by happening to remember from math that $y = Ax^2 + Bx + C$ is a parabola.)

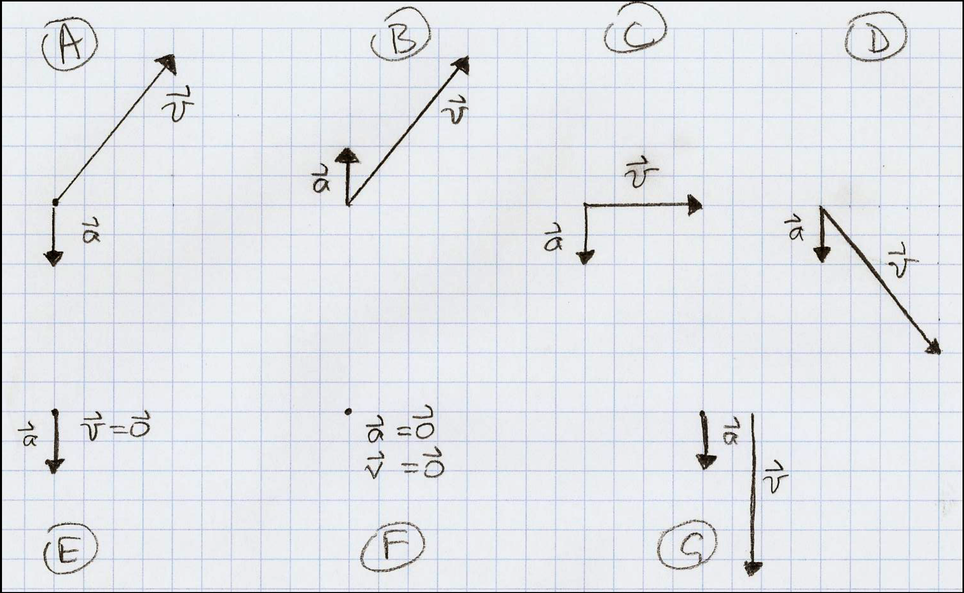
Let's draw and "decompose" the velocity vector at the moment the ball is launched from the cart.



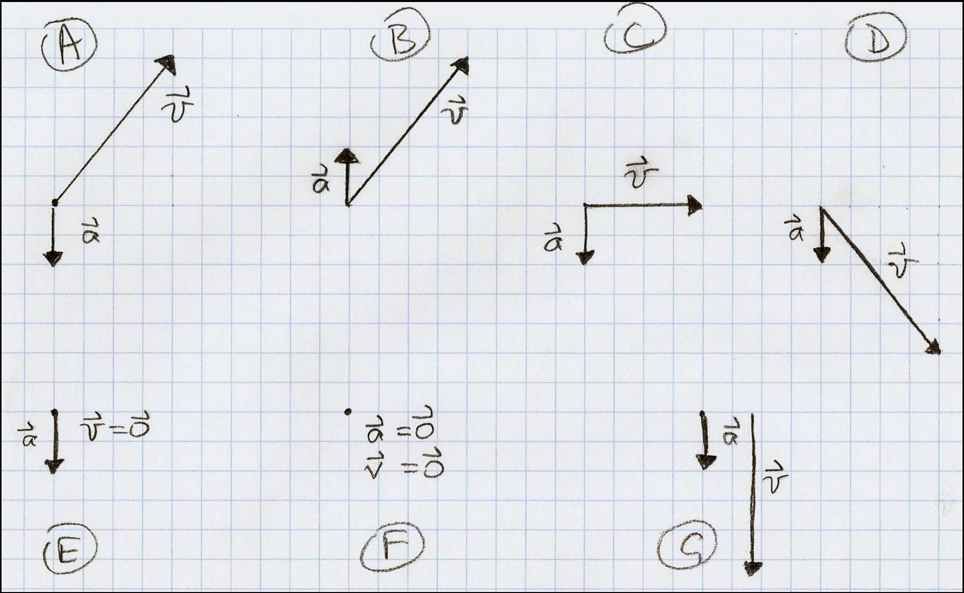
Now decompose into x and y components . . .



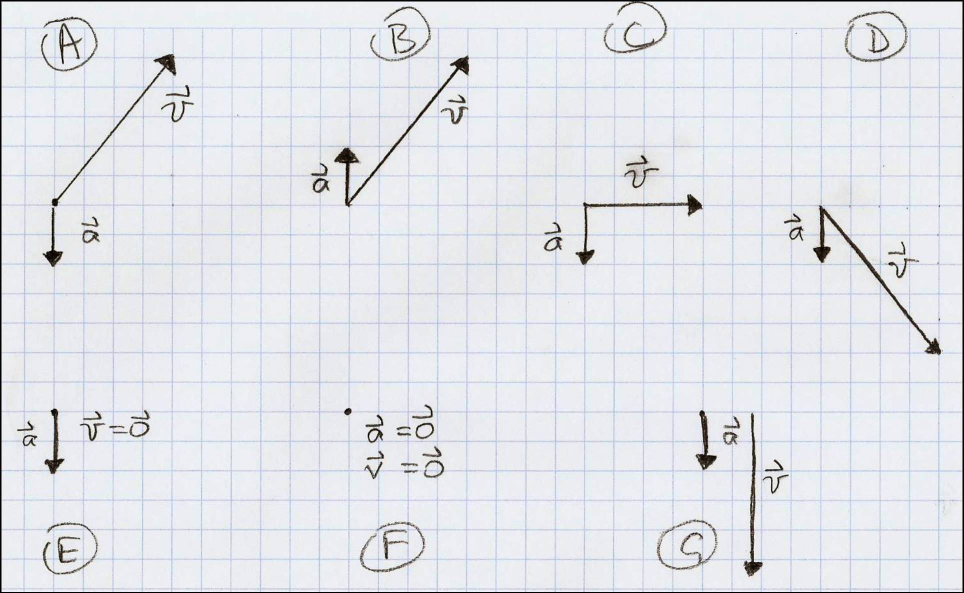
Notice (blackboard) that adding the two components together gives back the original vector.



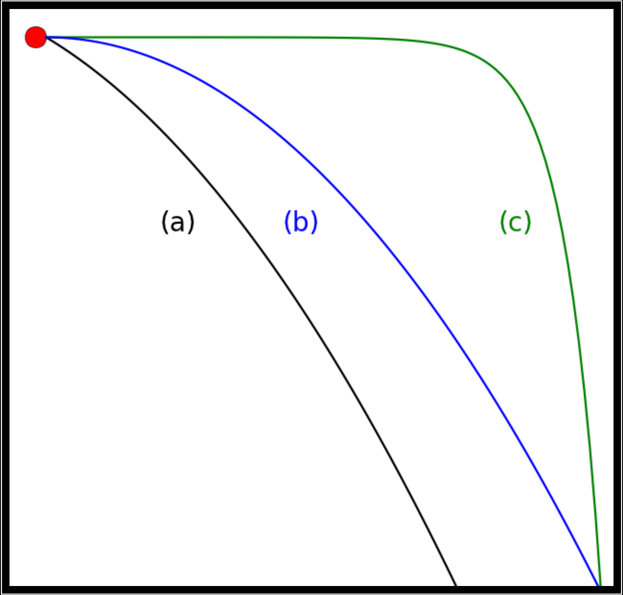
Which graph best represents the acceleration vector $\vec{a}$ and velocity vector $\vec{v}$ the instant **after** the ball is launched from the cart?



Which graph best represents the acceleration vector $\vec{a}$ and velocity vector $\vec{v}$ at the top of the ball's trajectory?

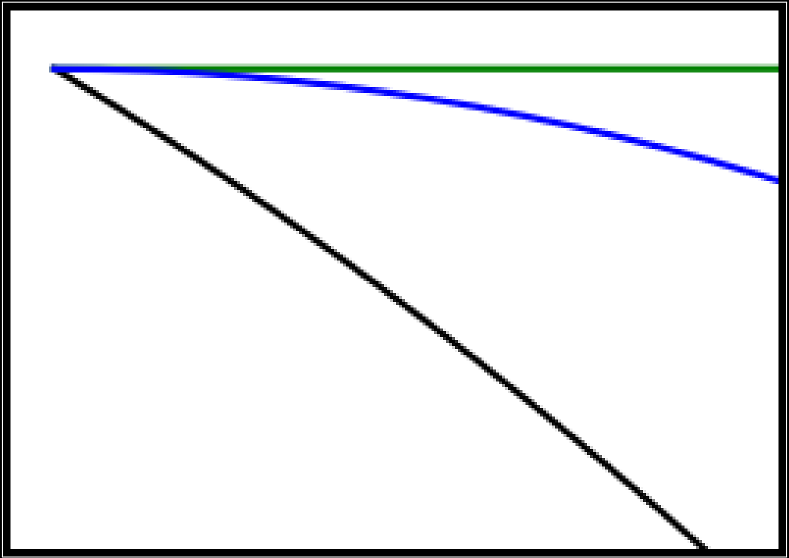


Which graph best represents the acceleration vector $\vec{a}$ and velocity vector $\vec{v}$ the instant before the ball lands in the cart?



Which path best represents the trajectory of a cantaloupe *thrown horizontally* off a bridge? (What's wrong with the other two?)
(Next slide zooms in on corner.)

zoom in on top-left corner (launch position)



Which path best represents the trajectory of a cantaloupe *thrown horizontally* off a bridge? (What's wrong with the other two?)

Two steel balls are released simultaneously from the same height above the ground.

One ball is simply dropped (zero initial velocity).

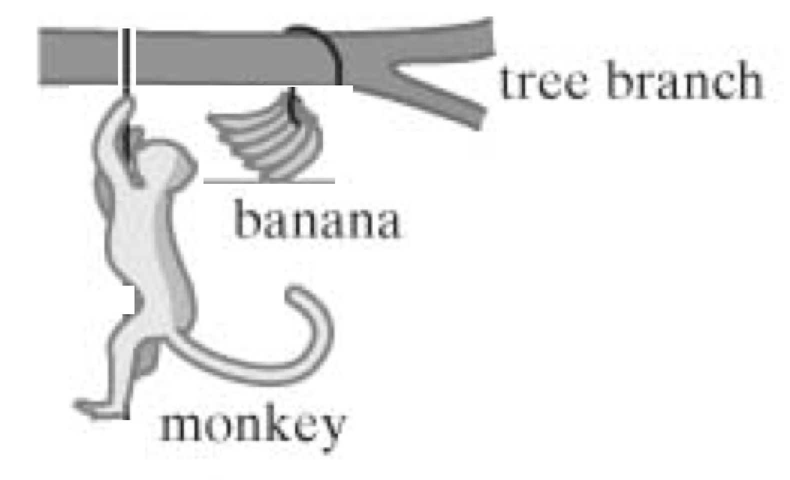
The other ball is thrown horizontally (initial velocity is nonzero, but is purely horizontal).

Which ball will hit the ground first?

(A) The ball thrown horizontally will hit the ground first.
(B) The ball released from rest will hit the ground first.
(C) Both balls will hit the ground at the same time.

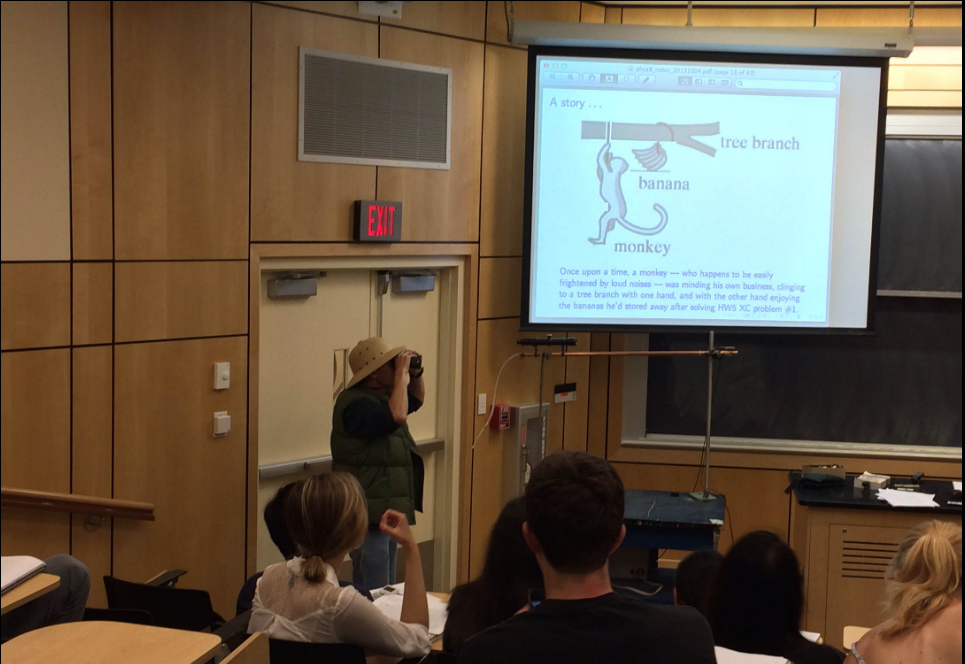
(I should draw a picture of both trajectories on the board.)

A story . . .



Once upon a time, a monkey — who happened to be easily frightened by loud noises — was minding his own business, clinging to a tree branch with one hand, and with the other hand enjoying the bananas he'd stored away after solving HW4 XC problem #7.

Look out . . .

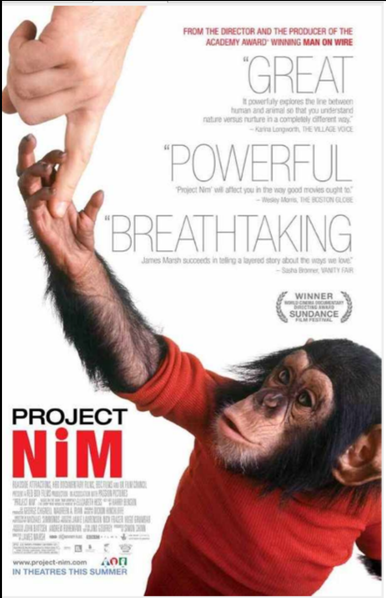

A story . . .
tree branch
banana
monkey
Once upon a time, a monkey — who happens to be easily frightened by loud noises — was minding his own business, clinging to a tree branch with one hand, and with the other hand enjoying the bananas he'd stored away after solving HW5 XC problem #1.
EXIT

Now let's move on to two questions of much more practical significance:

1. Should the "ecologist" shoot the "tranquilizer dart" at Nim Chimpsky, or at Mr. Bill? (She needs to collect a harmless DNA sample from one of these two characters for the Primate Genome Project.)

(A) Tranquilize Nim Chimpsky! (His DNA sample may explain why he was smart enough to learn all those words of American Sign Language.)

(B) Tranquilize Mr. Bill! (If you manage to find any real DNA in his sample, the result will definitely be a publishable paper, if not a Nobel Prize.)



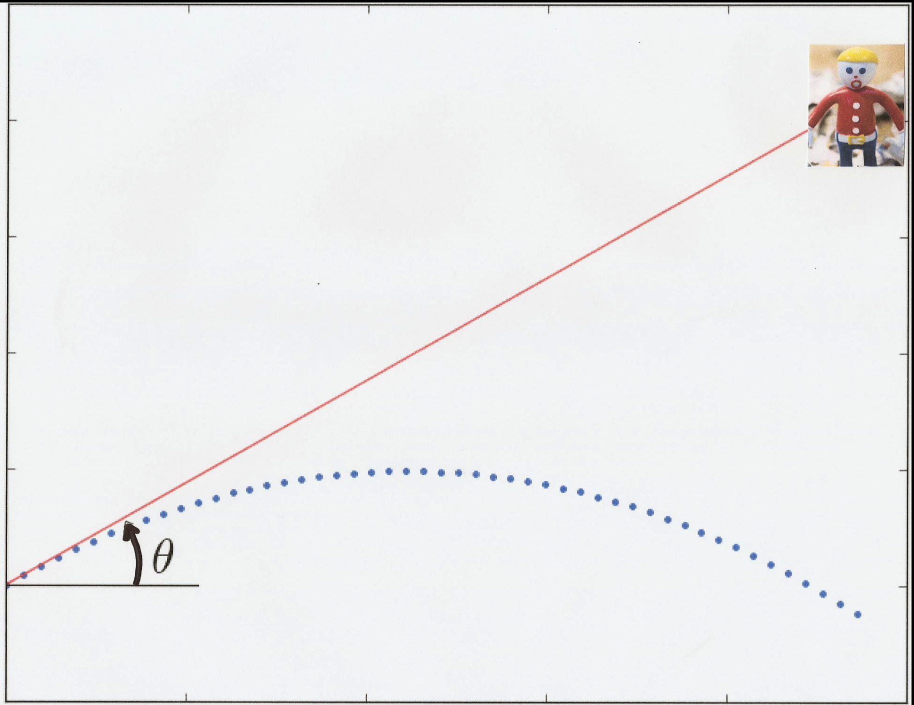
(A) study Nim Chimpsky.
(B) study Mr. Bill.

Now let's move on to two questions of much more practical significance:

1. Should the "ecologist" shoot the "tranquilizer pellet" at Nim Chimpsky, or at Mr. Bill?

2. It takes the pellet some time to travel across the width of the room.
   - In that time interval, gravity will cause Nim/Bill to fall.
   - So where should I aim the pea-shooter so that the pellet hits Nim/Bill as he drops?

Before you answer, let's explain in detail how this game works, why Nim/Bill lets go of the tree, what each trajectory will look like, etc.


θ

What shall I aim for?

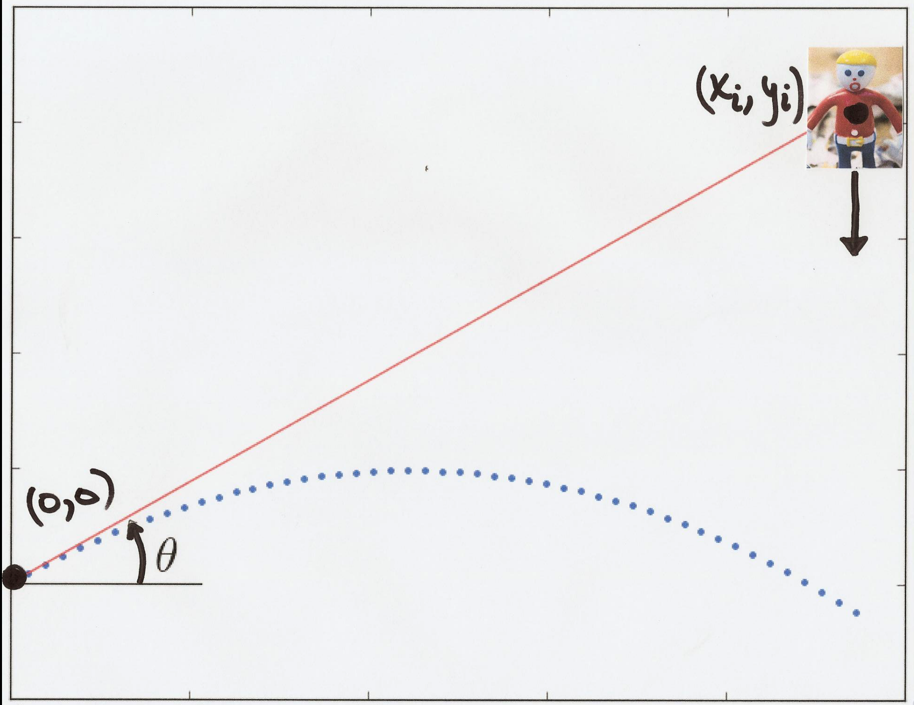
(A) Aim high, because the steel pellet is so much heavier than Mr. Bill, and will be pulled down more by gravity.

(B) Aim low, because Mr. Bill will be falling while the pellet travels.

(C) Aim directly for Mr. Bill. This is clearly what you would do if gravity were absent. The presence of gravity will affect Mr. Bill and the pellet in the same way (they experience the same downward gravitational acceleration), so aiming directly for Mr. Bill will result in a direct hit.

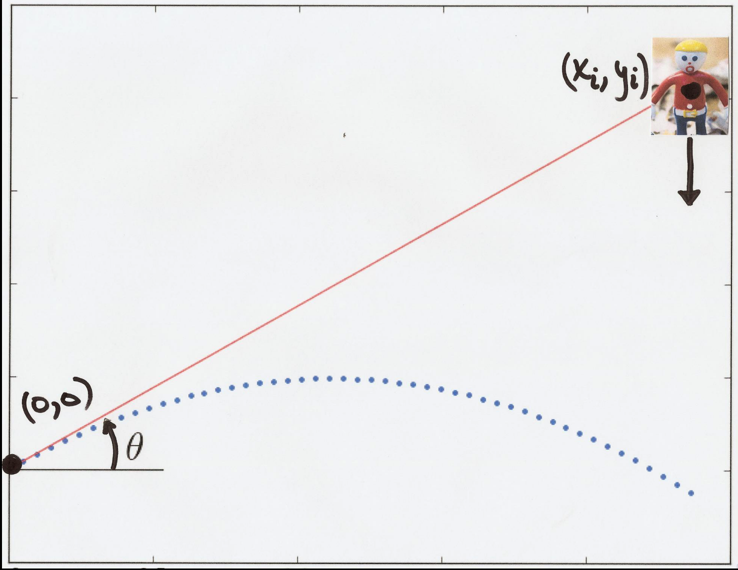
(D) How much below Mr. Bill you need to aim depends on the speed with which you fire the pellet, because the time that it takes the pellet to reach Mr. Bill will depend on how fast the pellet is shot.

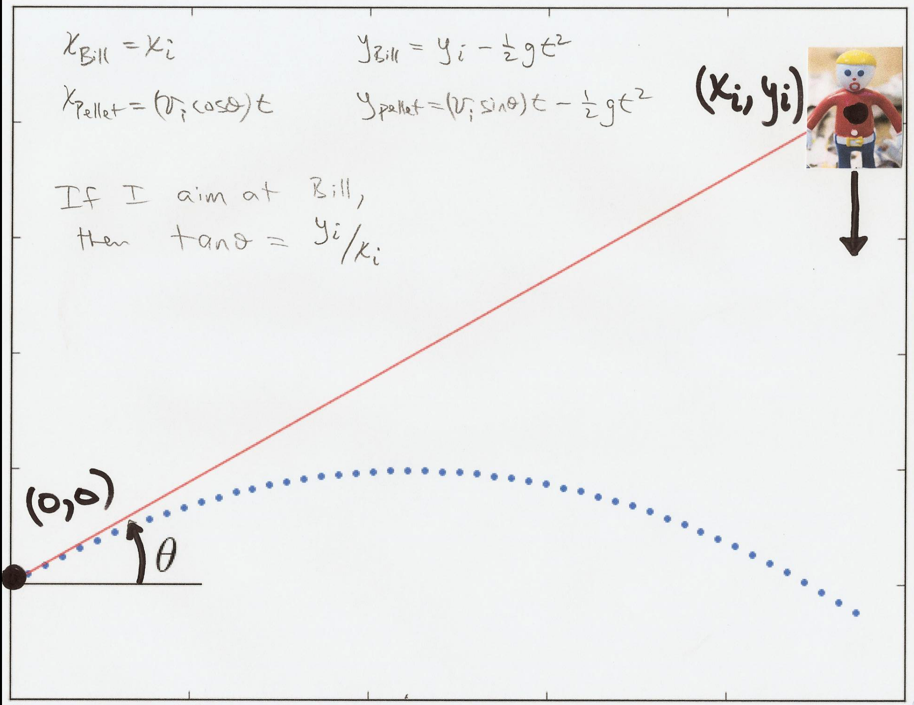
(I'm not going to give away my own answer yet!)



(x_i, y_i)
$(0,0)$
θ

Try writing equations for $\boxed{x_{\text{Bill}}(t),\ y_{\text{Bill}}(t),\ x_{\text{pellet}}(t),\ y_{\text{pellet}}(t)}$, in terms of x_i, y_i, θ (shown on diagram) and initial pellet speed v_i.




$x_{Bill} = x_i$
$y_{Bill} = y_i - \frac{1}{2} g t^2$
$x_{Pellet} = (v_i \cos\theta) t$
$y_{pellet} = (v_i \sin\theta) t - \frac{1}{2} g t^2$
(x_i, y_i)
If I aim at Bill,
then $\tan\theta = y_i / x_i$
$(0,0)$
θ

Anybody want to change his/her vote?

(A) Aim high, because the steel pellet is so much heavier than Mr. Bill, and will be pulled down more by gravity.

(B) Aim low, because Mr. Bill will be falling while the pellet travels.

(C) Aim directly for Mr. Bill. **This is clearly what you would do if gravity were absent.** The presence of gravity will affect Mr. Bill and the pellet in the same way, so aiming directly for Mr. Bill will result in a direct hit.

(D) How much below Mr. Bill you need to aim depends on the speed with which you fire the pellet, because the time that it takes the pellet to reach Mr. Bill will depend on how fast the pellet is shot.

Mr. Bill starts from rest at (x_i, y_i). Pellet starts at $(0, 0)$ with initial velocity $(v_i \cos\theta, v_i \sin\theta)$. Equations of motion:

$$x_{\text{bill}} = x_i$$

$$y_{\text{bill}} = y_i - \frac{1}{2}gt^2$$

$$x_{\text{pellet}} = v_i \cos\theta \ t$$

$$y_{\text{pellet}} = v_i \sin\theta \ t - \frac{1}{2}gt^2$$

When does pellet cross Mr. Bill's downward path?

$$x_{\text{pellet}} = x_{\text{bill}} \quad \Rightarrow \quad v_i \cos\theta \ t = x_i$$

$$t = \frac{x_i}{v_i \cos\theta}$$

Plugging in $t = \left(\frac{x_i}{v_i \cos\theta}\right)$:

$$x_{\text{bill}} = x_i$$

$$x_{\text{pellet}} = v_i \cos\theta \left(\frac{x_i}{v_i \cos\theta}\right) = x_i$$

$$y_{\text{bill}} = y_i - \frac{1}{2} g \left(\frac{x_i}{v_i \cos\theta}\right)^2$$

$$y_{\text{pellet}} = v_i \sin\theta \left(\frac{x_i}{v_i \cos\theta}\right) - \frac{1}{2} g \left(\frac{x_i}{v_i \cos\theta}\right)^2$$

What is vertical separation between Mr. Bill and the pellet at the instant when $x_{\text{pellet}} = x_{\text{bill}} = x_i$?

$$y_{\text{bill}} - y_{\text{pellet}} = y_i - v_i \sin\theta \left(\frac{x_i}{v_i \cos\theta}\right)$$

$$y_{\text{bill}} - y_{\text{pellet}} = y_i - x_i \tan\theta = y_i - y_i = 0$$

Anybody want to change his/her vote?

(A) Aim high, because the steel pellet is so much heavier than Mr. Bill, and will be pulled down more by gravity.

(B) Aim low, because Mr. Bill will be falling while the pellet travels.

(C) Aim directly for Mr. Bill. **This is clearly what you would do if gravity were absent.** The presence of gravity will affect Mr. Bill and the pellet in the same way, so aiming directly for Mr. Bill will result in a direct hit.

(D) How much below Mr. Bill you need to aim depends on the speed with which you fire the pellet, because the time that it takes the pellet to reach Mr. Bill will depend on how fast the pellet is shot.

Oh noooo ...

https://en.wikipedia.org/wiki/Mr._Bill

Physics 8 — Wednesday, October 2, 2019

From a height h above the ground, I throw a ball with an initial velocity that is nonzero only in the horizontal direction: $v_{xi} > 0$, $v_{yi} = 0$. How do I determine how long it takes to reach the ground?

(A) $h + v_{xi}t = 0$

(B) $h + v_{xi}t - \frac{1}{2}gt^2 = 0$

(C) $h + v_{yi}t = 0$

(D) $h - \frac{1}{2}gt^2 = 0$

From a height h above the ground, I throw a ball with an initial velocity that is nonzero only in the horizontal direction: $v_{xi} > 0$, $v_{yi} = 0$. If the ball's initial x coordinate is $x_i = 0$, how do I determine the x coordinate where the ball hits the ground?

(A) $x_f = h + v_{xi}t - \frac{1}{2}gt^2$, with t given on the previous page

(B) $x_f = h + v_{xi}t - \frac{1}{2}gt^2$, with $t = 0$

(C) $x_f = x_i + v_{xi}t$, with t given on the previous page

(D) $x_f = x_i + v_{xi}t$, with $t = 0$

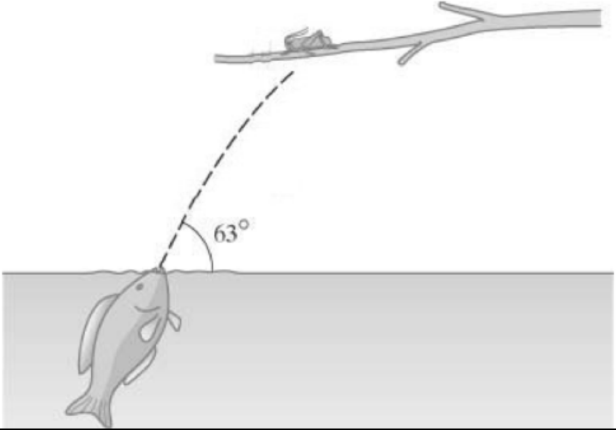
(E) $y_f = y_i + v_{xi}t$, with t given on the previous page

(F) $y_f = y_i + v_{xi}t$, with $t = 0$

From a height h above the ground, I throw a ball with an initial velocity that is nonzero only in the horizontal direction: $v_{xi} > 0$, $v_{yi} = 0$. How do I determine the x and y coordinates of the ball's velocity, v_x and v_y, at the instant before the ball hits the ground?

We stopped before we got here.

6. The archer fish shown in the figure, peering from just below the surface of the water, spits a drop of water at the grasshopper and knocks it into the water. The grasshopper's initial position is 0.45 m above the water surface and 0.25 m horizontally away from the fish's mouth. If the launch angle of the drop of water is 63° with respect to the horizontal water surface, how fast is the drop moving when it leaves the fish's mouth?



Physics 8 — Friday, October 4, 2019

Let's quickly revisit free-body diagrams in 1D

You push on a crate, and it starts to move but you don't. Draw a free-body diagram for you and one for the crate. Then use the diagrams and Newton's third law of motion to explain why the crate moves but you don't.

(A) The force I exert on the crate is larger than the force the crate exerts on me.

(B) The crate's force on me is equal and opposite to my force on the crate. The frictional force between my shoes and the floor is equal in magnitude to the crate's push on me, while the frictional force between the crate and the floor is smaller than my push on the crate.

(C) The crate and I exert equal and opposite forces on each other, but I don't move because I am much more massive than the crate.

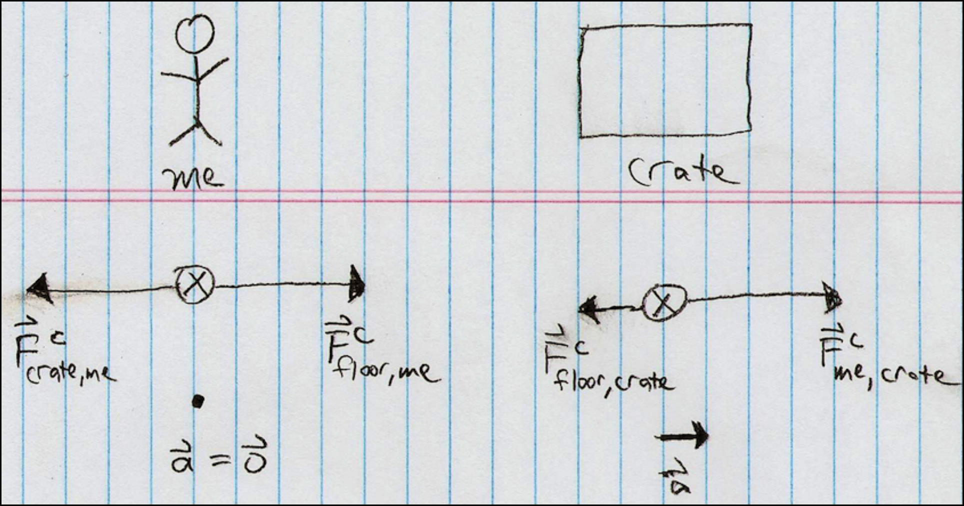
(free-body diagrams in one dimension)

If the crate and I were both standing on an ice rink, then it seems clear that we would both start to move. If the crate and I were both bolted to the floor, then it seems clear that neither one of us would start to move. So the grip of the floor's friction on my feet must be greater in magnitude than the grip of the floor's friction on the crate.

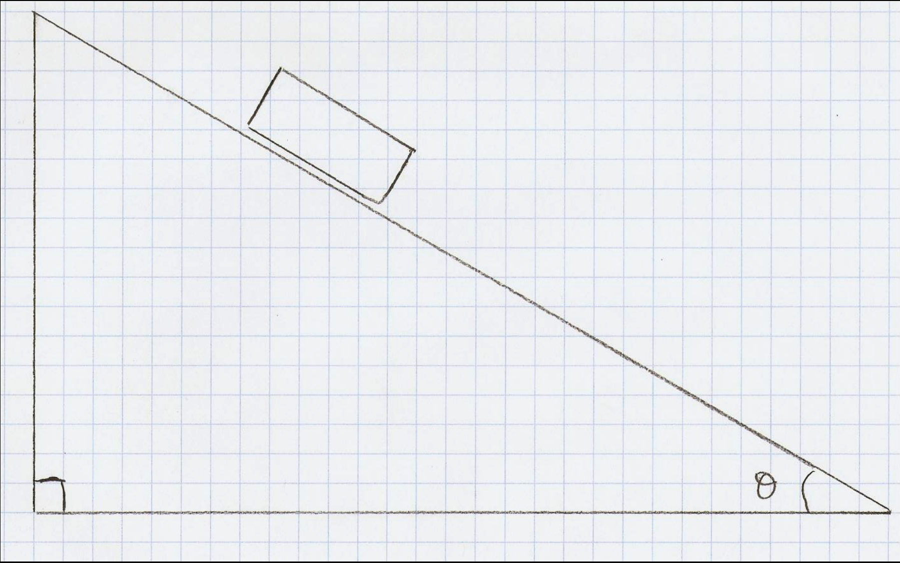
Let's say that I push to the right on the crate with a force $\vec{F}_{\mathrm{me,crate}}$, so the crate pushes to the left on me with a force $\vec{F}_{\mathrm{crate,me}} = -\vec{F}_{\mathrm{me,crate}}$. Meanwhile, the floor pushes to the right on me with a force $\vec{F}_{\mathrm{floor,me}}$, and the floor pushes (by a smaller amount) to the left on the crate with a force $\vec{F}_{\mathrm{floor,crate}}$.

It is reasonable that $|\vec{F}_{\mathrm{floor,crate}}| < |\vec{F}_{\mathrm{floor,me}}|$, because the bottom of the crate is wood, while the soles of my shoes are rubber.

(free-body diagrams in one dimension)

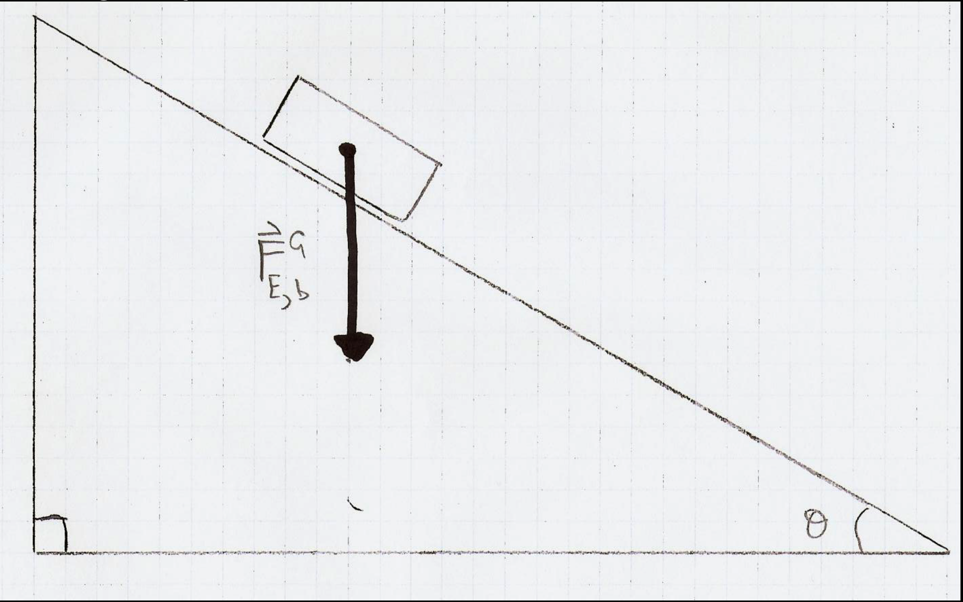


Block sliding down inclined plane: try drawing free-body diagram. Suppose some kinetic friction is present, but block still accelerates downhill. Try drawing this with a neighbor, one step ahead of me.



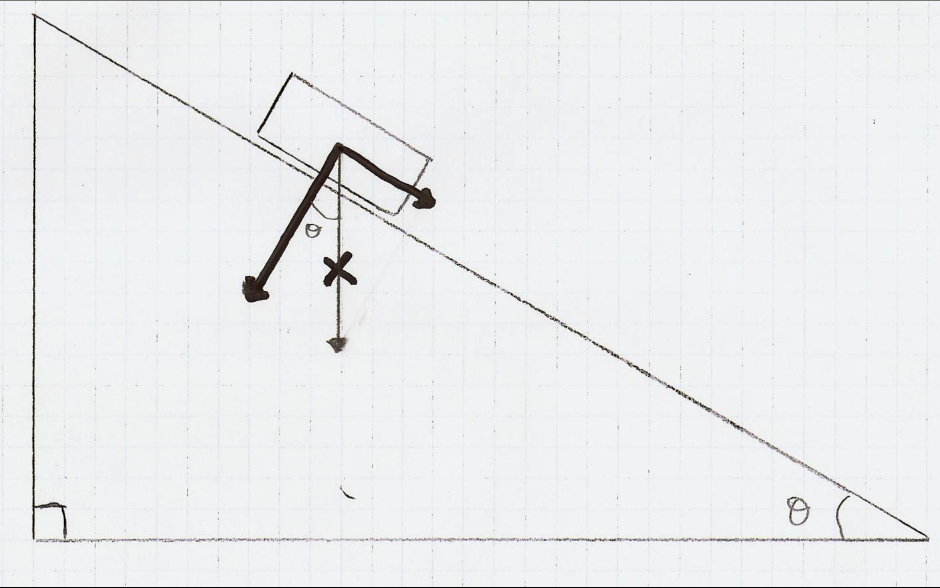
First: let's draw $\vec{F}^{G}_{E,b}$ for gravity.

Add gravity vector



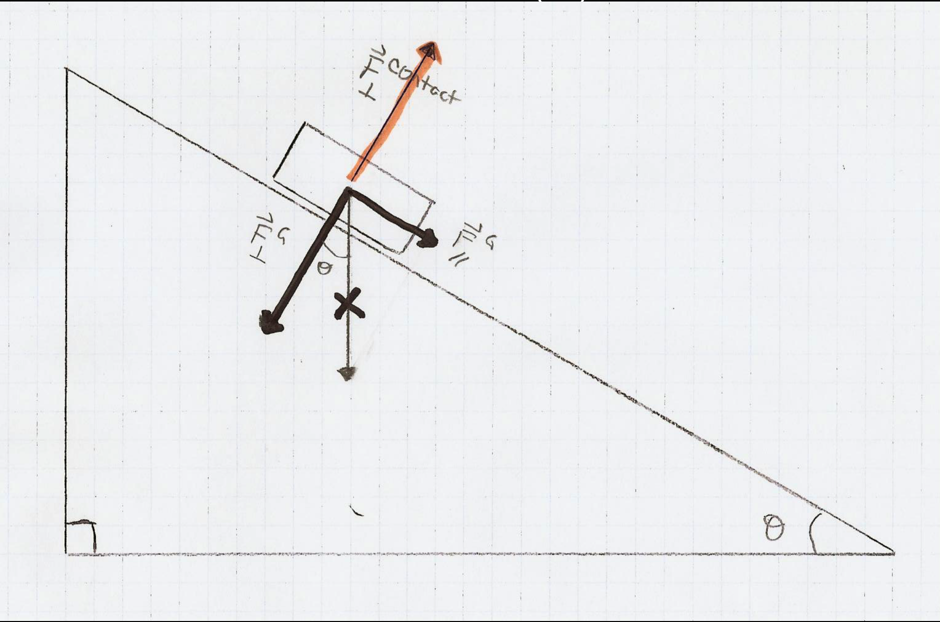
Next decompose $\vec{F}^G_{E,b}$ into components $\parallel$ and $\perp$ to surface.

Decompose gravity vector: $\parallel$ and $\perp$ to surface



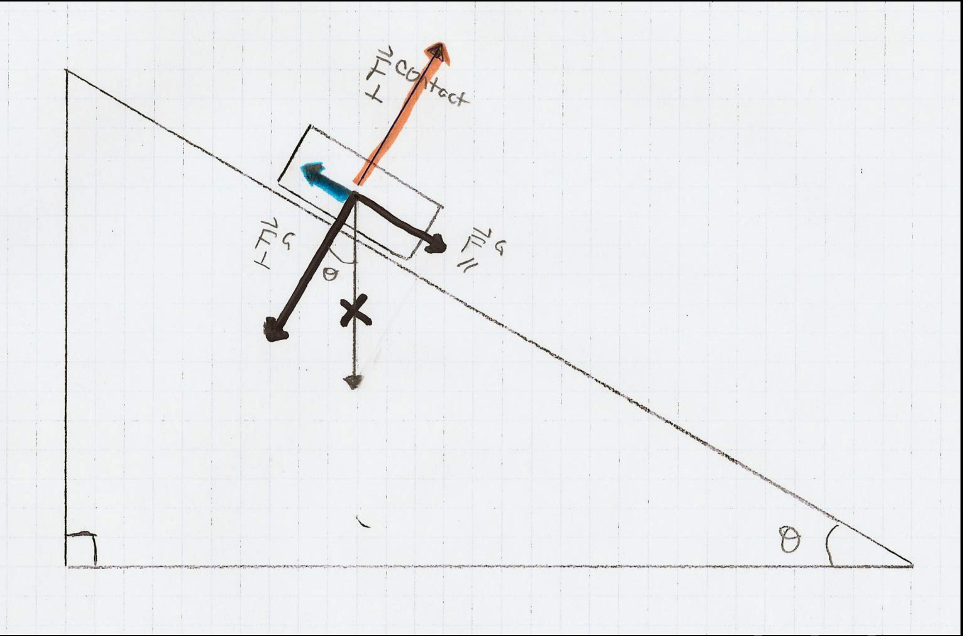
Next: add contact force "normal" ($\perp$) to surface.

Now add contact force "normal" (⊥) to surface



Next: add friction.

Now add friction (∥ to surface, opposing *relative* motion)



The block shown in this free-body diagram is

(A) at rest.

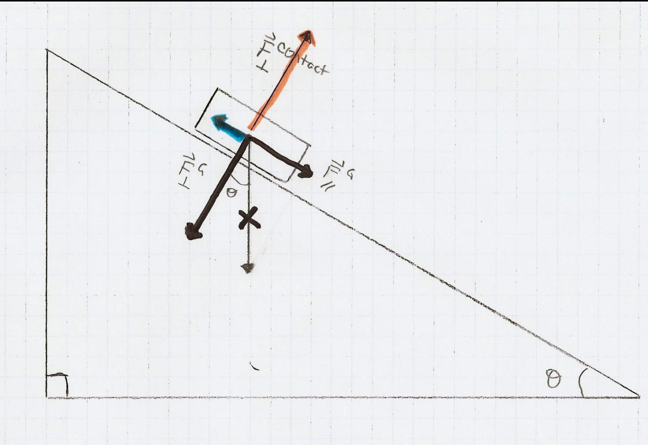
(B) sliding downhill at constant speed.

(C) sliding downhill and speeding up.

(D) sliding downhill and slowing down.

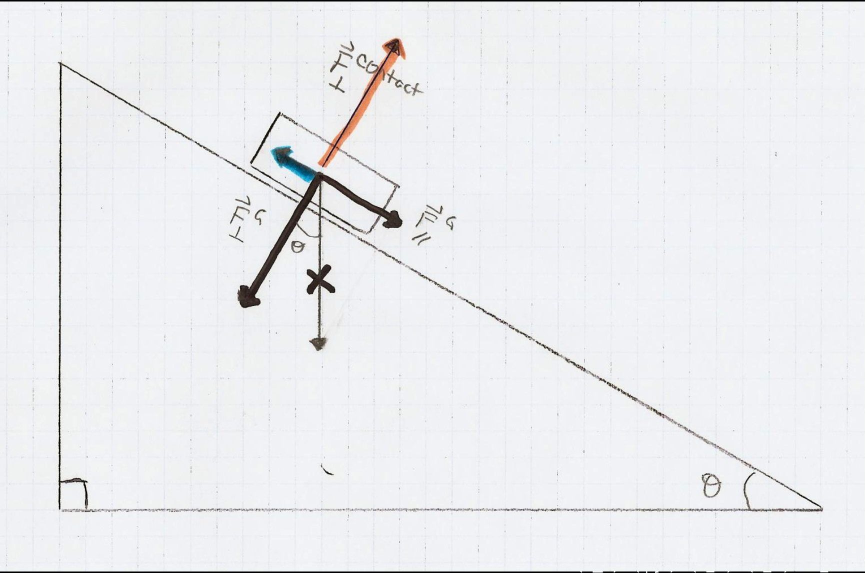
(E) sliding uphill and speeding up.

(F) sliding uphill and slowing down.



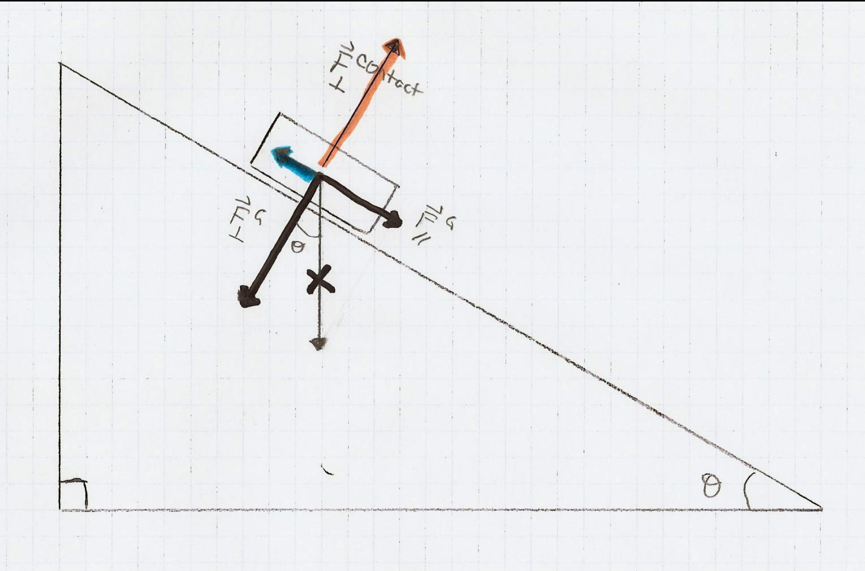
How would I change this free-body diagram . . .

if the block were at rest?



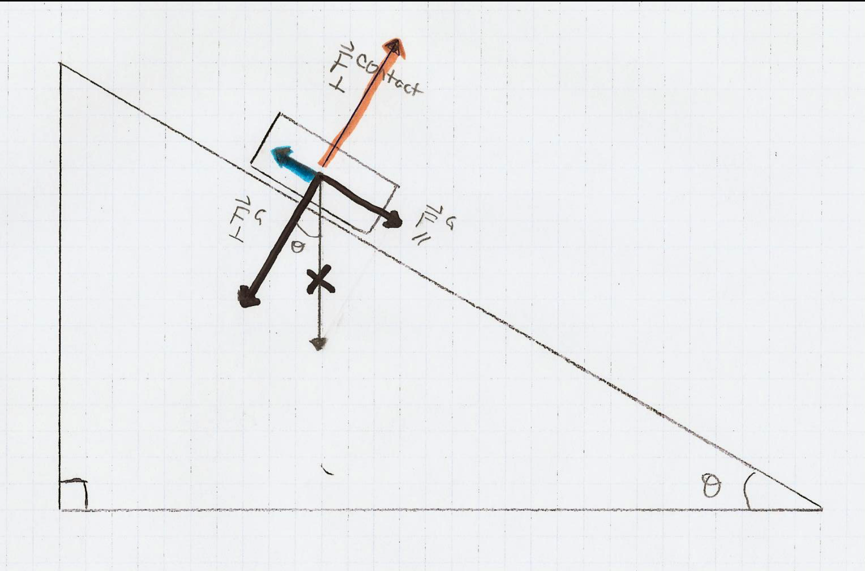
How would I change this free-body diagram . . .

if the block were sliding downhill at constant speed?



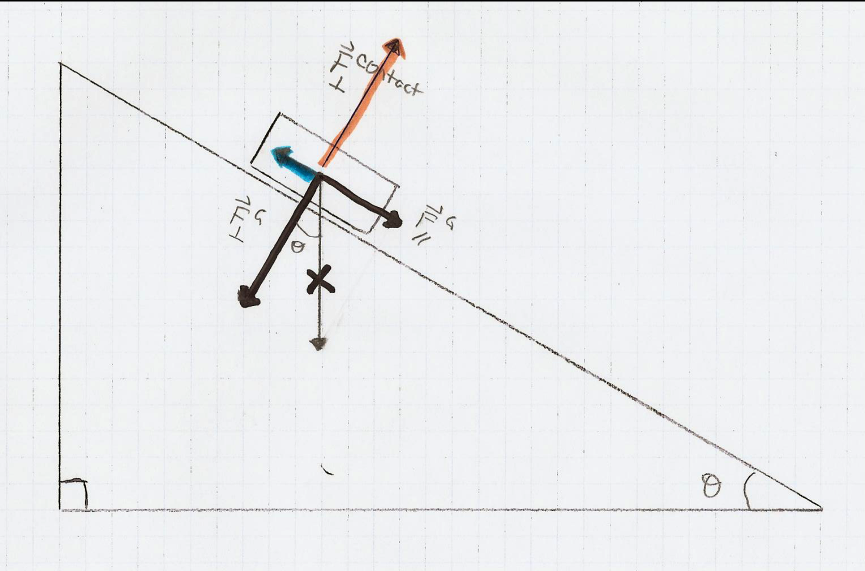
How would I change this free-body diagram . . .

if the block were sliding downhill and slowing down?



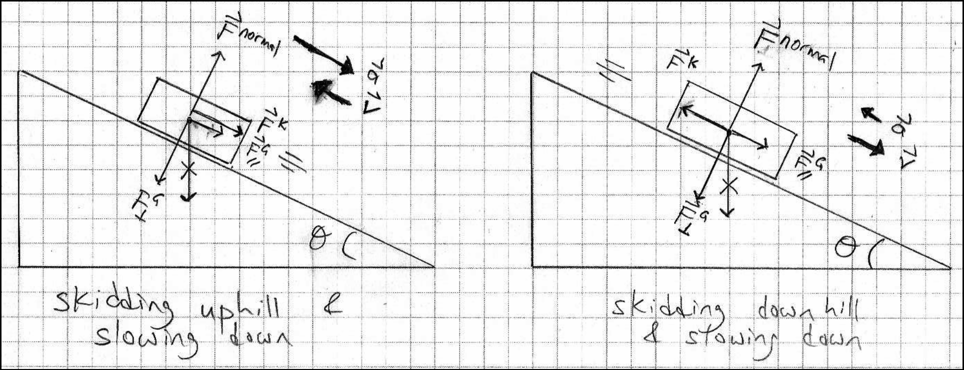
How would I change this free-body diagram . . .

if the block were sliding uphill and slowing down?



Another Chapter 10 reading question:

You've slammed on the brakes, and your car is skidding to a stop on a steep and slippery winter road. Other things being equal, will the car come to rest more quickly if it is traveling uphill or if it is traveling downhill? Why? (Consider FBD for each case.)



(We stopped on this page. Let's look again at FBDs then go on.)

A Ch10 problem that may not fit into HW6

The coefficient of static friction of tires on ice is about 0.10. (a) What is the steepest driveway on which you could park under those circumstances? (b) Draw a free-body diagram for the car when it is parked (successfully) on an icy driveway that is just a tiny bit less steep than this maximum steepness. [We might want to do (b) before we do (a).]

A Ch10 problem that may not fit into HW6

A fried egg of inertia m slides (at constant speed) down a Teflon frying pan tipped at an angle θ above the horizontal. (a) Draw the free-body diagram for the egg. Be sure to include friction.
(b) What is the "net force" (i.e. the vector sum of forces) acting on the egg? (c) How do these answers change if the egg is instead speeding up as it slides?

Physics 8 — Monday, October 7, 2019

If I gently step on my car's accelerator pedal, and the car starts to move faster (without any screeching sounds), the frictional force between the road and the rubber tire surface that causes my car to accelerate is

(A) static friction.

(B) kinetic friction.

(C) normal force.

(D) gravitational force.

(E) there is no frictional force between road and tire.

If I **slam down** on my car's accelerator pedal, and the car **screeches** forward noisily like a drag-race car, the frictional force between the road and the rubber tire surface that causes my car to accelerate is

(A) static friction.

(B) kinetic friction.

(C) normal force.

(D) gravitational force.

(E) there is no frictional force between road and tire.

Why do modern cars have anti-lock brakes?

(A) because the pumping action of the anti-lock brake mechanism keeps the brake pads from getting too hot.

(B) because pulsing the brakes on and off induces kinetic friction, which is preferable to static friction.

(C) because the cofficient of static friction is larger than the coefficient of kinetic friction, so you stop faster if your wheels roll on the ground than you would if your wheels were skidding on the ground.

(D) because the weird pulsating sensation you feel when the anti-lock brakes engage is fun and surprising!

Anti-Lock Brakes

(photo credit: Bill Berner)

Static friction and kinetic (sometimes confusingly called "sliding") friction:

$$F^{\mathrm{Static}} \leq \mu_S \, F^{\mathrm{Normal}}$$

$$F^{\mathrm{Kinetic}} = \mu_K \, F^{\mathrm{Normal}}$$

"normal" & "tangential" components are $\perp$ to and $\|$ to surface

Static friction is an example of what physicists call a "force of constraint" and engineers call a "reaction force." In most cases, you don't know its magnitude until you solve for the other forces in the problem and impose the condition that $\vec{a} = \vec{0}$. (An exception is if we're told that static friction "just barely holds on / just barely lets go," i.e. has its maximum possible value.)

TABLE 10.1
Coefficients of friction

Material1	Material 2	μ_s	μ_k
aluminum	aluminum	1.1–1.4	1.4
aluminum	steel	0.6	0.5
glass	glass	0.9–1.0	0.4
glass	nickel	0.8	0.6
ice	ice	0.1	0.03
oak	oak	0.6	0.5
rubber	concrete	1.0–4.0	0.8
steel	steel	0.8	0.4
steel	brass	0.5	0.4
steel	copper	0.5	0.4
steel	lead	0.95	0.95
wood	wood	0.25–0.5	0.2

The values given are for clean, dry, smooth surfaces.

TABLE 10.1
Coefficients of friction

Material1	Material 2	μ_s	μ_k
aluminum	aluminum	1.1–1.4	1.4
aluminum	steel	0.6	0.5
glass	glass	0.9–1.0	0.4
glass	nickel	0.8	0.6
ice	ice	0.1	0.03
oak	oak	0.6	0.5
rubber	concrete	1.0–4.0	0.8
steel	steel	0.8	0.4
steel	brass	0.5	0.4
steel	copper	0.5	0.4
steel	lead	0.95	0.95
wood	wood	0.25–0.5	0.2

The values given are for clean, dry, smooth surfaces.

- Steel on steel μ_K is about half that of rubber on concrete, and much less than that of μ_S for rubber on concrete.
- So a train can take a while to skid to a stop!
- Even more so if the tracks are wet: $\mu_K \approx 0.1$
- At $\mu = 0.1$ on level ground: 360 m to stop from 60 mph.
- At $\mu = 0.1$ on 6° slope: not possible to stop.

A car of mass 1000 kg travels at constant speed 20 m/s on dry, level pavement. The friction coeffs are $\mu_k = 0.8$ and $\mu_s = 1.2$. What is the **normal force** exerted by the road on the car?

(A) 1000 N downward

(B) 1000 N upward

(C) 1000 N forward

(D) 1000 N backward

(E) 9800 N downward

(F) 9800 N upward

(G) 11800 N downward

(H) 11800 N upward

A car of mass 1000 kg is traveling (in a straight line) at a constant speed of 20 m/s on dry, level pavement, with the cruise control engaged to maintain this speed. The friction coefficients are $\mu_k = 0.8$ and $\mu_s = 1.2$. The tires roll on the pavement without slipping. What is the frictional force exerted by the road on the car? (Let's use $g \approx 10 \text{ m/s}^2$ for simplicity here.)

(A) 8000 N backward

(B) 8000 N forward

(C) 8000 N upward

(D) 10000 N backward

(E) 10000 N forward

(F) 12000 N backward

(G) 12000 N forward

(H) It points forward, must have magnitude ≤ 12000 N, and has whatever value is needed to counteract air resistance.

A car of mass 1000 kg is initially traveling (in a straight line) at 20 m/s on dry, level pavement, when suddenly the driver jams on the (**non**-anti-lock) brakes, and the car skids to a stop with its wheels locked. The friction coefficients are $\mu_k = 0.8$ and $\mu_s = 1.2$. What is the frictional force exerted by the road on the car? (Let's use $g \approx 10\ \mathrm{m/s^2}$ for simplicity here.)

(A) 8000 N backward

(B) 8000 N forward

(C) 8000 N upward

(D) 10000 N backward

(E) 10000 N forward

(F) 12000 N backward

(G) 12000 N forward

(H) It points forward, must have magnitude $\leq 12000\ \mathrm{N}$, and has whatever value is needed to counteract air resistance.

Suppose that for rubber on dry concrete, $\mu_k = 0.8$ and $\mu_s = 1.2$. If a car of mass m traveling at initial speed v_i on a level road jams on its brakes and skids to a stop with its wheels locked, how do I solve for the length L of the skid marks? (Let's use $g \approx 10\ \mathrm{m/s^2}$ for simplicity here.)

(A) use $v_f^2 = v_i^2 + 2aL$ with $v_f = 0$ and $a = -2.0\ \mathrm{m/s^2}$

(B) use $v_f^2 = v_i^2 + 2aL$ with $v_f = 0$ and $a = -4.0\ \mathrm{m/s^2}$

(C) use $v_f^2 = v_i^2 + 2aL$ with $v_f = 0$ and $a = -6.0\ \mathrm{m/s^2}$

(D) use $v_f^2 = v_i^2 + 2aL$ with $v_f = 0$ and $a = -8.0\ \mathrm{m/s^2}$

(E) use $v_f^2 = v_i^2 + 2aL$ with $v_f = 0$ and $a = -10.0\ \mathrm{m/s^2}$

(F) use $v_f^2 = v_i^2 + 2aL$ with $v_f = 0$ and $a = -12.0\ \mathrm{m/s^2}$

(G) use $v_f^2 = v_i^2 + 2aL$ with $v_f = 0$ and $a = -14.0\ \mathrm{m/s^2}$

Suppose that for rubber tires on dry, level pavement, the friction coefficients are $\mu_k = 0.8$ and $\mu_s = 1.2$. If you assume that the forces between the ground and the tires are the same for all four tires (4-wheel drive, etc.), what is a car's maximum possible acceleration for this combination of tires and pavement? (Let's use $g \approx 10\ \mathrm{m/s^2}$ for simplicity here.)

(A) $1.0\ \mathrm{m/s^2}$

(B) $5.0\ \mathrm{m/s^2}$

(C) $8.0\ \mathrm{m/s^2}$

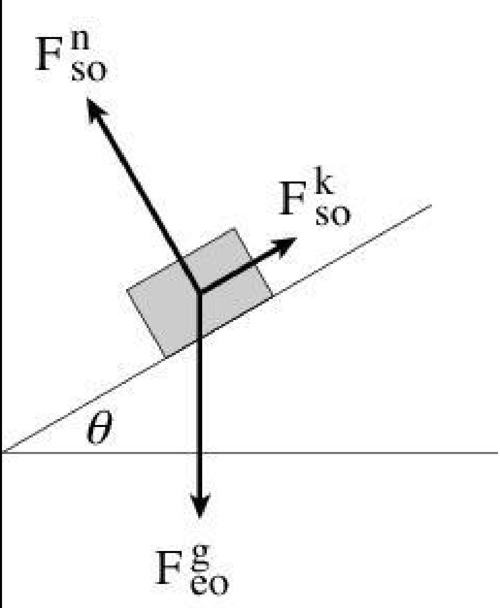
(D) $10.0\ \mathrm{m/s^2}$

(E) $12.0\ \mathrm{m/s^2}$

Physics 8 — Wednesday, October 9, 2019

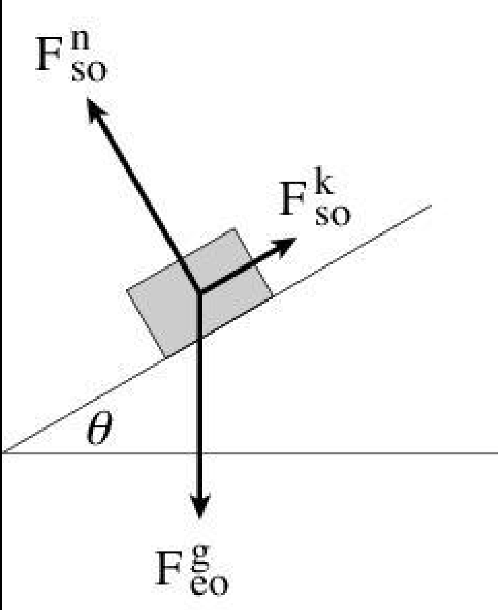
An object "O" of mass m slides down an inclined surface "S" at constant velocity. What is the magnitude of the (kinetic) **frictional force** F^k_{so} exerted by the surface on the object?

(A) $F^k_{so} = mg$
(B) $F^k_{so} = mg \sin\theta$
(C) $F^k_{so} = mg \cos\theta$
(D) $F^k_{so} = mg \tan\theta$
(E) $F^k_{so} = \mu_k mg$
(F) $F^k_{so} = \mu_k mg \sin\theta$
(G) $F^k_{so} = \mu_k mg \cos\theta$
(H) $F^k_{so} = \mu_k mg \tan\theta$



An object "O" of mass m slides down an inclined surface "S" at constant velocity. What is the magnitude of the **gravitational force** F^g_{eo} exerted by Earth on the object?

(A) $F^g_{eo} = mg$
(B) $F^g_{eo} = mg \sin\theta$
(C) $F^g_{eo} = mg \cos\theta$
(D) $F^g_{eo} = mg \tan\theta$
(E) $F^g_{eo} = \mu_k mg$
(F) $F^g_{eo} = \mu_k mg \sin\theta$
(G) $F^g_{eo} = \mu_k mg \cos\theta$
(H) $F^g_{eo} = \mu_k mg \tan\theta$



An object "O" of mass m slides down an inclined surface "S" at constant velocity. Let the x-axis point downhill. What is the magnitude of the **downhill (tangential) component** $F^g_{eo,x}$ of the gravitational force exerted by Earth on the object?

(A) $F^g_{eo,x} = mg$

(B) $F^g_{eo,x} = mg \sin\theta$

(C) $F^g_{eo,x} = mg \cos\theta$

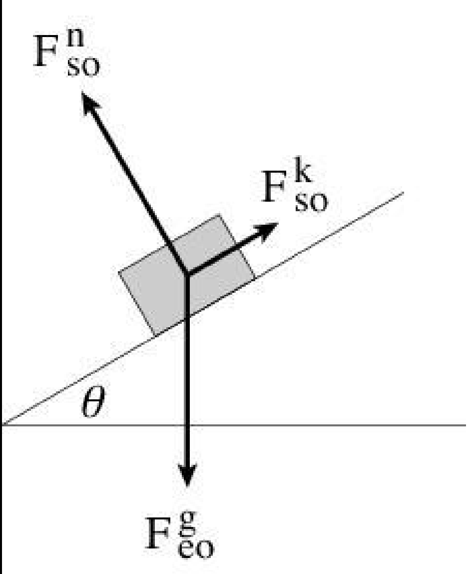
(D) $F^g_{eo,x} = mg \tan\theta$

(E) $F^g_{eo,x} = \mu_k mg$

(F) $F^g_{eo,x} = \mu_k mg \sin\theta$

(G) $F^g_{eo,x} = \mu_k mg \cos\theta$

(H) $F^g_{eo,x} = \mu_k mg \tan\theta$



An object "O" of mass m slides down an inclined surface "S" at constant velocity. What is the magnitude of the **normal force** F_{so}^n exerted by the surface on the object?

(A) $F_{so}^n = mg$

(B) $F_{so}^n = mg \sin\theta$

(C) $F_{so}^n = mg \cos\theta$

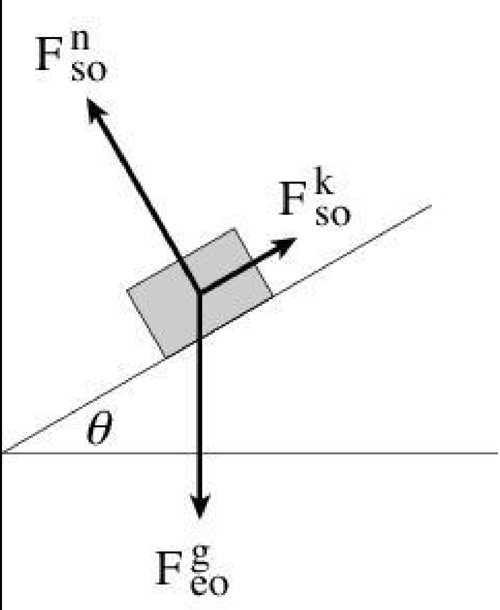
(D) $F_{so}^n = mg \tan\theta$

(E) $F_{so}^n = \mu_k mg$

(F) $F_{so}^n = \mu_k mg \sin\theta$

(G) $F_{so}^n = \mu_k mg \cos\theta$

(H) $F_{so}^n = \mu_k mg \tan\theta$



Since object "O" slides down surface "S" at constant velocity, the forces on O must sum vectorially to zero. How do I express this fact for the forces acting along the downhill (tangential) axis?

(A) $\mu_k mg = mg\cos\theta$

(B) $\mu_k mg = mg\sin\theta$

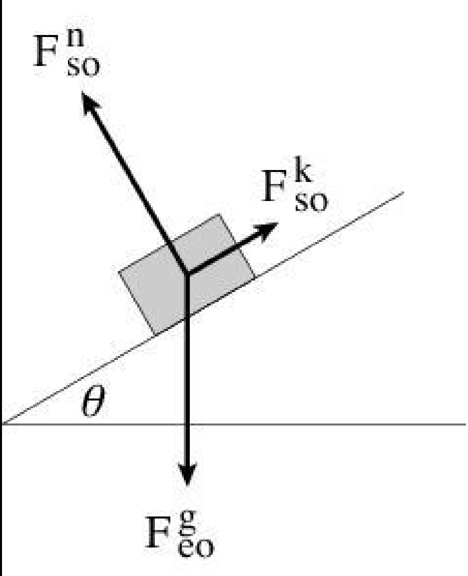
(C) $\mu_k mg\cos\theta = mg$

(D) $\mu_k mg\sin\theta = mg$

(E) $\mu_k mg\cos\theta = mg\sin\theta$

(F) $\mu_k mg\sin\theta = mg\cos\theta$

(G) $mg\sin\theta = mg\cos\theta$



Suppose friction holds object "O" at rest on surface "S." Which statement is true?

(A) $mg \sin\theta = F^s_{so} = \mu_k mg \cos\theta$

(B) $mg \sin\theta = F^s_{so} = \mu_s mg \cos\theta$

(C) $mg \sin\theta = F^s_{so} \leq \mu_k mg \cos\theta$

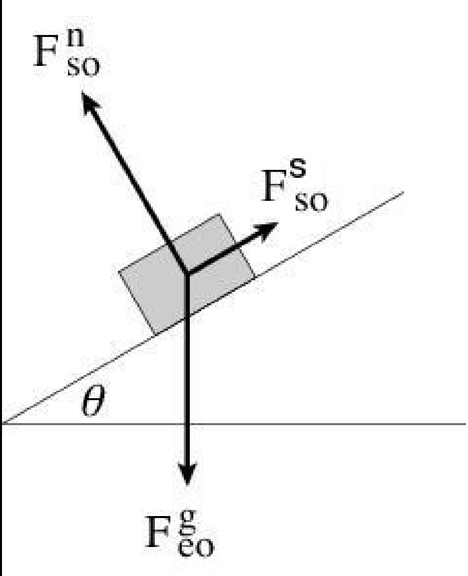
(D) $mg \sin\theta = F^s_{so} \leq \mu_s mg \cos\theta$

(E) $mg \cos\theta = F^s_{so} = \mu_k mg \sin\theta$

(F) $mg \cos\theta = F^s_{so} = \mu_s mg \sin\theta$

(G) $mg \cos\theta = F^s_{so} \leq \mu_k mg \sin\theta$

(H) $mg \cos\theta = F^s_{so} \leq \mu_s mg \sin\theta$



Suppose friction holds object "O" at rest on surface "S." Then I gradually increase θ until the block just begins to slip. Which statement is true at the instant when the block starts slipping?

(A) $mg \sin\theta = F^s_{so} = \mu_k mg \cos\theta$

(B) $mg \sin\theta = F^s_{so} = \mu_s mg \cos\theta$

(C) $mg \sin\theta = F^s_{so} \leq \mu_k mg \cos\theta$

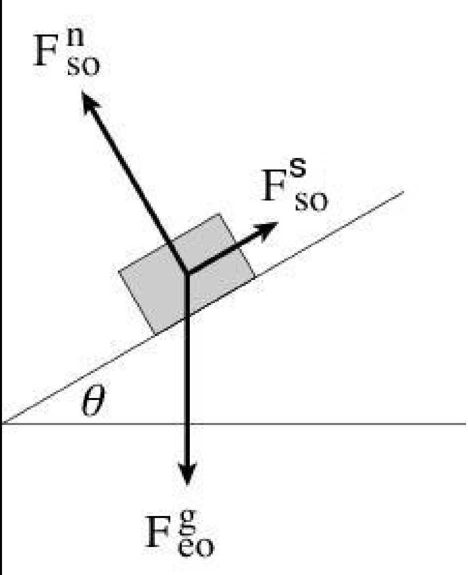
(D) $mg \sin\theta = F^s_{so} \leq \mu_s mg \cos\theta$

(E) $mg \cos\theta = F^s_{so} = \mu_k mg \sin\theta$

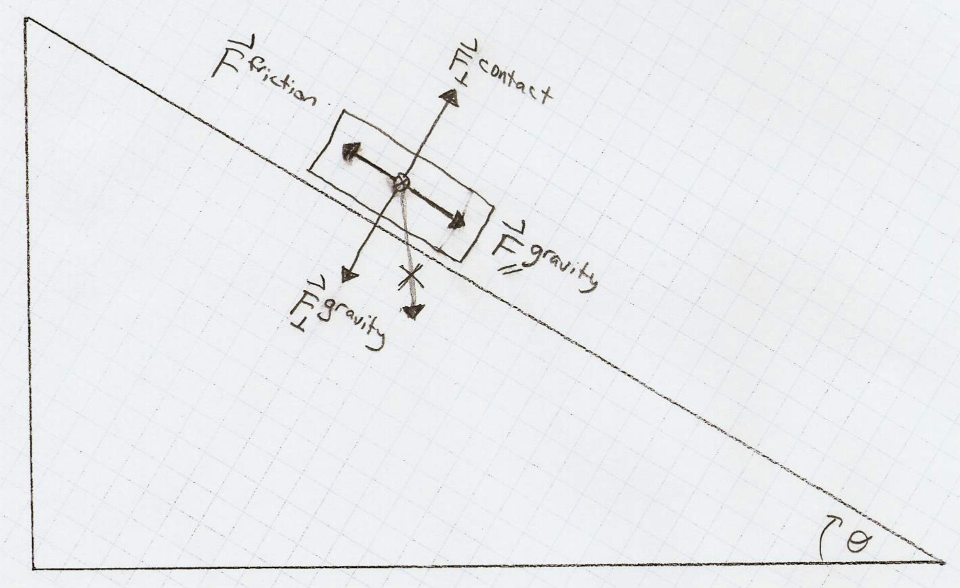
(F) $mg \cos\theta = F^s_{so} = \mu_s mg \sin\theta$

(G) $mg \cos\theta = F^s_{so} \leq \mu_k mg \sin\theta$

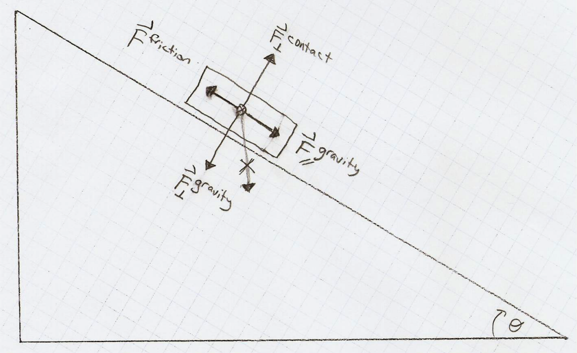
(H) $mg \cos\theta = F^s_{so} \leq \mu_s mg \sin\theta$



Friction on inclined plane



Why do I "cross off" the downward gravity arrow?



Take x-axis to be downhill, y-axis to be upward $\perp$ from surface.

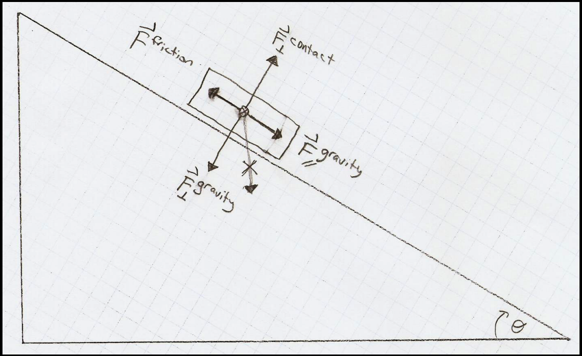
$$\vec{F}^G_{\perp} = -mg\cos\theta\,\hat{j}, \qquad \vec{F}^N = +mg\cos\theta\,\hat{j}$$

$$\vec{F}^G_{\parallel} = +mg\sin\theta\,\hat{i}$$

If block is not sliding then friction balances downhill gravity:

$$\vec{F}^S = -mg\sin\theta\,\hat{i}$$

(I'll skip this slide, but it's here for reference.)



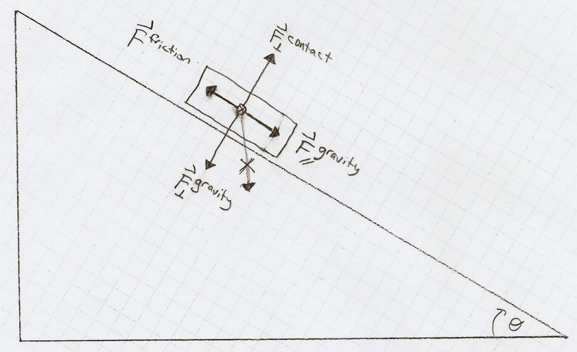
Magnitude of "normal" force ("normal" is a synonym for "perpendicular") between surfaces is

$$F^N = mg\cos\theta$$

Magnitude of static friction must be less than maximum:

$$F^S \ \le \ \mu_S F^N = \mu_S \ mg\cos\theta$$

Block begins sliding when downhill component of gravity equals maximum magnitude of static friction ...



Block begins sliding when downhill component of gravity equals maximum magnitude of static friction:

$$\mu_S \; mg \cos\theta = mg \sin\theta$$

$$\mu_S = \frac{mg \sin\theta}{mg \cos\theta}$$

$$\mu_S = \tan\theta$$

A Ch10 problem that may not fit into HW6

The coefficient of static friction of tires on ice is about 0.10. (a) What is the steepest driveway on which you could park under those circumstances? (b) Draw a free-body diagram for the car when it is parked (successfully) on an icy driveway that is just a tiny bit less steep than this maximum steepness. [We might want to do (b) before we do (a).]

Answering part (a) starts by expressing (in math) which statement:

(A) (total gravitational force on car) equals (kinetic friction)

(B) (total gravitational force on car) equals (largest possible value of static friction)

(C) (downhill component of gravity) equals (kinetic friction)

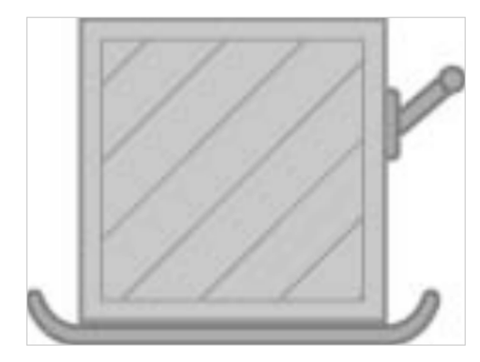
(D) (downhill component of gravity) equals (largest possible value of static friction)

A heavy crate has plastic skid plates beneath it and a tilted handle attached to one side. Which requires a smaller force (directed along the diagonal rod of the handle) to move the box? Why?

(A) Pushing the crate is easier than pulling.

(B) Pulling the crate is easier than pushing.

(C) There is no difference.



find tension in rope

Step two: what is the frictional force exerted by the floor on the box (which is sliding across the floor at constant speed)?

(A) $F^K = \mu_K(mg - T\sin\theta)$
(B) $F^K = \mu_K(mg - T\cos\theta)$
(C) $F^K = \mu_S(mg - T\sin\theta)$
(D) $F^K = \mu_S(mg - T\cos\theta)$
(E) $F^K = (mg - T\sin\theta)$
(F) $F^K = (mg - T\cos\theta)$

We stopped here.

A Ch10 problem that may not fit into HW6

Calculate $\vec{C} \cdot (\vec{B} - \vec{A})$ if $\vec{A} = 3.0\hat{\imath} + 2.0\hat{\jmath}$, $\vec{B} = 1.0\hat{\imath} - 1.0\hat{\jmath}$, and $\vec{C} = 2.0\hat{\imath} + 2.0\hat{\jmath}$. Remember that there are two ways to compute a dot product—choose the easier method in a given situation: one way is $\vec{P} \cdot \vec{Q} = |\vec{P}||\vec{Q}| \cos\varphi$, where φ is the angle between vectors $\vec{P}$ and $\vec{Q}$, and the other way is $\vec{P} \cdot \vec{Q} = P_x Q_x + P_y Q_y$.

A Ch10 problem that may not fit into HW6

A child rides her bike 1.0 block east and then $\sqrt{3} \approx 1.73$ blocks north to visit a friend. It takes her 10 minutes, and each block is 60 m long. What are (a) the magnitude of her displacement, (b) her average velocity (magnitude and direction), and (c) her average speed?

A Ch10 problem that may not fit into HW6

A fried egg of inertia m slides (at constant speed) down a Teflon frying pan tipped at an angle θ above the horizontal. [This only works if the angle θ is just right.] (a) Draw the free-body diagram for the egg. Be sure to include friction. (b) What is the "net force" (i.e. the vector sum of forces) acting on the egg? (c) How do these answers change if the egg is instead speeding up as it slides?

Physics 8 — Monday, October 14, 2019

Example (tricky!) problem

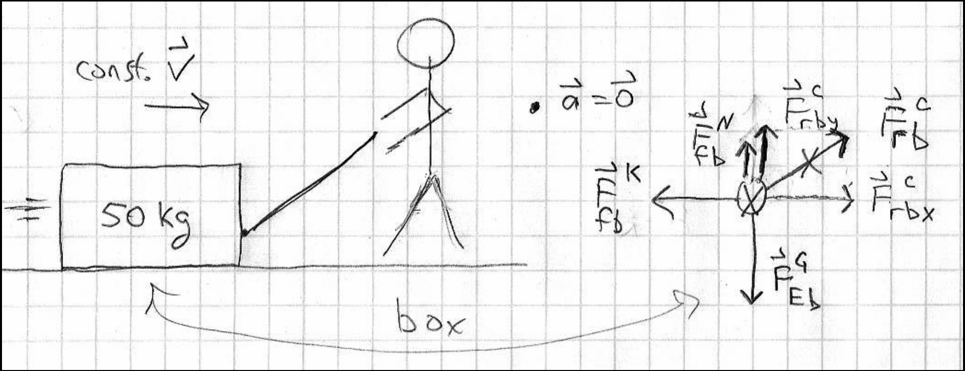
A woman applies a constant force to pull a 50 kg box across a floor **at constant speed**. She applies this force by pulling on a rope that makes an angle of 37° above the horizontal. The friction coefficient between the box and the floor is $\mu_k = 0.10$.

(a) Find the tension in the rope.

(b) How much work does the woman do in moving the box 10 m?

free-body diagram for box

What are all of the forces acting on the box? Try drawing your own FBD for the box. It's tricky!



(I should redraw the RHS of this diagram on the board.)

find tension in rope

Step one: If T is the tension in the rope, then what is the normal force (by floor on box)?

(A) $F^N = mg$
(B) $F^N = mg + T\cos\theta$
(C) $F^N = mg + T\sin\theta$
(D) $F^N = mg - T\cos\theta$
(E) $F^N = mg - T\sin\theta$

find tension in rope

Step two: what is the frictional force exerted by the floor on the box (which is sliding across the floor at constant speed)?

(A) $F^K = \mu_K(mg - T\sin\theta)$
(B) $F^K = \mu_K(mg - T\cos\theta)$
(C) $F^K = \mu_S(mg - T\sin\theta)$
(D) $F^K = \mu_S(mg - T\cos\theta)$
(E) $F^K = (mg - T\sin\theta)$
(F) $F^K = (mg - T\cos\theta)$

find tension in rope

Step three: how do I use the fact that the box is moving at constant velocity (and hence is not accelerating)?

(A) $T = F^K = \mu_K(mg - T\sin\theta)$
(B) $T\cos\theta = F^K = \mu_K(mg - T\sin\theta)$
(C) $T\sin\theta = F^K = \mu_K(mg - T\sin\theta)$

solution (part a): find tension in rope

Force by rope on box has upward vertical component $T\sin\theta$. So the normal force (by floor on box) is $\boxed{F^N = mg - T\sin\theta}$.

Force of friction is $\boxed{F^K = \mu_K(mg - T\sin\theta)}$. To keep box sliding at constant velocity, horizontal force by rope on box must balance F^K.

$$\boxed{T\cos\theta = F^K = \mu_K(mg - T\sin\theta)} \quad \Rightarrow \quad T = \frac{\mu_K mg}{\cos\theta + \mu_K \sin\theta}$$

This reduces to familiar $T = \mu_K mg$ if $\theta = 0^\circ$ (pulling horizontally) and even reduces to a sensible $T = mg$ if $\theta = 90^\circ$ (pulling vertically).

Plugging in $\theta = 37^\circ$, so $\cos\theta = 4/5 = 0.80$, $\sin\theta = 3/5 = 0.60$,

$$T = \frac{(0.10)(50\ \mathrm{kg})(9.8\ \mathrm{m/s^2})}{(0.80) + (0.10)(0.60)} = 57\ \mathrm{N}$$

solution (part b): work done by pulling for 10 meters

In part (a) we found tension in rope is $T = 57\ \mathrm{N}$ and is oriented at an angle $\theta = 36.9^\circ$ above the horizontal.

In 2D, work is displacement times **component of force along direction of displacement** (which is horizontal in this case). So the work done by the rope on the box is

$$W = \vec{F}_{rb} \cdot \Delta\vec{r}_b$$

This is the dot product (or "scalar product") of the force $\vec{F}_{rb}$ (by rope on box) with the displacement $\Delta\vec{r}_b$ of the point of application of the force.

In part (a) we found tension in rope is $T = 57\ \mathrm{N}$ and is oriented at an angle $\theta = 36.9^\circ$ above the horizontal.

What is the work done by the rope on the box by pulling the box across the floor for 10 meters? (Assume my arithmetic is correct.)

(In two dimensions, work is the dot product of the force $\vec{F}_{rb}$ with the displacement $\Delta\vec{r}_b$ of the point of application of the force.)

(A) $W = (10\ \mathrm{m})(T) = (10\ \mathrm{m})(57\ \mathrm{N}) = 570\ \mathrm{J}$
(B) $W = (10\ \mathrm{m})(T\cos\theta) = (10\ \mathrm{m})(57\ \mathrm{N})(0.80) = 456\ \mathrm{J}$
(C) $W = (10\ \mathrm{m})(T\sin\theta) = (10\ \mathrm{m})(57\ \mathrm{N})(0.60) = 342\ \mathrm{J}$
(D) $W = (8.0\ \mathrm{m})(T\cos\theta) = (8.0\ \mathrm{m})(57\ \mathrm{N})(0.80) = 365\ \mathrm{J}$
(E) $W = (8.0\ \mathrm{m})(T\sin\theta) = (8.0\ \mathrm{m})(57\ \mathrm{N})(0.60) = 274\ \mathrm{J}$

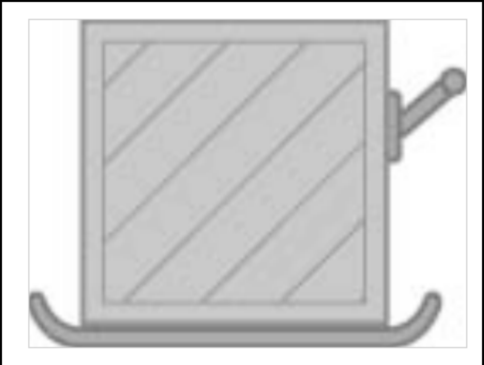
Repeat, now that we've analyzed this quantitatively

A heavy crate has plastic skid plates beneath it and a tilted handle attached to one side. Which requires a smaller force (directed along the diagonal rod of the handle) to move the box? Why?

(A) Pushing the crate is easier than pulling.

(B) Pulling the crate is easier than pushing.

(C) There is no difference.



Easier example (quickly, or skip)

How hard do you have to push a 1000 kg car (with brakes on, all wheels, on level ground) to get it to start to slide? Let's take $\mu_S \approx 1.2$ for rubber on dry pavement.

$$F^{\mathrm{Normal}} = mg = 9800\ \mathrm{N}$$

$$F^{\mathrm{Static}} \leq \mu_S F^N = (1.2)(9800\ \mathrm{N}) \approx 12000\ \mathrm{N}$$

So the static friction gives out (hence car starts to slide) when your push exceeds 12000 N.

How hard do you then have to push to keep the car sliding at constant speed? Let's take $\mu_K \approx 0.8$ for rubber on dry pavement.

$$F^{\mathrm{Kinetic}} = \mu_K F^N = (0.8)(9800\ \mathrm{N}) \approx 8000\ \mathrm{N}$$

How far does your car slide on dry, level pavement if you jam on the brakes, from 60 mph (27 m/s)?

$$F^N = mg, \quad F^K = \mu_K mg$$

$$a = ? \qquad \Delta x = ?$$

(The math is worked out on the next slides, but we won't go through them in detail. It's there for you to look at later.)

How far does your car slide on dry, level pavement if you jam on the brakes, from 60 mph (27 m/s)?

$$F^N = mg, \quad F^K = \mu_K mg$$

$$a = -F^K/m = -\mu_K g = -(0.8)(9.8\ \mathrm{m/s^2}) \approx -8\ \mathrm{m/s^2}$$

Constant force $\rightarrow$ constant acceleration from 27 m/s down to zero:

$$v_f^2 = v_i^2 + 2ax$$

$$x = \frac{v_i^2}{-2a} = \frac{(27\ \mathrm{m/s})^2}{2 \times (8\ \mathrm{m/s^2})} \approx 45\ \mathrm{m}$$

How much time elapses before you stop?

$$v_f = v_i + at \quad \Rightarrow \quad t = \frac{27\ \mathrm{m/s}}{8\ \mathrm{m/s^2}} = 3.4\ \mathrm{s}$$

How does this change if you have anti-lock brakes (or good reflexes) so that the tires never skid? Remember $\mu_S > \mu_K$. For rubber on dry pavement, $\mu_S \approx 1.2$ (though there's a wide range) and $\mu_K \approx 0.8$. The best you can do is *maximum* static friction:

$$F^S \leq \mu_S mg$$

$$a = -F^S/m = -\mu_S g = -(1.2)(9.8\ \mathrm{m/s^2}) \approx -12\ \mathrm{m/s^2}$$

Constant force $\rightarrow$ constant acceleration from 27 m/s down to zero:

$$v_f^2 = v_i^2 + 2ax$$

$$x = \frac{v_i^2}{-2a} = \frac{(27\ \mathrm{m/s})^2}{2 \times (12\ \mathrm{m/s^2})} \approx 30\ \mathrm{m}$$

How much time elapses before you stop?

$$v_f = v_i + at \quad \Rightarrow \quad t = \frac{27\ \mathrm{m/s}}{15\ \mathrm{m/s^2}} = 2.2\ \mathrm{s}$$

So you can stop in about 2/3 the time (and 2/3 the distance) if you don't let your tires skid. Or whatever μ_K/μ_S ratio is.

video segment break

- begin video preceding ws13 = mz11

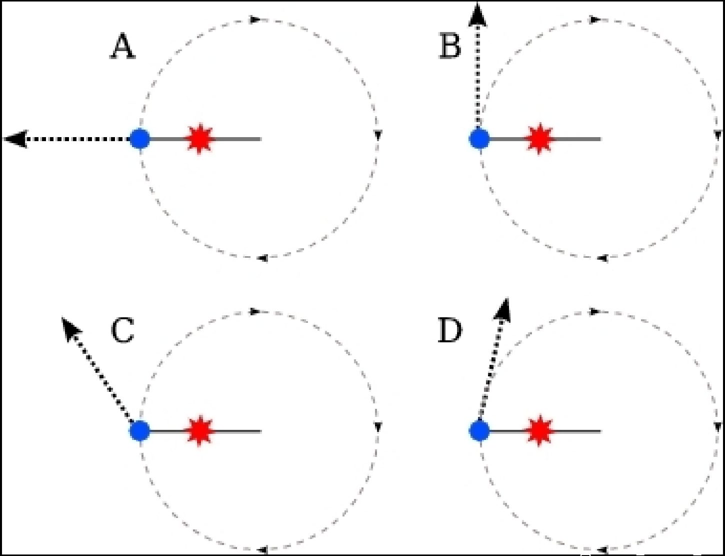
Chapter 11: motion in a circle

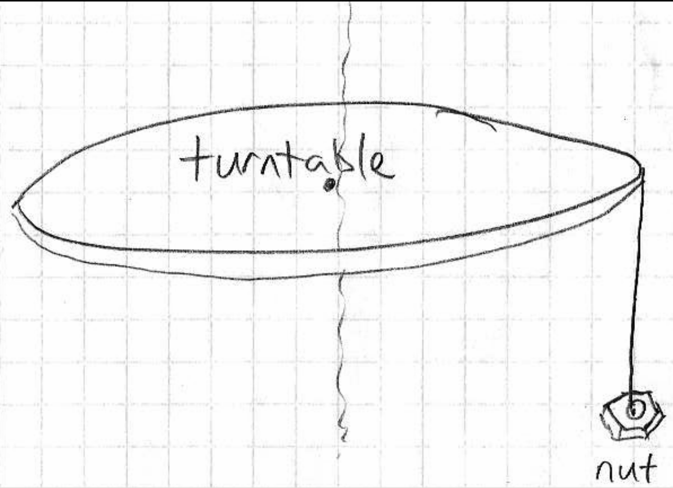
- If you go around in a circle at constant speed, your velocity vector is always changing direction.
- A change in velocity (whether magnitude, direction, or both) requires acceleration.
- For motion in a circle of radius R at constant speed v

$$a = \frac{v^2}{R}$$

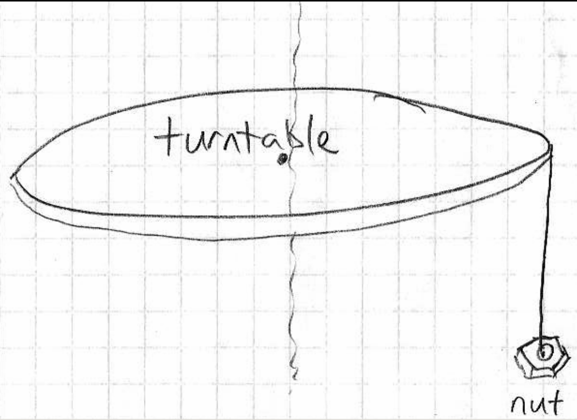
- This is called **centripetal acceleration**, and points toward the center of the circle.
- In the absence of a force (i.e. if vector sum of forces (if any) is zero), there is no acceleration, hence no change in velocity.

You are looking down (plan view) as I spin a (blue) ball on a string above my head in a circle at constant speed. The string breaks at the instant shown below. Which picture depicts the subsequent motion of the ball?





If the nut has mass m and the turntable is sitting idle, what is the tension in the string?

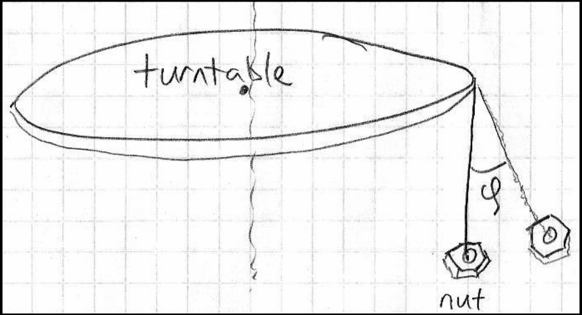


What will happen when I start the turntable spinning?

(A) The nut will continue to hang down vertically.

(B) The nut will move inward somewhat, making some angle φ w.r.t. the vertical axis.

(C) The nut will move outward somewhat, making some angle φ w.r.t. the vertical axis.



What will happen if I spin the turntable faster? Let T be the tension in the string.

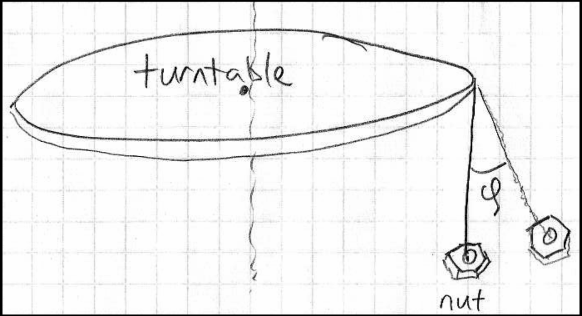
(A) The nut will move farther outward. $T \sin\varphi$ provides the centripetal force mv/R^2, while $T\cos\varphi$ balances gravity mg.

(B) The nut will move farther outward. $T\cos\varphi$ provides the centripetal force mv/R^2, while $T\sin\varphi$ balances gravity mg.

(C) The nut will move farther outward. $T\sin\varphi$ provides the centripetal force mv^2/R, while $T\cos\varphi$ balances gravity mg.

(D) The nut will move farther outward. $T\cos\varphi$ provides the centripetal force mv^2/R, while $T\sin\varphi$ balances gravity mg.

Physics 8 — Wednesday, October 16, 2019



$$m\vec{a}_{\text{nut}} = \vec{F}^{\text{tension}}_{s,\text{nut}} + \vec{F}^{G}_{E,\text{nut}}$$

$$0 = ma_y = T\cos\varphi - mg$$

$$T\cos\varphi = mg$$

$$-\frac{mv^2}{R} = ma_x = -T\sin\varphi$$

$$\frac{T\sin\varphi}{T\cos\varphi} = \tan\varphi = \frac{mv^2/R}{mg} = \frac{v^2}{gR}$$

Now suppose that friction provides the centripetal force

Suppose that a highway offramp that I often use bends with a radius of 20 meters. I notice that my car tires allow me (in good weather) to take this offramp at 15 m/s without slipping. How large does the offramp's bending radius need to be for me to be able to make the turn at 30 m/s instead?

(Assume that the frictional force between the road and my tires is the same in both cases and that the offramp is level (horizontal), i.e. not "banked.")

(A) 5 meters
(B) 10 meters
(C) 20 meters
(D) 30 meters
(E) 40 meters
(F) 80 meters

Now suppose that friction provides the centripetal force

- If velocity gets too large, penny flies off of turntable, as friction is no longer large enough to hold it in place.
- What if there are several pennies placed on the turntable at several different radii?
- As I slowly increase the speed at which the turntable rotates, do all of the pennies fly off at the same time?! Discuss!

If I put several pennies on the turntable at several different radii and turn the turntable slowly enough that all pennies stay put, which of the following statements is true?

(A) All pennies have the same velocity.

(B) All pennies make the same number of revolutions per second.

(C) All pennies have the same "angular velocity" $\omega = v/r$, but r will vary from penny to penny, so v will also vary.

(D) B and C are both true.

(E) A, B, and C are all true.

How can I best express the centripetal acceleration for each penny on the turntable?

(A) $a = v^2/r$

(B) $a = v^2/r = (r\omega)^2/r = \omega^2 r$

(C) a is the same for all pennies on the turntable

(D) (A) and (B) are both true, but (B) is a more useful way to describe what is happening on the turntable, because v varies from penny to penny, while ω is the same for all pennies.

(E) (A), (B), and (C) are all true

(F) (A), (B), and (C) are all false

Now we know that the centripetal acceleration can be written $a = \omega^2 r$ and varies with the radius of each penny. We also know that static friction must provide the force $m\omega^2 r$ to keep each penny going in a circle. Can we predict which pennies will slide off of the turntable first as I gradually increase the rotational velocity of the turntable?

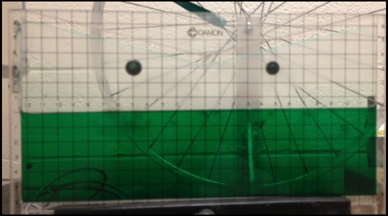
(A) The inside pennies will fly off first. This makes sense, because (for a given speed v) your tires screech more when you go around a turn with small radius than when you go around a turn with large radius.

(B) The outside pennies will fly off first. This makes sense because ω is the same for all pennies (v is not the same for all pennies), but $m\omega^2 r$ is largest for the outermost pennies.

(C) They all slide off at the same time.

(D) No way to predict.

What happens to the surface of this liquid if I center the tank atop the turntable and spin the turntable? (You'll have to make a leap of intuition, by analogy with the spinning nut-on-string, as we haven't studied fluids in this course.)



(A) The water surface will stay horizontal.

(B) The water surface is (when not spinning) perpendicular to the vector $(0, -g)$, i.e. it is horizontal. When spinning, the water surface will be perpendicular to the vector $(\omega^2 r, -g)$, with slope $= \omega^2 r/g$. So the surface will be triangular.

(C) The water surface is (when not spinning) perpendicular to the vector $(0, -mg)$, i.e. it is horizontal. When spinning, the water surface will be perpendicular to the vector $(\omega^2 r, -g)$, with slope $= \omega^2 r/g$. So the surface will be a parabola.

(D) All of the water will be stuck against the outer walls, as if trying to escape from a salad spinner.

Why does a salad spinner work?

(A) The outer wall of the spinner provides the centripetal force that pushes the lettuce toward the center of rotation, but the water feels no such force, because it can flow through the holes in the outer wall, thus separating water from lettuce.

(B) I really want to say "centrifugal force," even though my high-school teachers told me that there is really no such thing as "centrifugal force" — it's just a pseudo-force that one perceives when observing from the confusing perspective of a non-inertial reference frame.

(C) I guess you could say (A) or (B), but (A) is the way we've learned to analyze the situation methodically from Earth's reference frame. We haven't learned how to do calculations in non-inertial reference frames.

(D) While the obvious answer is (A), I am so fascinated by the pseudo-forces that appear in non-inertial reference frames that I went and read the Wikipedia article on the Coriolis effect!

(We stopped after this.)

Physics 8 — Friday, October 18, 2019

Suppose I try to spin a pail of water in a vertical circle at constant rotational speed ω, with the water a distance R from the pivot point at my shoulder. So the water is moving at speed $v = \omega R$. (I'll demonstrate first with an empty pail.) Will the water fall out of the pail?

(A) The water will fall out while the pail is upside down, no matter how fast you spin it around.

(B) The water will stay in the pail, no matter how slowly you spin it around.

(C) The water will stay in the pail as long as you spin it fast enough. "Fast enough" means $v/R^2 > g$ (or equivalently $\omega^2/R > g$) when the bucket is upside-down.

(D) The water will stay in the pail as long as you spin it fast enough. "Fast enough" means $v^2/R > g$ (or equivalently $\omega^2 R > g$) when the bucket is upside-down.

The way to think about the water-in-bucket problem is

(A) The bottom surface of the bucket can both push and pull on the water, as if the water and bucket were glued together.

(B) The bottom surface of the bucket can push on the water (compressive force) but cannot pull on the water (no tensile force). If the required centripetal acceleration is large enough that the bucket must push on the water to keep it moving in a circle (even when Earth's gravity is pulling down on the water), then the water will stay in the bucket.

(C) When the bucket is upside down, the bottom surface of the bucket must "pull up" on the water to keep it inside the bucket, or else the water will spill out.

(D) The water stays in the upside-down bucket if the outward "centrifugal pseudo-force" (magnitude mv^2/R or $m\omega^2 R$) is at least as large as the downward force of gravity.

(E) I think you could say (B) or (D), but we haven't learned in this course how to analyze the "pseudo-forces" that one perceives when working in a non-inertial reference frame. So I prefer (B), which uses the Earth reference frame.

Here is a good answer to the salad-spinner question: "The explanation for the physics going on as the spinner does its job is centripetal acceleration. The centripetal acceleration of an object in circular motion at constant speed tells us that the vector sum of the forces exerted on the object must be directed toward the center of the circle, continuously adjusting the object's direction. Without this inward pointing vector sum of forces, the object would move in a straight line. Centripetal force between the lettuce and the inside of the spinner pushes the lettuce around in a circle. On the other hand, the water can slip through the drain holes, so there's nothing to give it the same kind of push (and consequently there's no centripetal force to make it go in a circle). Thus, the lettuce experiences centripetal force while the water doesn't. In this way, the spinner manages to separate the two as the lettuce goes round in a circle and the water in a straight line through the holes."

Several people pointed out that we expect the water to shoot out *tangentially* from the spinner, since the water, once it loses contact with the lettuce, should travel in a straight line in the absence of a centripetal force. Need transparent salad spinner to verify!

How does this thing work? (Discuss!)

`http://www.youtube.com/watch?v=oh9sn5gn2fk`

Can you tell me what movie this is from?
(Hints: directed by Stanley Kubrick, story by A.C. Clarke.)

An ice cube and a rubber ball are both placed at one end of a warm cookie sheet, and the sheet is then tipped up. The ice cube slides down with virtually no friction, and the ball rolls down without slipping. Which one makes it to the bottom first?

(A) They reach the bottom at the same time.

(B) The ball gets there slightly faster, because the ice cube's friction (while very small) is kinetic and dissipates some energy, while the rolling ball's friction is static and does not dissipate energy.

(C) The ice cube gets there substantially faster, because the ball's initial potential energy mgh gets shared between $\frac{1}{2}mv^2$ (translational) and $\frac{1}{2}I\omega^2$ (rotational), while essentially all of the ice cube's initial mgh goes into $\frac{1}{2}mv^2$ (translational).

(D) The ice cube gets there faster because the ice cube's friction is negligible, while the frictional force between the ball and the cookie sheet dissipates the ball's kinetic energy into heat.

A hollow cylinder and a solid cylinder both roll down an inclined plane without slipping. Does friction play an important role in the cylinders' motion?

(A) No, friction plays a negligible role.

(B) Yes, (kinetic) friction dissipates a substantial amount of energy as the objects roll down the ramp.

(C) Yes, (static) friction is what causes the objects to roll rather than to slide. Without static friction, they would just slide down, so there would be no rotational motion (if you just let go of each cylinder from rest at the top of the ramp).

Why are people who write physics problems (e.g. about cylinders rolling down inclined planes) so fond of the phrase "rolls without slipping?"

(A) Because Nature abhors the frictional dissipation of energy.

(B) Because "rolls without slipping" implies that $v = \omega R$, where v is the cylinder's (translational) speed down the ramp. This lets you directly relate the rotational and translational parts of the motion.

(C) No good reason. You could analyze the problem just as easily if the cylinders were slipping somewhat while they roll.

How do I write the total kinetic energy of an object that has both translational motion at speed v and rotational motion at speed ω?

(Note that the symbol I is a capital I (for rotational "inertia") in the sans-serif font that I use to make my slides. Sorry!)

(A) $K = \frac{1}{2}mv^2$
(B) $K = \frac{1}{2}I\omega^2$
(C) $K = \frac{1}{2}I^2\omega$
(D) $K = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$
(E) $K = \frac{1}{2}mv^2 + \frac{1}{2}I^2\omega$
(F) $K = \frac{1}{2}m\omega^2 + \frac{1}{2}Iv^2$

While you discuss, I'll throw a familiar object across the room, for you to look at now from the perspective of Chapter 11 (and 12).

Sliding vs. rolling downhill:

For translational motion with no friction, $v_f = \sqrt{2gh}$ because

$$mgh_i = \frac{1}{2}mv_f^2$$

For rolling without slipping, we can write $\omega_f = v_f/R$:

$$mgh_i = \frac{1}{2}mv_f^2 + \frac{1}{2}I\omega_f^2$$

$$mgh_i = \frac{1}{2}mv_f^2 + \frac{1}{2}I\left(\frac{v_f}{R}\right)^2$$

$$mgh_i = \frac{1}{2}mv_f^2\left(1 + \frac{I}{mR^2}\right)$$

So the final velocity is slower (as are all intermediate velocities):

$$v_f = \sqrt{\frac{2gh}{1 + \frac{I}{mR^2}}}$$

A hollow cylinder and a solid cylinder both roll down an inclined plane without slipping. Assuming that the two cylinders have the same mass and same outer radius, which one has the larger rotational inertia?

(A) The hollow cylinder has the larger rotational inertia, because the material is concentrated at larger radius.

(B) The solid cylinder has the larger rotational inertia, because the material is distributed over more area.

(C) The rotational inertias are the same, because the masses and radii are the same.

The rolling object's downhill acceleration is smaller by a factor

$$\left(\frac{1}{1+\frac{I}{mR^2}}\right)$$

$I = mR^2$ for hollow cylinder. $\frac{1}{1+1} = 0.5$

$I = \frac{2}{3}mR^2$ for hollow sphere. $\frac{1}{1+(2/3)} = 0.60$

$I = \frac{1}{2}mR^2$ for solid cylinder. $\frac{1}{1+(1/2)} = 0.67$

$I = \frac{2}{5}mR^2$ for solid sphere. $\frac{1}{1+(2/5)} = 0.71$

Using Chapter 11 ideas, we know how to analyze the rolling objects' motion using energy arguments. (With Chapter 12 ideas, we will look again at the same problem using torque arguments, and directly find each object's downhill acceleration.)

Rotational inertia

For an extended object composed of several particles, with particle j having mass m_j and distance r_j from the rotation axis,

$$I = \sum_{j \in \text{particles}} m_j r_j^2$$

For a continuous object like a sphere or a solid cylinder, you have to integrate (or more often just look up the answer):

$$I = \int r^2 \, \mathrm{d}m$$

If you rearrange the same total mass to put it at larger distance from the axis of rotation, you get a larger rotational inertia.

(In which configuration does this adjustable cylinder-like object have the larger rotational inertia?)

inertia

$$m$$

translational velocity

$$v$$

translational K.E.

$$K = \frac{1}{2}mv^2$$

momentum

$$p = mv$$

rotational inertia

$$I = \sum mr^2$$

rotational velocity

$$\omega$$

rotational K.E.

$$K = \frac{1}{2}I\omega^2$$

angular momentum

$$L = I\omega$$

We learned earlier that momentum can be transferred from one object to another, but cannot be created or destroyed.

Consequently, a system on which no external forces are exerted (an "isolated system") has a constant momentum ($\vec{p} = m\vec{v}$):

$$\Delta \vec{p} = 0$$

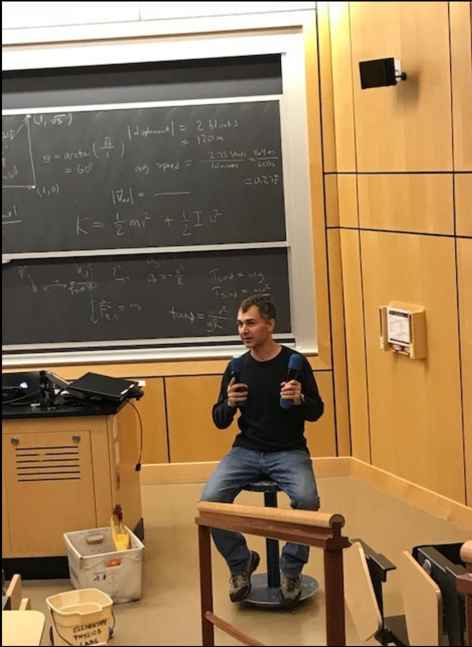
We now also know that angular momentum can be transferred from one object to another, but cannot be created or destroyed.

So a system on which no external torques are exerted has a constant angular momentum ($L = I\omega$):

$$\Delta \vec{L} = 0$$

If I spin around while sitting on a turntable (so that I am rotationally "isolated") and suddenly decrease my own rotational inertia, what happens to my rotational velocity?

In which photo is this character spinning faster (larger ω)?



position

$$\vec{r} = (x, y)$$

velocity

$$\vec{v} = (v_x, v_y) = \frac{\mathrm{d}\vec{r}}{\mathrm{d}t}$$

acceleration

$$\vec{a} = (a_x, a_y) = \frac{\mathrm{d}\vec{v}}{\mathrm{d}t}$$

if a_x is constant then:

$$v_{x,f} = v_{x,i} + a_x t$$

$$x_f = x_i + v_{x,i} t + \frac{1}{2} a_x t^2$$

$$v_{x,f}^2 = v_{x,i}^2 + 2 a_x \Delta x$$

rotational coordinate

$$\vartheta = s/r$$

rotational velocity

$$\omega = \frac{\mathrm{d}\vartheta}{\mathrm{d}t}$$

rotational acceleration

$$\alpha = \frac{\mathrm{d}\omega}{\mathrm{d}t}$$

if α is constant then:

$$\omega_f = \omega_i + \alpha t$$

$$\vartheta_f = \vartheta_i + \omega_i t + \frac{1}{2} \alpha t^2$$

$$\omega_f^2 = \omega_i^2 + 2\alpha \Delta\vartheta$$

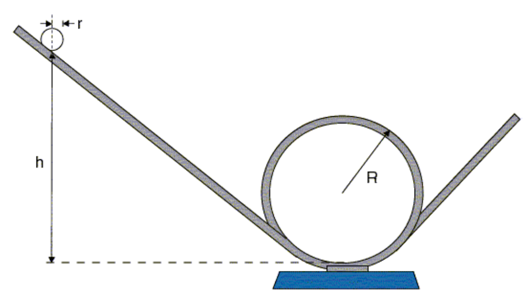
4. An automobile accelerates from rest starting at t = 0 such that its tires undergo a constant rotational acceleration $\alpha = 5.9\ s^{-2}$. The radius of each tire is 0.29 m. At t = 11 s after the acceleration begins, find (a) the instantaneous rotational speed ω of the tires, (b) the total rotational displacement $\Delta\vartheta$ of each tire, (c) the linear speed v of the automobile (assuming the tires stay perfectly round) and (d) the total distance the car travels in the 11 s.

(Let's not spend time solving this today. But think about which equations from the previous slide would be useful.)

We stopped here.

Physics 8 — Monday, October 21, 2019

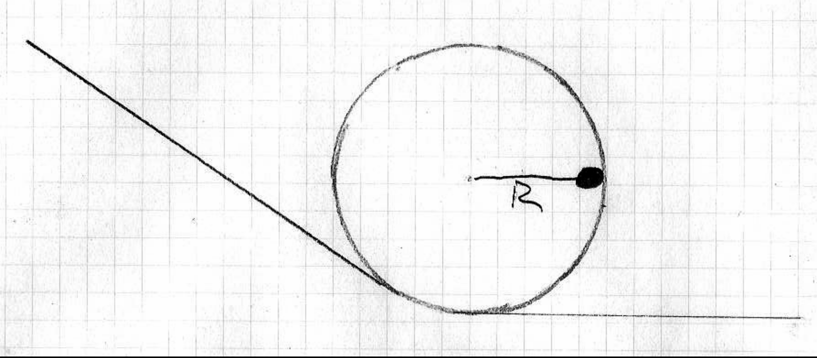
Let R be the **radius** of the circle in this loop-the-loop demo. I want the ball to make it all the way around the loop without falling off. What is the lowest height h at which I can start the ball (from rest)?



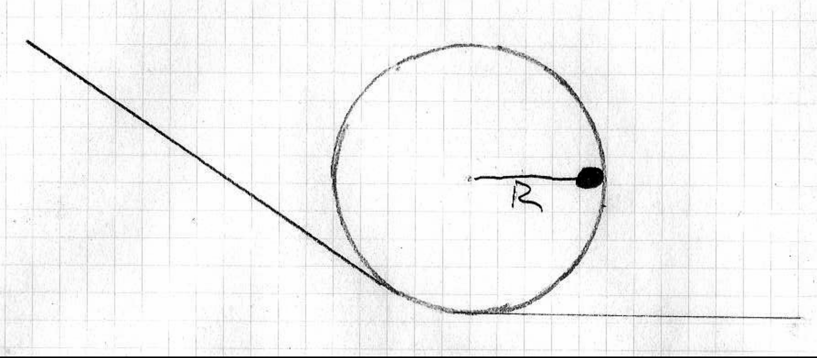
(A) The ball will make it all the way around if $h \geq R$.

(B) The ball will make it all the way around if $h \geq 2R$.

(C) If $h = 2R$, the ball will just make it to the top and will then fall down (assuming, for the moment, that it slides frictionlessly along the track). When the ball is at the top of the circle, its velocity must still be large enough to require a downward normal force exerted by the track on the ball. So the minimum h is even larger than $2R$. My neighbor and I are discussing now just how much higher that should be.

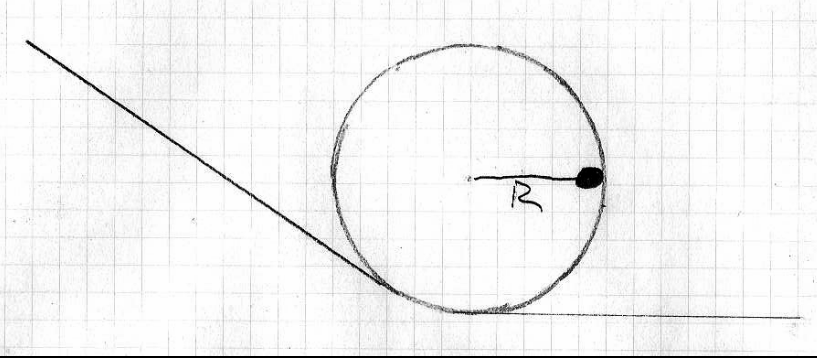


The ball clearly slows down as it makes its way up from the bottom toward the top of the circle. At the instant when the ball is at the position shown (i.e. it is at the same level as the center of the circle), in what direction does its velocity vector point?



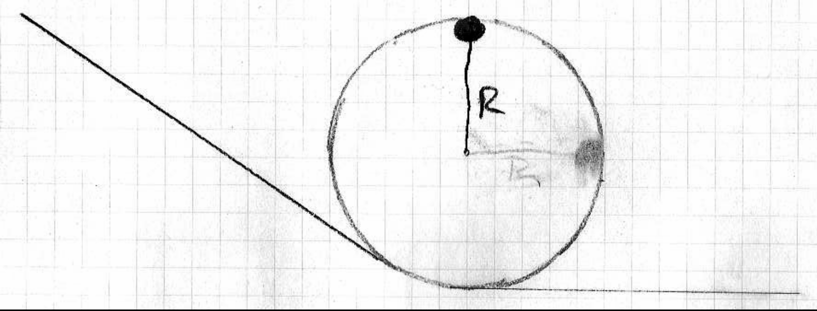
The ball clearly slows down as it makes its way up from the bottom toward the top of the circle. At the instant when the ball is at the position shown (i.e. it is at the same level as the center of the circle), what do we know about the vertical (y axis points up) component, a_y, of the ball's acceleration vector?

If $a_y \neq 0$, what vertical force(s) F_y is/are responsible?



The ball clearly slows down as it makes its way up from the bottom toward the top of the circle. At the instant when the ball is at the position shown (i.e. it is at the same level as the center of the circle), what do we know about the horizontal (x axis points right) component, a_x, of the ball's acceleration vector?

If $a_x \neq 0$, what horizontal force(s) F_x is/are responsible?



Suppose the ball makes it all the way around the circle without falling off. At the instant when the ball is at the position shown (at top of circle), what do we know about the vertical component, a_y, of the ball's acceleration vector?

If $a_y \neq 0$, what vertical force(s) F_y is/are responsible?

Suppose the ball makes it all the way around the loop-the-loop with much more than sufficient speed to stay on the circular track. Let the y-axis point upward, and let v_{top} be the ball's speed when it reaches the top of the loop. What is the y component, a_y, of the ball's acceleration when it is at the very top of the loop?

(A) $a_y = -g$

(B) $a_y = +g$

(C) $a_y = +v_{\text{top}}^2/R$

(D) $a_y = -v_{\text{top}}^2/R$

(E) $a_y = +g + v_{\text{top}}^2/R$

(F) $a_y = -g - v_{\text{top}}^2/R$

(G) $a_y = +g + v_{\text{top}}/R^2$

(H) $a_y = -g - v_{\text{top}}/R^2$

The track can push on the ball, but it can't pull on the ball! How do I express the fact that the track is still pushing on the ball even at the very top of the loop?

(A) Write the equation of motion for the ball: $m\vec{a} = \sum \vec{F}_{\text{on ball}}$, and require the normal force exerted by the track on the ball to point inward, even at the very top. (At the very top, "inward" is "downward.") If the equation $m\vec{a} = \sum \vec{F}_{\text{on ball}}$ gave us an outward-pointing normal force (exerted by track on ball), that would be inconsistent with the ball's staying in contact with the track.

(B) Use conservation of angular momentum.

(C) Draw a free-body diagram for the ball, and require that gravity and the normal force point in opposite directions.

(D) Draw a free-body diagram for the ball, and require that the magnitude of the normal force be at least as large as the force of Earth's gravity on the ball.

For the ball to stay in contact with the track when it is at the top of the loop, there must still be an inward-pointing normal force exerted by the track on the ball, even at the very top. How can I express this fact using $\boxed{ma_y = \sum F_y}$? Let $v_{\rm top}$ be the ball's speed at the top of the loop.

(A) $+mv_{\rm top}^2/R = +mg + F_{tb}^N$ with $F_{tb}^N > 0$

(B) $+mv_{\rm top}^2/R = +mg - F_{tb}^N$ with $F_{tb}^N > 0$

(C) $+mv_{\rm top}^2/R = -mg + F_{tb}^N$ with $F_{tb}^N > 0$

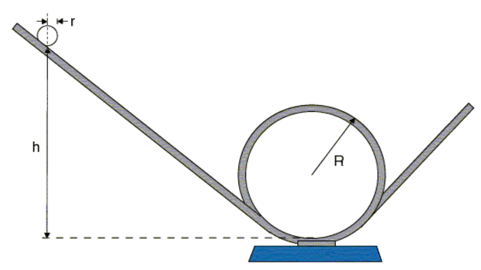
(D) $+mv_{\rm top}^2/R = -mg - F_{tb}^N$ with $F_{tb}^N > 0$

(E) $-mv_{\rm top}^2/R = +mg + F_{tb}^N$ with $F_{tb}^N > 0$

(F) $-mv_{\rm top}^2/R = +mg - F_{tb}^N$ with $F_{tb}^N > 0$

(G) $-mv_{\rm top}^2/R = -mg + F_{tb}^N$ with $F_{tb}^N > 0$

(H) $-mv_{\rm top}^2/R = -mg - F_{tb}^N$ with $F_{tb}^N > 0$



How do I decide the minimum height h from which the ball will make it all the way around the loop without losing contact with the track? For simplicity, assume that the track is very slippery, so that you can neglect the ball's rotational kinetic energy.

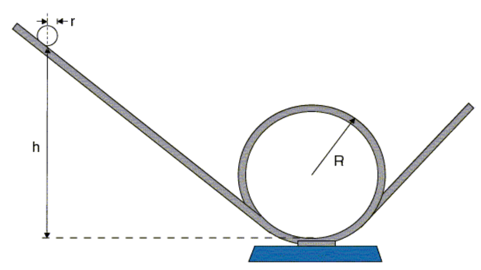
(A) $2mgR = \frac{1}{2}mv_{\rm top}^2 + mgh$ with $v_{\rm top} = \sqrt{gR}$

(B) $mgR = \frac{1}{2}mv_{\rm top}^2 + mgh$ with $v_{\rm top} = \sqrt{gR}$

(C) $mgh = \frac{1}{2}mv_{\rm top}^2 + 2mgR$ with $v_{\rm top} = \sqrt{gR}$

(D) $mgh = \frac{1}{2}mv_{\rm top}^2 + mgR$ with $v_{\rm top} = \sqrt{gR}$

(By the way, how would the answer change if I said instead that the (solid) ball rolls without slipping on the track?)



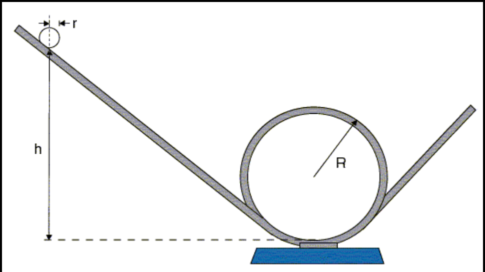
How do I decide the minimum height h from which the ball will make it all the way around the loop without losing contact with the track? Let's now be realistic: the ball is a solid sphere that rolls without slipping on the track.

(A) $mgh = \frac{1}{2} m v_{\rm top}^2 + 2mgR$

(B) $mgh = \frac{1}{2} m v_{\rm top}^2 + \frac{1}{2} I \omega_{\rm top}^2 + 2mgR$

with $v_{\rm top} = \sqrt{gR}$ and $\omega_{\rm top} = v_{\rm top}/r_{\rm ball}$

(Little "$r_{\rm ball}$" is the radius of the ball. Big "R" is the radius of the loop-the-loop.)



$$mgh = \frac{1}{2}mv_{\mathrm{top}}^2 + \frac{1}{2}I\omega_{\mathrm{top}}^2 + 2mgR$$

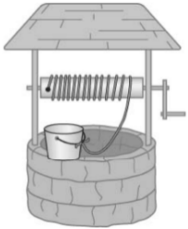
with $v_{\mathrm{top}} = \sqrt{gR}$ and $\omega_{\mathrm{top}} = v_{\mathrm{top}}/r_{\mathrm{ball}}$ and $I = \frac{2}{5}mr_{\mathrm{ball}}^2$.

$$mgh = \frac{1}{2}m(gR) + \frac{\frac{2}{5}mr_{\mathrm{ball}}^2}{2r_{\mathrm{ball}}^2}(gR) + 2mgR = 2.7mgR$$

We stopped after this.

Physics 8 — Wednesday, October 23, 2019

6*. You accidentally knock a full bucket of water off the side of the well shown in the figure at right. The bucket plunges 18 m to the bottom of the well. Attached to the bucket is a light rope that is wrapped around the crank cylinder. How fast is the handle turning (rotational speed) when the bucket hits bottom? The inertia of the bucket plus water is 13 kg. The crank cylinder is a solid cylinder of radius 0.65 m and inertia 5.0 kg. (Assume the small handle's inertia is negligible in comparison with the crank cylinder.)



How would you approach this problem? Discuss with your neighbor while I set up a demonstration along the same lines . . .

(A) initial angular momentum of bucket equals final angular momentum of cylinder + bucket

(B) initial G.P.E. equals final K.E. (translational for bucket + rotational for cylinder)

(C) initial G.P.E. equals final K.E. of bucket

(D) initial G.P.E. equals final K.E. of cylinder

(E) initial K.E. of bucket equals final G.P.E.

(F) use torque = mgR to find constant angular acceleration

- What is the rotational inertia for a solid cylinder?
- How do you relate v of the bucket with ω of the cylinder? Why is this true?
- What is the expression for the total kinetic energy?
- Why is angular momentum not the same for the initial and final states?
- What are the two expressions for angular momentum used in Chapter 11?
- Does anyone know (though this is in Chapter 12 and is tricky) why using $\tau = mgR$ would not give the correct angular acceleration? What if you used $\tau = TR$, where T is the tension in the rope?

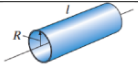
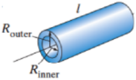
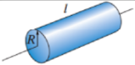
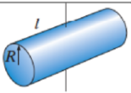
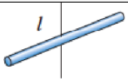
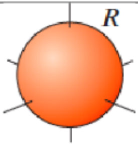
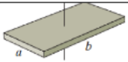
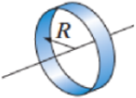
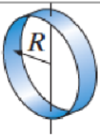
configuration		rotational inertia
thin cylindrical shell about its axis		mR^2
thick cylindrical shell about its axis		$(1/2)m(R_i^2 + R_o^2)$
solid cylinder about its axis		$(1/2)mR^2$
solid cylinder $\perp$ to axis		$(1/4)mR^2 + (1/12)m\ell^2$
thin rod $\perp$ to axis		$(1/12)m\ell^2$

Table 11.3. Also in "equation sheet"

http://positron.hep.upenn.edu/p8/files/equations.pdf

hollow sphere		$(2/3)mR^2$
solid sphere		$(2/5)mR^2$
rectangular plate		$(1/12)m(a^2+b^2)$
thin hoop about its axis		mR^2
thin hoop $\perp$ to axis		$(1/2)mR^2$

5. A long, thin rod is pivoted from a hinge such that it hangs vertically from one end. (The hinge is at the top.) The length of the rod is 1.23 m. You want to hit the lower end of the rod just hard enough to get the rod to swing all the way up and over the pivot (i.e. to swing more than 180°). How fast do you have to make the end go?

How would you approach this problem? Discuss with neighbors!
Which (if any) of these statements is **false** ?

(A) I know the change in G.P.E from the initial to the desired final states. So the initial K.E. (which is rotational) of the rod needs to be at least this large.

(B) The book (or equation sheet) gives rotational inertia I for a long, thin rod about its center. So I can use the parallel-axis theorem to get I for the rod about one end.

(C) The angular momentum, $L = I\omega$, is the same for the initial and final states.

(D) Because the rod pivots about one end, the speed of the other end is $v = \omega\ell$ (where ℓ is length of rod)

(E) None. (All of the above statements are true.)

The rotational inertia for a long, thin rod of length ℓ about a perpendicular axis through its center is

$$I = \frac{1}{12} m\ell^2$$

What is its rotational inertia about one end?

(A) $\frac{1}{12} m\ell^2$
(B) $\frac{1}{24} m\ell^2$
(C) $\frac{1}{2} m\ell^2$
(D) $\frac{1}{3} m\ell^2$
(E) $\frac{1}{4} m\ell^2$
(F) $\frac{1}{6} m\ell^2$

(We'll repeat this question after some explanation.)

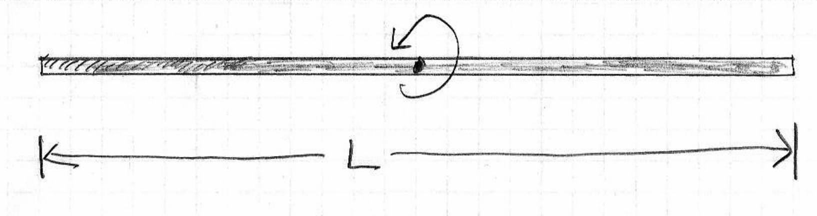
If an object revolves about an axis that does not pass through the object's center of mass (suppose axis has $\perp$ distance $d_\perp$ from CoM), the rotational inertia is larger, because the object's CoM revolves around a circle of radius $d_\perp$ and in addition the object rotates about its own CoM.

This larger rotational inertia is given by the *parallel axis theorem*:

$$I = I_{\rm cm} + M d_\perp^2$$

where $I_{\rm cm}$ is the object's rotational inertia about an axis (which must be parallel to the new axis of rotation) that passes through the object's CoM.

(We'll go over the parallel-axis theorem again next time. First I want to make sure you know what you need for this week's HW.)



The rotational inertia of a long, thin rod (whose thickness is negligible compared with its length) of mass M and length L, for rotation about its CoM, is

$$I = \frac{1}{12} ML^2$$

Using the parallel axis theorem, what is the rod's rotational inertia for rotation about one end? (Click next page.)

The rotational inertia for a long, thin rod of length ℓ about a perpendicular axis through its center is

$$I = \frac{1}{12} m\ell^2$$

What is its rotational inertia about one end?

(A) $\frac{1}{12} m\ell^2$

(B) $\frac{1}{24} m\ell^2$

(C) $\frac{1}{2} m\ell^2$

(D) $\frac{1}{3} m\ell^2$

(E) $\frac{1}{4} m\ell^2$

(F) $\frac{1}{6} m\ell^2$

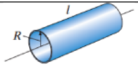
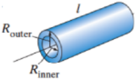
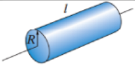
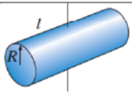
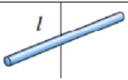
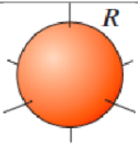
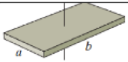
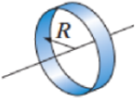
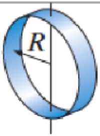
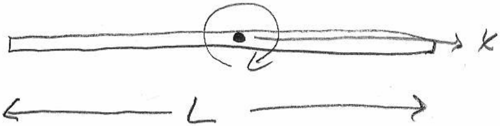
configuration		rotational inertia
thin cylindrical shell about its axis		mR^2
thick cylindrical shell about its axis		$(1/2)m(R_i^2 + R_o^2)$
solid cylinder about its axis		$(1/2)mR^2$
solid cylinder $\perp$ to axis		$(1/4)mR^2 + (1/12)m\ell^2$
thin rod $\perp$ to axis		$(1/12)m\ell^2$

Table 11.3. Also in "equation sheet"

http://positron.hep.upenn.edu/p8/files/equations.pdf

hollow sphere		$(2/3)mR^2$
solid sphere		$(2/5)mR^2$
rectangular plate		$(1/12)m(a^2+b^2)$
thin hoop about its axis		mR^2
thin hoop $\perp$ to axis		$(1/2)mR^2$

(In case you're curious where that $I = ML^2/12$ comes from.)



$$I = \sum m r^2 \rightarrow \int r^2\, dm$$

$$dm = \frac{M}{L}\, dx \qquad r = |x|$$

$$I = \int_{-L/2}^{L/2} x^2 \left(\frac{M}{L}\right) dx = \frac{M}{L}\left[\frac{x^3}{3}\right]_{x=-\frac{L}{2}}^{x=+\frac{L}{2}}$$

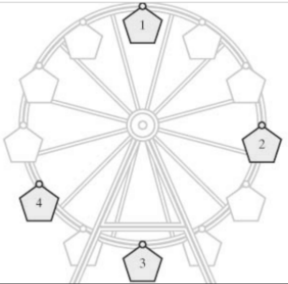
$$I = \frac{M}{L}\left[\frac{(L/2)^3}{3} - \frac{(-L/2)^3}{3}\right] = \frac{ML^2}{12}$$

3*. You have a weekend job selecting speed-limit signs to put at road curves. The speed limit is determined by the radius of the curve and the bank angle of the road w.r.t. horizontal. Your first assignment today is a turn of radius 250 m at a bank angle of 4.8°. (a) What speed limit sign should you choose for that curve such that a car traveling at the speed limit negotiates the turn successfully even when the road is wet and slick? (So at this speed, it stays on the road even when friction is negligible.) (b) Draw a free-body diagram showing all of the forces acting on the car when it is moving at this maximum speed. (Your diagram should also indicate the direction of the car's acceleration vector.)

Let's start by drawing an FBD (for the car) for the case where the car's speed is at exactly the value for which no friction at all is needed to keep the car moving in its circular path. In that case, what are the forces acting on the car?

Alongside the FBD, let's draw (elevation view) the car on the banked road. Let's assume that the road curves to the left.

Physics 8 — Friday, October 25, 2019



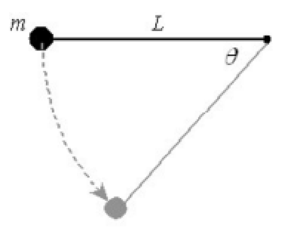
A Ferris wheel rotates at constant speed. Draw an FBD for each numbered carriage. For scale, draw $m\vec{a}$ on each FBD instead of the usual $\vec{a}$. Then the two force vectors must sum to $m\vec{a}$. For a realistic Ferris wheel, ma would be much smaller than mg, but to make drawings that bring out the physics, let's make ma be half the size of mg. In other words, for the sake of illustration, we'll spin the Ferris wheel fast enough that $v^2/R = g/2$.

I'll try to use these values for some ad-hoc questions:

(A) E (B) NE (C) N (D) NW (E) W (F) SW (G) S (H) SE

(A) 0.5 (B) 1.0 (C) $\sqrt{\frac{5}{4}} = 1.118$ (D) 1.32 (E) 1.5 (F) 2.0

2. You attach one end of a string of length L to a small ball of inertia m. You attach the string's other end to a pivot that allows free revolution. You hold the ball out to the side with the string taut along a horizontal line, as the in figure (below, left). (a) If you release the ball from rest, what is the tension in the string as a function of angle ϑ swept through? (b) What should the tensile strength of the string be (the maximum tension it can sustain without breaking) if you want it not to break through the ball's entire motion (from $\vartheta = 0$ to $\vartheta = \pi$)?

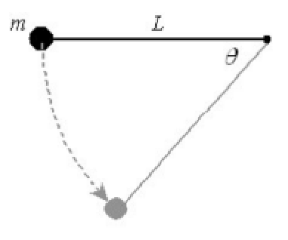


- Since the string's length L stays constant, what shape does the ball's path trace out as it moves?
- Does the ball's acceleration have a component that points along the axis of the string? If so, does its magnitude depend on the ball's speed?
- What two forces are acting on the ball?
- Assuming that no energy is dissipated, how can we relate the ball's speed v to its height y ?
- Can you write $m\vec{a} = \sum \vec{F}$ for the component of $\vec{a}$ and $\sum \vec{F}$ that points along the string?

("small" ball $\Rightarrow$ neglect the ball's rotation about its own CoM)

2. You attach one end of a string of length L to a small ball of inertia m. You attach the string's other end to a pivot that allows free revolution. You hold the ball out to the side with the string taut along a horizontal line, as the in figure (below, left). (a) If you release the ball from rest, what is the tension in the string as a function of angle ϑ swept through? (b) What should the tensile strength of the string be (the maximum tension it can sustain without breaking) if you want it not to break through the ball's entire motion (from $\vartheta = 0$ to $\vartheta = \pi$)?

How do I relate angle θ to speed v? $E_i = mgL \quad \rightarrow \quad E = \frac{1}{2}mv^2 + mgy$



(A) $\frac{1}{2}mv^2 = mg(L - y) = mgL(1 - \cos\theta)$

(B) $\frac{1}{2}mv^2 = mg(L - y) = mgL(1 - \sin\theta)$

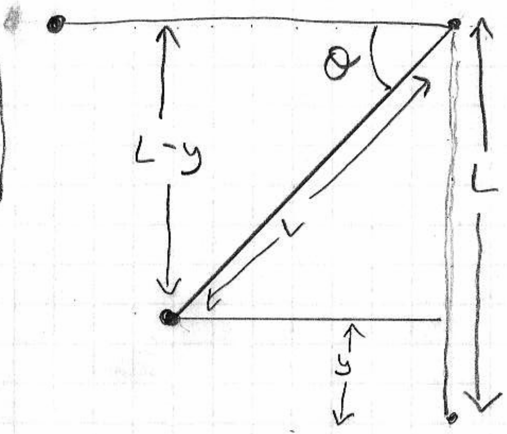
(C) $\frac{1}{2}mv^2 = mg(L - y) = mgL\cos\theta$

(D) $\frac{1}{2}mv^2 = mg(L - y) = mgL\sin\theta$

Hint: draw on the figure a vertical line of length $L - y = y_i - y$

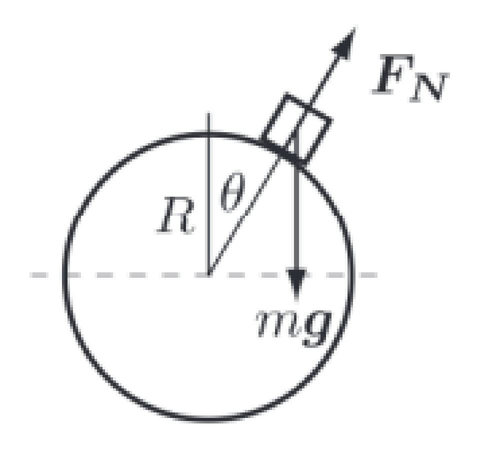
Next: write "radial" component of $m\vec{a} = \sum \vec{F}$ to find T

$$\sin\theta = \frac{L-y}{L}$$

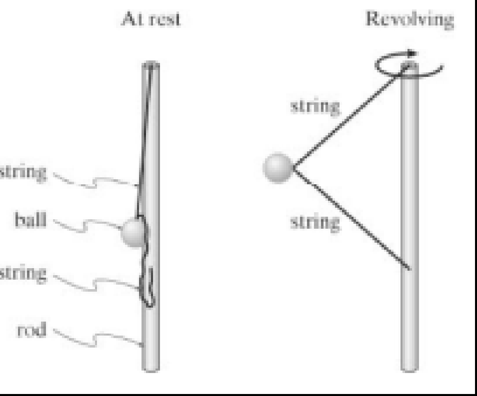


When does the block lose contact with the sphere?

A small block of mass m slides down a sphere of radius R, starting from rest at the top. The sphere is immobile, and friction between the block and the sphere is negligible. In terms of m, g, R, and θ, determine:
(a) the K.E. of the block;
(b) the centripetal acceleration of the block;
(c) the normal force exerted by the sphere on the block.
(d) At what value of θ does the block lose contact with the sphere?



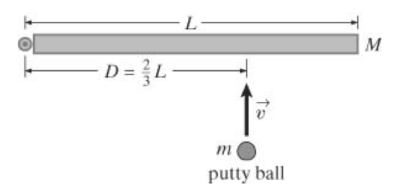
8*. A ball is attached to a vertical rod by two strings of equal strength and equal length. (See figure, below left.) The strings are very light and do not stretch. The rod begins to rotate under the influence of a constant rotational acceleration. (a) Which string breaks first? (b) Draw a free-body diagram for the ball, indicating all forces and their relative magnitudes, to justify your answer to (a).



- Are the angles of the two strings w.r.t. horizontal equal?
- Are the tensions in the two strings equal? How do you know?
- What three forces act on the ball?
- Is the ball accelerating vertically? Horizontally?
- Draw a FBD for the ball, showing both horizontal (radial) and vertical component of each force.

Notice that the ball's speed v increases with time, until finally one string breaks. Which one? (Which string's tension is larger?)

(plan view — from above)



9*. An open door of inertia M and width L is at rest when it is struck by a thrown putty ball of inertia m that is moving at linear speed v at the instant it strikes the door. (See figure, above right.) The impact point is a distance $D = \frac{2}{3}L$ from the rotation axis through the hinges. The putty ball strikes at a right angle to the door face and sticks after it hits. What is the rotational speed of the door and putty? Do not ignore the inertia m.

How would you approach this problem? Discuss with neighbors!

(A) The final K.E. (rotational+translational) equals the initial K.E. of the ball.

(B) The initial momentum $m\vec{v}$ of the ball equals the final momentum $(m+M)\vec{v}$ of the door+ball.

(C) The initial angular momentum $L = r_\perp mv$ of the ball w.r.t. the hinge axis equals the final angular momentum $L = I\omega$ of the door+ball.

9*. An open door of inertia M and width L is at rest when it is struck by a thrown putty ball of inertia m that is moving at linear speed v at the instant it strikes the door. (See figure, above right.) The impact point is a distance $D = \frac{2}{3}L$ from the rotation axis through the hinges. The putty ball strikes at a right angle to the door face and sticks after it hits. What is the rotational speed of the door and putty? Do not ignore the inertia m.

I know that the rotational inertia of a thin rod of length L about a perpendicular axis through its center is $I = \frac{1}{12}mL^2$. The rotational inertia I to use for the final state here is

(A) $I = ML^2 + mL^2$

(B) $I = \frac{1}{12}ML^2 + M(\frac{L}{2})^2 + m(\frac{2}{3}L)^2$

(C) $I = \frac{1}{12}ML^2 + \frac{2}{3}mL^2$

(D) $I = \frac{1}{12}ML^2 + m(\frac{2}{3}L)^2$

(E) $I = \frac{1}{12}ML^2 + mL^2$

(Challenge: Also think how the answer would change if the radius of the putty ball were non-negligible. What if the thickness of the door were non-negligible? Does the height of the door matter?)

Physics 8 — Monday, October 28, 2019

- After spending this week's class time on torque, we'll spend 4 weeks applying the ideas of forces, vectors, and torque to the analysis of architectural structures. Fun reward for your work!

Three different expressions for angular momentum:

$$L = I\omega$$

$$L = r_\perp \; mv$$

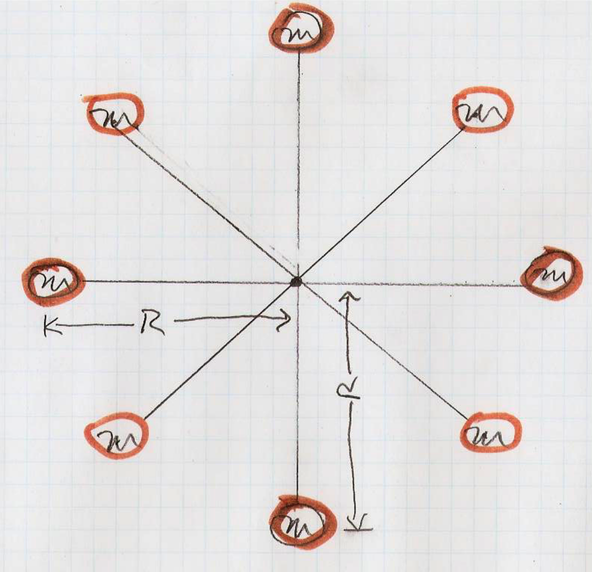
$$L = r \; mv_\perp$$

The second expression is telling you that momentum times lever arm (w.r.t. the relevant pivot axis) equals angular momentum.

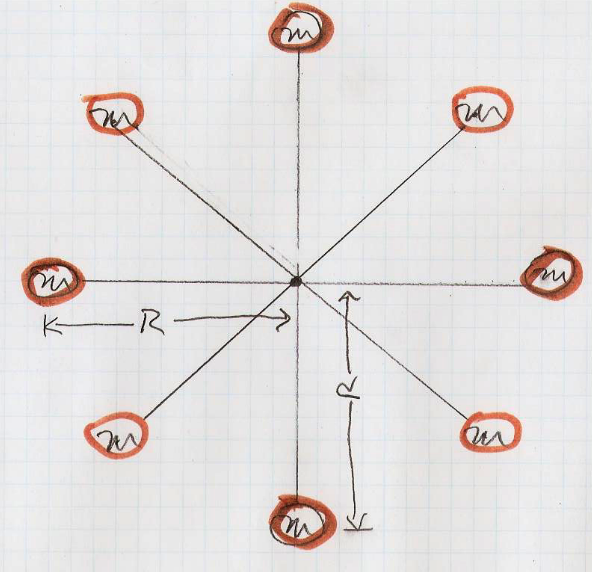
The second and third expressions are both simplified ways of writing the more general (but more difficult) expression

$$\vec{L} = \vec{r} \times \vec{p}$$

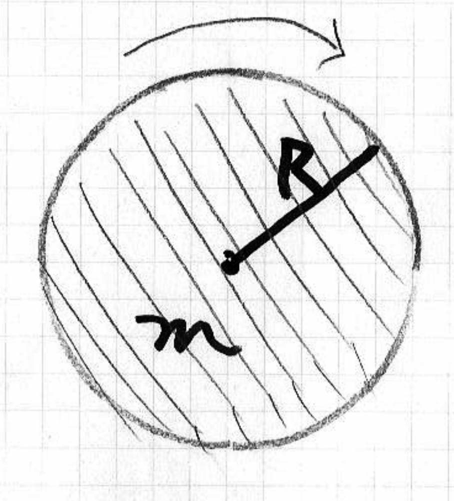
10*. Two skaters skate toward each other, each moving at 3.1 m/s. Their lines of motion are separated by a perpendicular distance of 1.8 m. Just as they pass each other (still 1.8 m apart), they link hands and spin about their common center of mass. What is the rotational speed of the couple about the center of mass? Treat each skater as a point particle, each with an inertia of 52 kg.



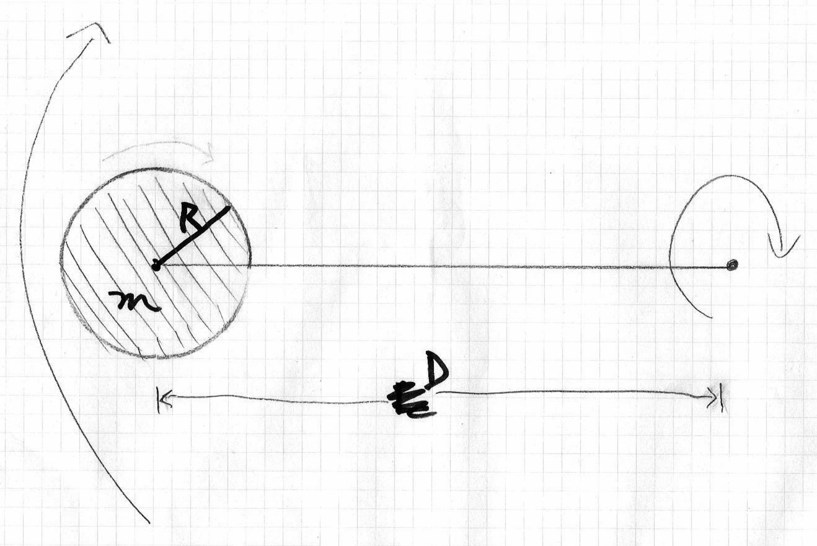
Where is the center of mass of this pinwheel-like object?



What is this object's rotational inertia, for rotation about its center of mass? Assume that all of the mass is concentrated in the orange blobs, and assume that the orange blobs are "point masses," i.e. that their size is much smaller than R.



Suppose I have a solid disk of radius R and mass m. I rotate it about its CoM, about an axis $\perp$ to the plane of the page. What is its rotational inertia? (If you don't happen to remember — is it bigger than, smaller than, or equal to mR^2 ?)



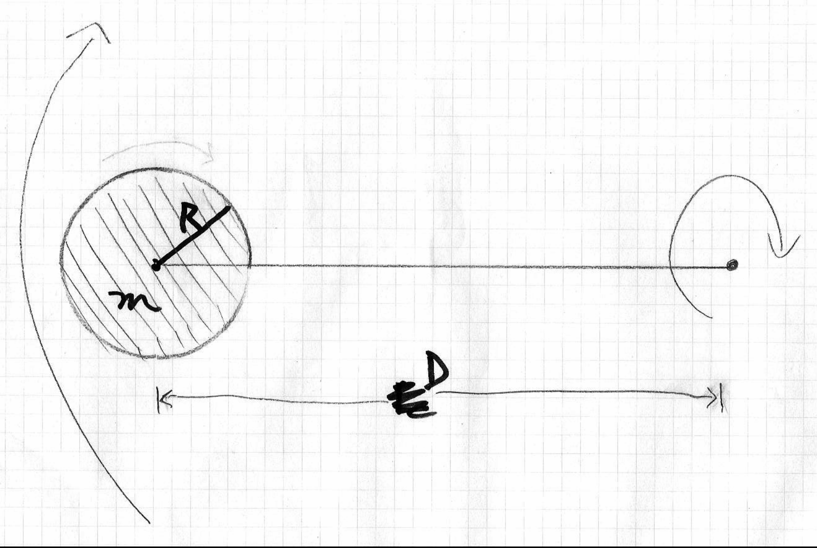
Now I take the same disk, attach it to a string or a lightweight stick of length D, and make the disk's CoM go around in circles of radius D. Is the mass now farther than or closer to the rotation axis than in the original rotation (about CoM)? What happens to I ?

If an object revolves about an axis that does not pass through the object's center of mass (suppose axis has $\perp$ distance D from CoM), the rotational inertia is larger, because the object's CoM revolves around a circle of radius D and in addition the object rotates about its own CoM.

This larger rotational inertia is given by the *parallel axis theorem*:

$$I = I_{\rm cm} + MD^2$$

where $I_{\rm cm}$ is the object's rotational inertia about an axis (which must be parallel to the new axis of rotation) that passes through the object's CoM.



Using the parallel axis theorem, what is the disk's rotational inertia about the displaced axis (the axis that is distance D away from the CoM)?

video segment break

- begin video preceding ws15 = mz12

Torque: the rotational analogue of force

Just as an unbalanced force causes linear acceleration

$$\vec{F} = m\vec{a}$$

an unbalanced torque causes rotational acceleration

$$\tau = I\alpha$$

Torque is **(lever arm)** $\times$ **(force)**

$$\tau = r_{\perp} \, F$$

where $r_{\perp}$ is the "perpendicular distance" from the rotation axis to the line-of-action of the force.

position

$$\vec{r} = (x, y)$$

velocity

$$\vec{v} = (v_x, v_y) = \frac{\mathrm{d}\vec{r}}{\mathrm{d}t}$$

acceleration

$$\vec{a} = (a_x, a_y) = \frac{\mathrm{d}\vec{v}}{\mathrm{d}t}$$

momentum

$$\vec{p} = m\vec{v}$$

force

$$\vec{F} = m\vec{a}$$

rotational coordinate

$$\vartheta = s/r$$

rotational velocity

$$\omega = \mathrm{d}\vartheta/\mathrm{d}t$$

rotational acceleration

$$\alpha = \mathrm{d}\omega/\mathrm{d}t$$

angular momentum

$$L = I\omega$$

$$L = r_\perp\, mv$$

torque

$$\tau = I\alpha$$

$$\tau = r_\perp\, F$$

I wind a string around a coffee can of radius $R = 0.05\ \mathrm{m}$. (That's 5 cm.) Friction prevents the string from slipping. I apply a tension $T = 20\ \mathrm{N}$ to the free end of the string. The free end of the string is tangent to the coffee can, so that the radial direction is perpendicular to the force direction. What is the magnitude of the torque exerted by the string on the coffee can?

(A) $1\ \mathrm{N \cdot m}$

(B) $2\ \mathrm{N \cdot m}$

(C) $5\ \mathrm{N \cdot m}$

(D) $10\ \mathrm{N \cdot m}$

(E) $20\ \mathrm{N \cdot m}$

Suppose that the angular acceleration of the can is $\alpha = 2\ \mathrm{s}^{-2}$ when the string exerts a torque of $1\ \mathrm{N} \cdot \mathrm{m}$ on the can. What would the angular acceleration of the can be if the string exerted a torque of $2\ \mathrm{N} \cdot \mathrm{m}$ instead?

(A) $\alpha = 0.5\ \mathrm{s}^{-2}$

(B) $\alpha = 1\ \mathrm{s}^{-2}$

(C) $\alpha = 2\ \mathrm{s}^{-2}$

(D) $\alpha = 4\ \mathrm{s}^{-2}$

(E) $\alpha = 5\ \mathrm{s}^{-2}$

(F) $\alpha = 10\ \mathrm{s}^{-2}$

I apply a force of 5.0 N at a perpendicular distance of 5 cm ($r_\perp = 0.05\ \mathrm{m}$) from this rotating wheel, and I observe some angular acceleration α. What force would I need to apply to this same wheel at $r_\perp = 0.10\ \mathrm{m}$ (that's 10 cm) to get the same angular acceleration α?

(A) $F = 1.0\ \mathrm{N}$

(B) $F = 2.5\ \mathrm{N}$

(C) $F = 5.0\ \mathrm{N}$

(D) $F = 10\ \mathrm{N}$

(E) $F = 20\ \mathrm{N}$

Suppose that I use the tension T in the string to apply a given torque $\tau = r_\perp T$ to this wheel, and it experiences a given angular acceleration α. Now I **increase** the rotational inertia I of the wheel and then apply the same torque. The new angular acceleration $\alpha_{\rm new}$ will be

(A) larger: $\alpha_{\rm new} > \alpha$

(B) the same: $\alpha_{\rm new} = \alpha$

(C) smaller: $\alpha_{\rm new} < \alpha$

I want to tighten a bolt to a torque of 1.0 newton-meter, but I don't have a torque wrench. I do have an ordinary wrench, a ruler, and a 1.0 kg mass tied to a string. How can I apply the correct torque to the bolt?

(A) Orient the wrench horizontally and hang the mass at a distance 0.1 m from the axis of the bolt

(B) Orient the wrench horizontally and hang the mass at a distance 1.0 m from the axis of the bolt

If the wrench is at 45° w.r.t. horizontal, will the 1.0 kg mass suspended at a distance 0.1 m along the wrench still exert a torque of 1.0 newton-meter on the bolt?

(A) Yes. The force of gravity has not changed, and the distance has not changed.

(B) No. The torque is now smaller — about 0.71 newton-meter — because the "perpendicular distance" is now smaller by a factor of $1/\sqrt{2}$.

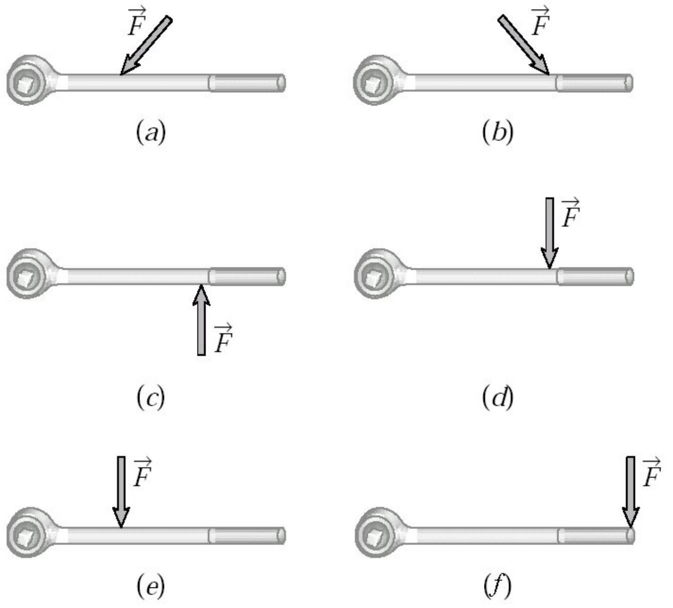
(C) No. The torque is now larger — about 1.4 newton-meter.

$$\tau \;=\; r_\perp F \;=\; rF_\perp \;=\; rF\sin\theta_{rF} \;=\; |\vec{r}\times\vec{F}|$$

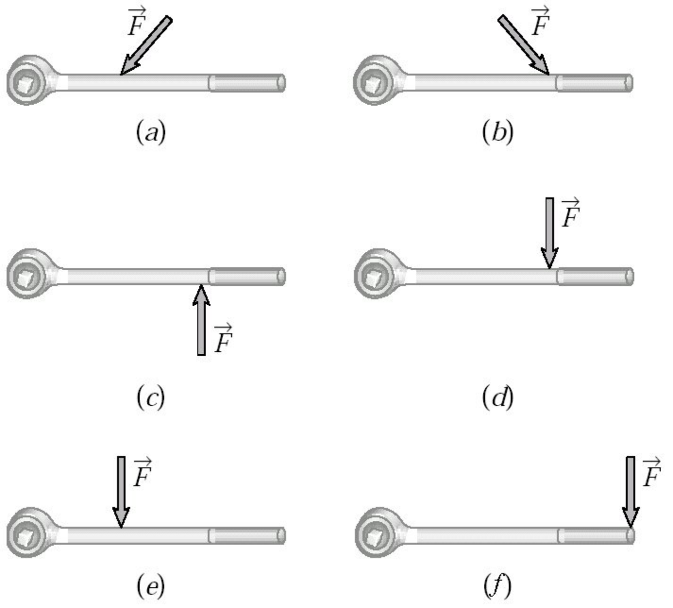
Four ways to get the magnitude of the torque

- (perpendicular component of distance) $\times$ (force)
- (distance) $\times$ (perpendicular component of force)
- (distance) (force) ($\sin\theta$ between $\vec{r}$ and $\vec{F}$)
- use magnitude of "vector product" $\vec{r}\times\vec{F}$ (a.k.a. "cross product")

To tighten a bolt, I apply a force of the same magnitude F at different positions and angles. Which torque is *largest*?



To tighten a bolt, I apply a force of magnitude F at different positions and angles. Which torque is *smallest*?



I want to apply to this meter stick two torques of the same magnitude and opposite sense, so that the stick has zero rotational acceleration. I apply one force of 5 N at a lever arm of 0.5 m. I want to apply an opposing force at a lever arm of 0.2 m, so that the second torque balances the first torque. How large must this second force be?

(A) 1.0 N

(B) 2.0 N

(C) 12.5 N

(D) 25 N

we stopped here

Physics 8 — Wednesday, October 30, 2019

- This week, you're reading Ch2 (statics) and Ch3 (determinate systems: equilibrium, trusses, arches) of Onouye/Kane. Feel free to buy one of my $10 used copies if you wish. At the end of the term, you can keep it, or sell it back to me for $10.
- HW8 due this Friday. HW help: Wed 4–6 (4–7?) [Bill] 3C4, Thu 6–8pm [Grace] 2C4.

I want to apply to a meter stick two torques of the same magnitude and opposite sense, so that the stick has zero rotational acceleration. I apply one force of 5 N at a lever arm of 0.5 m. I want to apply an opposing force at a lever arm of 0.2 m, so that the second torque balances the first torque. How large must this second force be? (Both forces' lines of action are perpendicular to the axis of the meter stick.)

(A) 1.0 N (B) 2.0 N (C) 12.5 N (D) 25 N

I want to apply to this meter stick two torques of the same magnitude and opposite sense, so that the stick has zero rotational acceleration. I apply one force of 10 N at a lever arm of 0.5 m. I tie a second string on the opposite end, 0.5 m from the pivot point. **The second force is applied at a** 45° **angle w.r.t. the vertical.** How large must this second force be?

(A) 5 N

(B) 7 N

(C) 10 N

(D) 14 N

(E) 20 N

If the rod doesn't accelerate (rotationally, about the pivot), what force does the scale read?

(A) 1.0 N

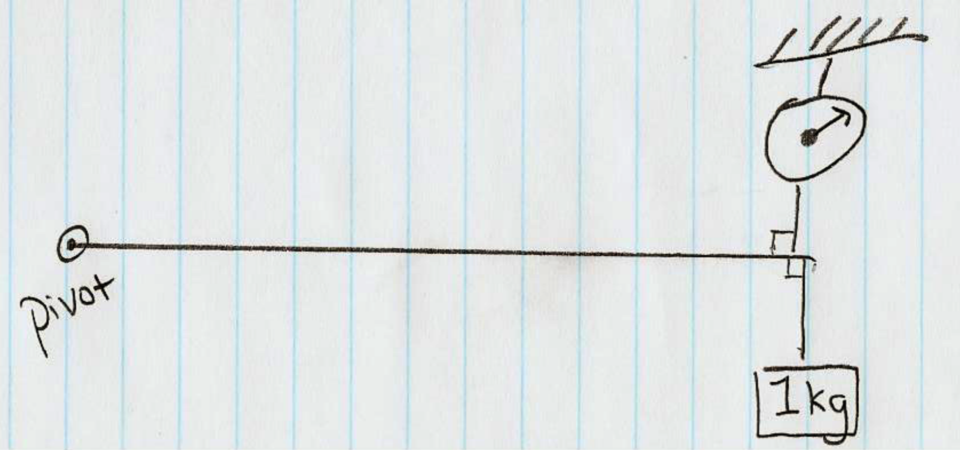
(B) 5.0 N

(C) 7.1 N

(D) 10 N

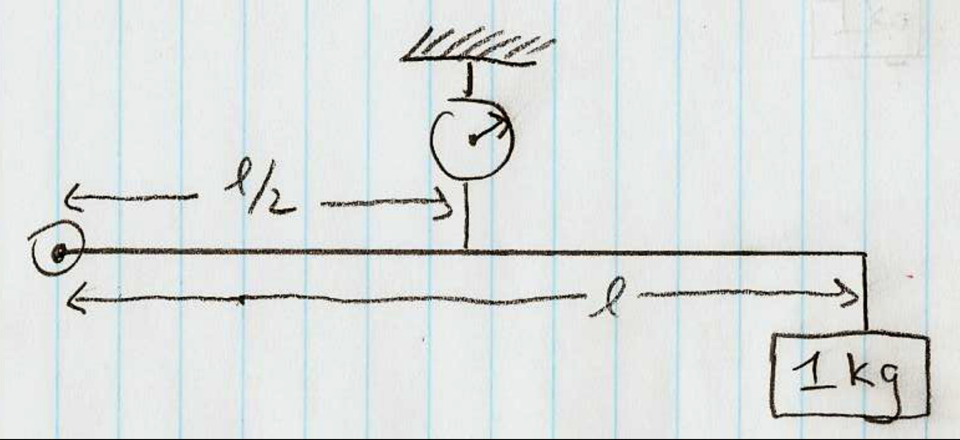
(E) 14 N

(F) 20 N



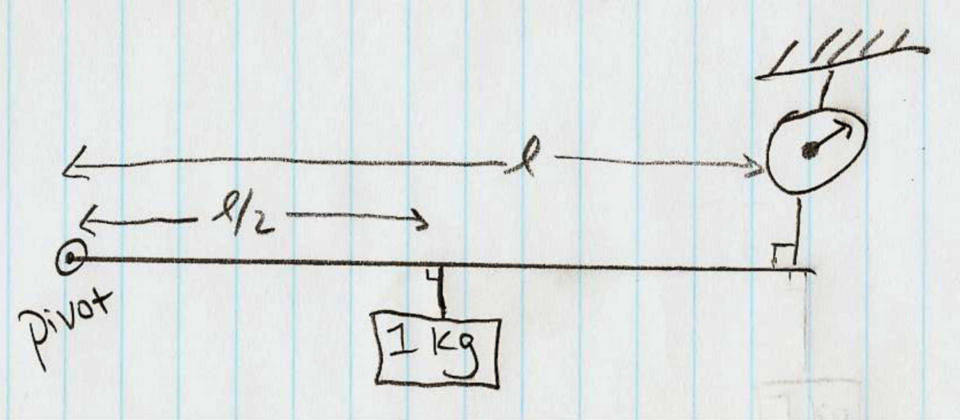
If the rod doesn't accelerate, what force does the scale read?

(A) 1.0 N
(B) 5.0 N
(C) 7.1 N
(D) 10 N
(E) 14 N
(F) 20 N



If the rod doesn't accelerate, what force does the scale read?

(A) 1.0 N
(B) 5.0 N
(C) 7.1 N
(D) 10 N
(E) 14 N
(F) 20 N



If the rod doesn't accelerate, what force does the scale read?

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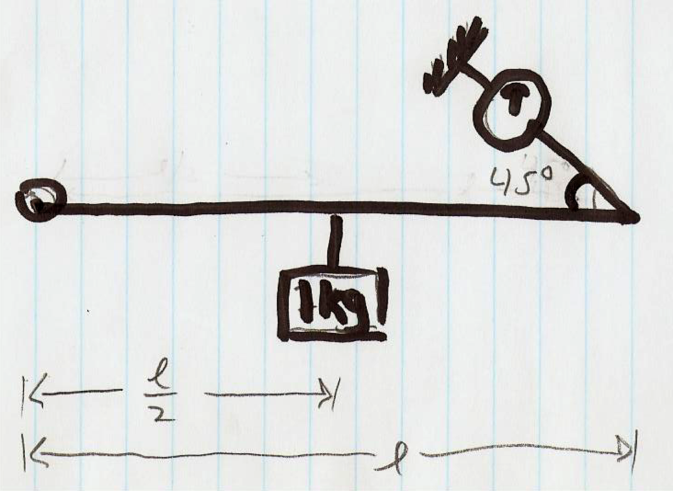
(E) 14 N
(F) 20 N



If the rod doesn't accelerate, what force does the scale read?

(A) 1.0 N
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(C) 7.1 N
(D) 10 N
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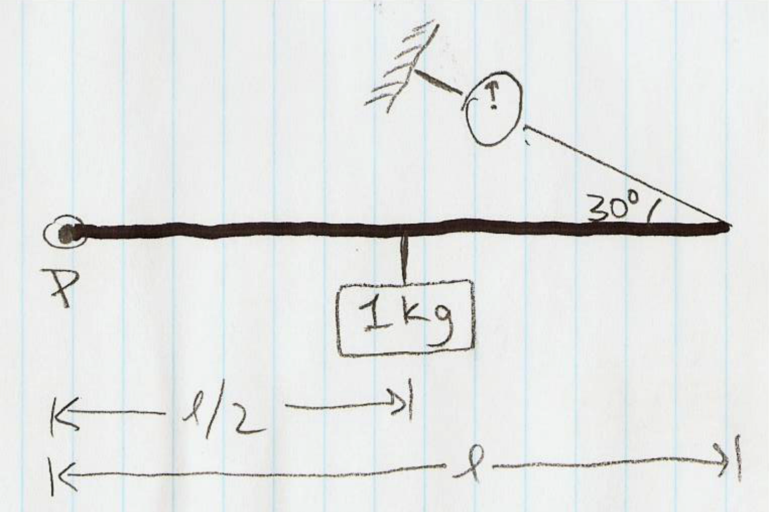
(F) 20 N



$$\tau = r_\perp F = r F_\perp = rF\sin\theta_{rF} = |\vec{r} \times \vec{F}|$$

Four ways to get the magnitude of the torque due to a force:

- (perpendicular component of distance) $\times$ (force)
- (distance) $\times$ (perpendicular component of force)
- (distance) (force) ($\sin\theta$ between $\vec{r}$ and $\vec{F}$)
- use magnitude of "vector product" $\vec{r} \times \vec{F}$ (a.k.a. "cross product")

To get the "direction" of a torque, use the right-hand rule.

Note right-hand rule for vector product $\vec{\tau} = \vec{r} \times \vec{F}$.

Note that most screws have “right-handed” threads.

Turn “right” (clockwise) to tighten, turn “left” (counterclockwise) to loosen.

If you look at the face of a clock, whose hands are moving clockwise, do the rotational velocity vectors of the clock's hands point toward you or toward the clock?

(A) Toward me

(B) Toward the clock

(C) Neither — when I curl the fingers of my right hand toward the clock, my thumb points to the left, in the 9 o'clock direction

Let's use forces and torques to analyze the big red wheel that we first saw on Monday. The wheel has rotational inertia I.

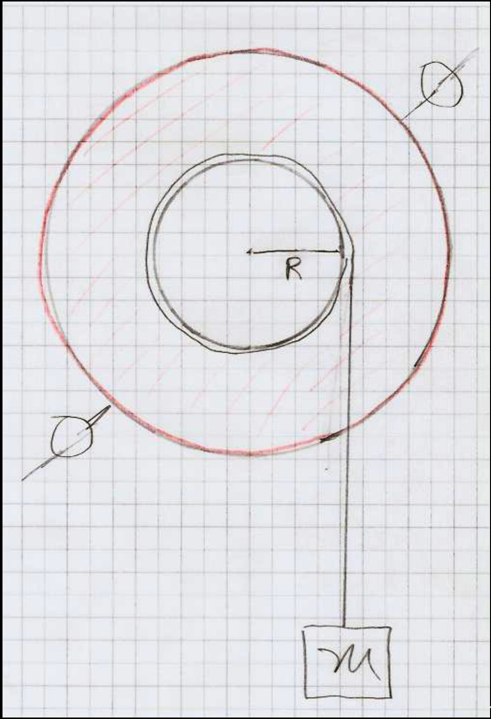
The string is wrapped at radius R, with an object of mass m dangling on the string.

For the dangling object, write

$$ma_y = \sum F_y$$

For the cylinder, write

$$I\alpha = \sum \tau$$



(Let's postpone this math until Friday.)

After some math, I get

$$\alpha = \frac{mgR}{I_{\text{wheel}} + mR^2} \approx \frac{mgR}{I_{\text{wheel}}}$$

(The approximation is for the limit where the object falls at $a \ll g$, so the string tension is $T = (mg - ma) \approx mg$.)

$$I\alpha = \tau = RT \Rightarrow T = \frac{I\alpha}{R}$$

$$ma = mg - T \Rightarrow m(\alpha R) = mg - \left(\frac{I\alpha}{R}\right)$$

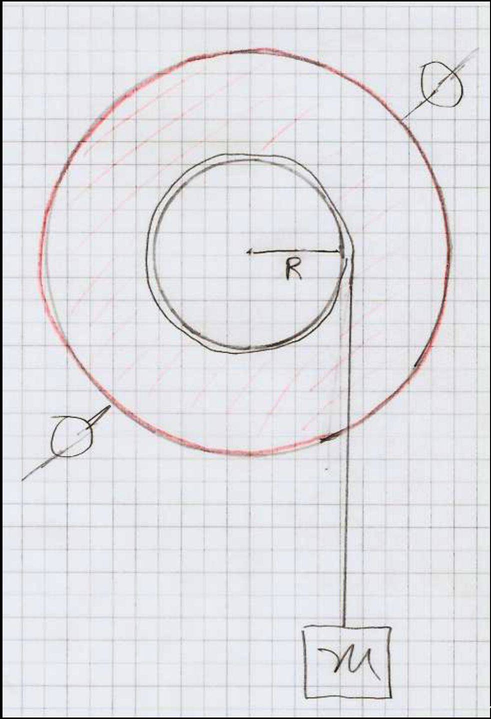
$$\alpha = \frac{g}{R} - \left(\frac{I}{mR^2}\right)\alpha \Rightarrow \alpha\left(1 + \frac{I}{mR^2}\right) = \frac{g}{R}$$

$$\alpha = \frac{g}{R\left(1 + \frac{I}{mR^2}\right)} = \frac{mgR}{I + mR^2}$$

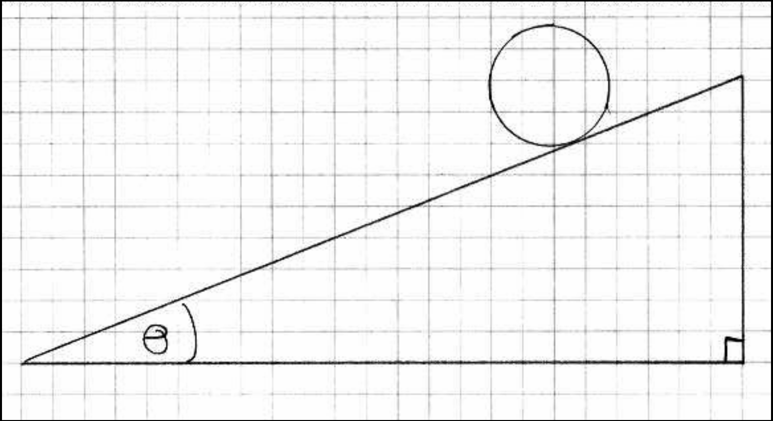
Why did increasing the dangling mass *m* **increase** the wheel's rotational acceleration α ?

Why did increasing the radius *R* from which the dangling mass was suspended **increase** the wheel's rotational acceleration?

Why did sliding the big rotating masses farther out on the extended "arms" **decrease** the wheel's rotational acceleration?



Let's go back and use torque to analyze another problem that last week we were only able to analyze using energy conservation: a cylinder rolling (without slipping) down an inclined plane.



What 3 forces act on the cylinder? What is the rotation axis? Draw FBD and extended FBD. What are the torque(s) about this axis? How are α and a related? Write $\vec{F} = m\vec{a}$ and $\tau = I\alpha$.

(Let's also postpone the math for this until Friday.)

$$I\alpha = \sum \tau$$

$$I\left(\frac{a_x}{R}\right) = RF^s$$

$$F^s = \left(\frac{I}{R^2}\right) a_x$$

$$ma_x = mg\sin\theta - F^s$$

$$ma_x = mg\sin\theta - \left(\frac{I}{R^2}\right) a_x$$

$$\left(m + \frac{I}{R^2}\right) a_x = mg\sin\theta$$

$$a_x = \frac{mg\sin\theta}{m + \frac{I}{R^2}} = \frac{g\sin\theta}{1 + \left(\frac{I}{mR^2}\right)}$$

Remember that the object with the larger "shape factor" $I/(mR^2)$ rolls downhill more slowly.

Torques are important in architecture because they allow us to determine the conditions for a structure to stay put.

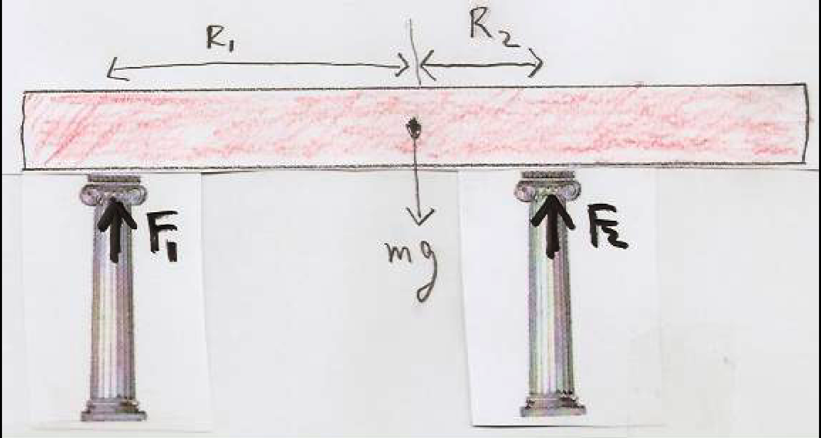
For an object (such as a structure or a part of a structure) to stay put, it must have zero acceleration, and it must have zero rotational acceleration.

So the vector sum of all forces must add to zero, and the sum of all torques (about any axis) must also be zero (to keep $\vec{a} = 0$ and $\alpha = 0$).

If these conditions are met, the object is in **equilibrium**: no unbalanced forces or torques.

Which column supports more of the beam's weight?

(A) Left column supports more than half of the beam's weight.
(B) Right column supports more than half of the beam's weight.
(C) Same. Each column supports half of the beam's weight.



Let's analyze this configuration, then demonstrate using two scales.

- How do I write $\sum F_y = 0$?
- What is a good choice of "rotation" axis here?
- How do I write $\sum \tau = 0$?
- What if I picked a different axis?

While we here, let's revisit the "center-of-mass chalkline" demonstration from a few weeks ago.

Now that we know about torque, we can see why the CoM always winds up directly beneath the pivot, once we understand that the line-of-action for gravity passes through the CoM.

(Depending on the time, we may do this Friday instead.)

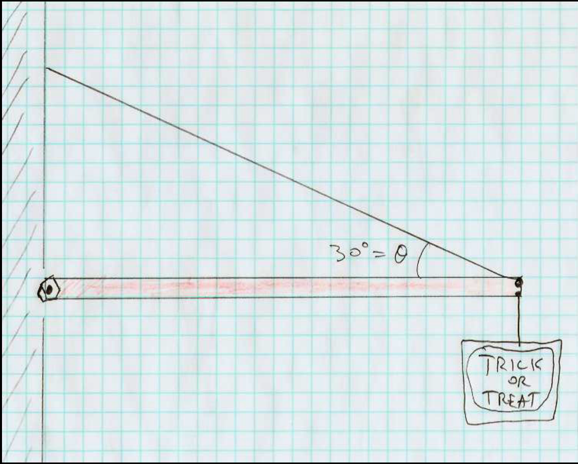
A beam of mass $M = 20$ kg and length $L = 2$ m is attached to a wall by a hinge. A sign of mass $m = 10$ kg hangs from the end of the beam. The end of the beam is supported by a cable (at $\theta = 30^\circ$ angle w.r.t. horizontal beam), which is anchored to the wall above the hinge.

What forces act on the beam? (Draw EFBD.)

Find the cable tension T.

Find the "reaction" forces F_x and F_y exerted by the hinge on the beam.

What 3 equations can we write for the beam? (Next few slides.)



(Redraw this on the board.)

Beam mass M, length L. Sign mass m. Cable angle θ from horizontal. Hinge exerts forces F_x, F_y on beam.

How do we write "sum of horizontal forces (on beam) $= 0$" ?

(A) $F_x + T\cos\theta = 0$

(B) $F_x + T\sin\theta = 0$

(C) $F_x - T\cos\theta = 0$

(D) $F_x - T\sin\theta = 0$

Beam mass M, length L. Sign mass m. Cable angle θ from horizontal. Hinge exerts forces F_x, F_y on beam.

How do we write "sum of vertical forces (on beam) = 0" ?

(A) $F_y + T\cos\theta + (M+m)g = 0$

(B) $F_y + T\cos\theta - (M+m)g = 0$

(C) $F_y + T\cos\theta - (M+m)g = 0$

(D) $F_y + T\sin\theta + (M+m)g = 0$

(E) $F_y + T\cos\theta - (M+m)g = 0$

(F) $F_y + T\sin\theta - (M+m)g = 0$

(G) $F_y - T\cos\theta - (M+m)g = 0$

(H) $F_y - T\sin\theta - (M+m)g = 0$

Beam mass M, length L. Sign mass m. Cable angle θ from horizontal. Hinge exerts forces F_x, F_y on beam.

How do we write "sum of torques (about hinge) = 0" ?

(A) $+\frac{L}{2}Mg + Lmg + LT\cos\theta = 0$

(B) $+\frac{L}{2}Mg + Lmg + LT\sin\theta = 0$

(C) $-\frac{L}{2}Mg + Lmg + LT\cos\theta = 0$

(D) $-\frac{L}{2}Mg + Lmg + LT\sin\theta = 0$

(E) $-\frac{L}{2}Mg - Lmg + LT\cos\theta = 0$

(F) $-\frac{L}{2}Mg - Lmg + LT\sin\theta = 0$

The 3 equations for static equilibrium in the *xy* plane

sum of horizontal forces $= 0$:

$$F_x - T\cos\theta = 0$$

sum of vertical forces $= 0$:

$$F_y + T\sin\theta - (M+m)g = 0$$

sum of torques (a.k.a. moments) about hinge $= 0$:

$$-\frac{L}{2}Mg - Lmg + LT\sin\theta = 0$$

Here's my solution: let's compare with the demonstration

$$F_x - T\cos\theta = 0$$

$$F_y - Mg - mg + T\sin\theta = 0$$

$$-\left(\frac{L}{2}\right)(Mg) - (L)(mg) + (L)(T\sin\theta) = 0$$

$$LT\sin\theta = Lmg + \frac{1}{2}LMg = L\left(m + \frac{M}{2}\right)g$$

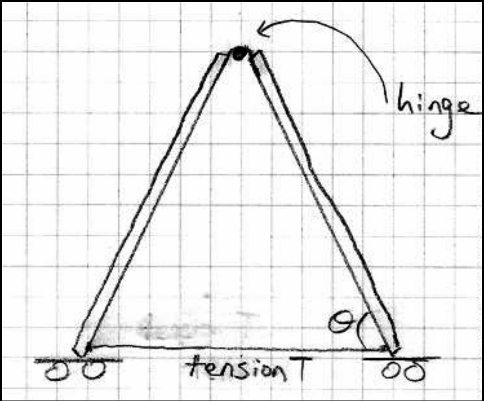
$$T = \frac{\left(m + \frac{M}{2}\right)g}{\sin\theta}$$

$$F_x = T\cos\theta = \frac{\left(m + \frac{M}{2}\right)g}{\tan\theta}$$

$$F_y = (M+m)g - T\sin\theta = (M+m)g - \left(m + \frac{M}{2}\right)g = \frac{Mg}{2}$$

Let's build & measure a simplified arch

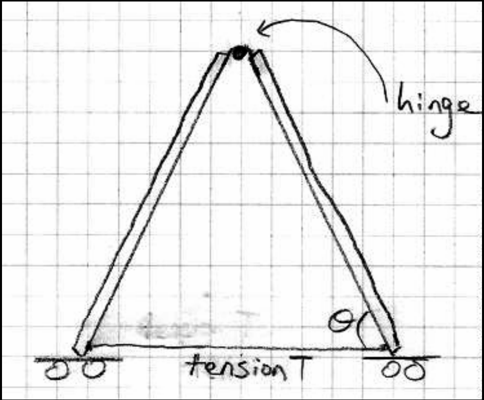
Often the essence of physics is to reduce a complicated problem to a similar problem that is easier to analyze.



(Does this make the function of a "roller support" more obvious?!)

(We'll emphasize function over form here . . .)

Often the essence of physics is to reduce a complicated problem to a similar problem that is easier to analyze. Use a cable to hold bottom together so that we can use scale to measure tension.

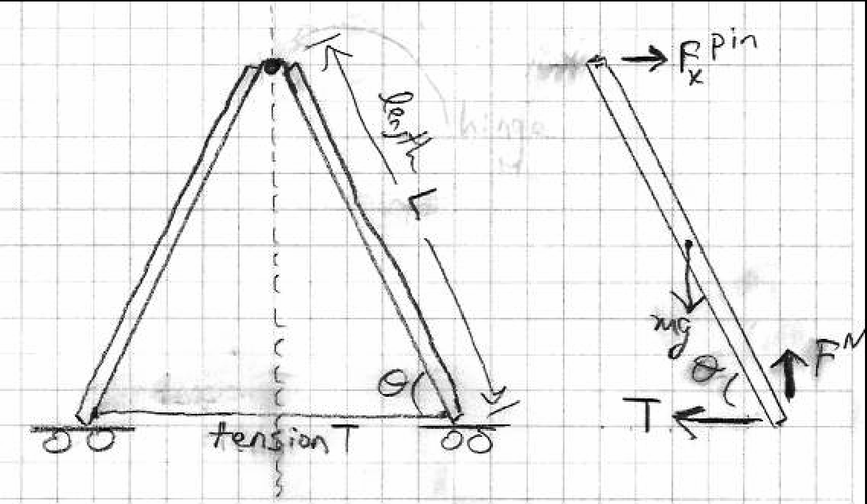


Weight (mg) of each side is 20 N.

We'll exploit mirror symmetry and analyze just one side of arch.

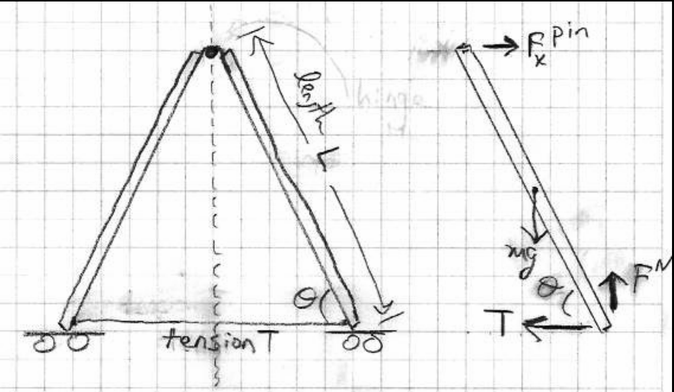
What forces act (and where) on the r.h.s. of the arch? **(Draw EFBD for the right-hand board.)**

Use a cable to hold bottom of "arch" together so that we can use scale to measure tension. Weight (mg) of each side is 20 N. We'll exploit mirror symmetry and analyze just one side of arch.



Right side shows EFBD for right-hand board.

How many unknown variables is it possible to determine using the equations for static equilibrium in a plane?



(A) one

(B) two

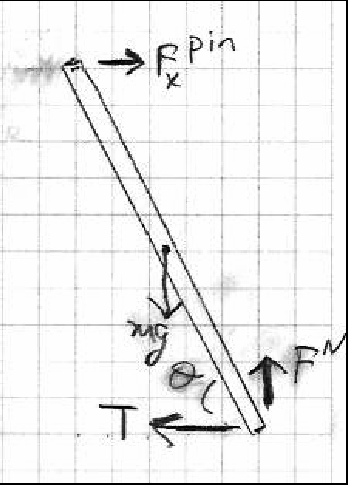
(C) three

(D) four

(E) five

Static equilibrium lets us write down three equations for a given object: $\sum F_x = 0$, $\sum F_y = 0$, $\sum M_z = 0$. Let's first sum up the "moments" (a.k.a. torques) **about the top hinge.**

Which statement correctly expresses $\sum M_z = 0$ (a.k.a. $\sum \tau = 0$)? (Let the mass and length of each wooden board be L and m.)



(A) $-mg\left(\frac{L}{2}\right)\cos\theta + F^N L\cos\theta - TL\sin\theta = 0$

(B) $-mg\left(\frac{L}{2}\right)\cos\theta + F^N L\cos\theta + TL\sin\theta = 0$

(C) $-mg\left(\frac{L}{2}\right)\sin\theta + F^N L\sin\theta - TL\cos\theta = 0$

(D) $-mg\left(\frac{L}{2}\right)\sin\theta + F^N L\sin\theta + TL\cos\theta = 0$

(E) $+mg\left(\frac{L}{2}\right)\cos\theta + F^N L\cos\theta - TL\sin\theta = 0$

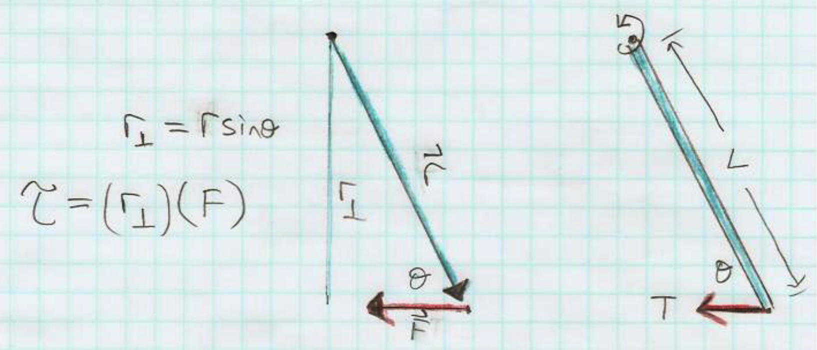
(F) $+mg\left(\frac{L}{2}\right)\cos\theta + F^N L\cos\theta + TL\sin\theta = 0$

(G) $+mg\left(\frac{L}{2}\right)\sin\theta + F^N L\sin\theta - TL\cos\theta = 0$

(H) $+mg\left(\frac{L}{2}\right)\sin\theta + F^N L\sin\theta + TL\cos\theta = 0$

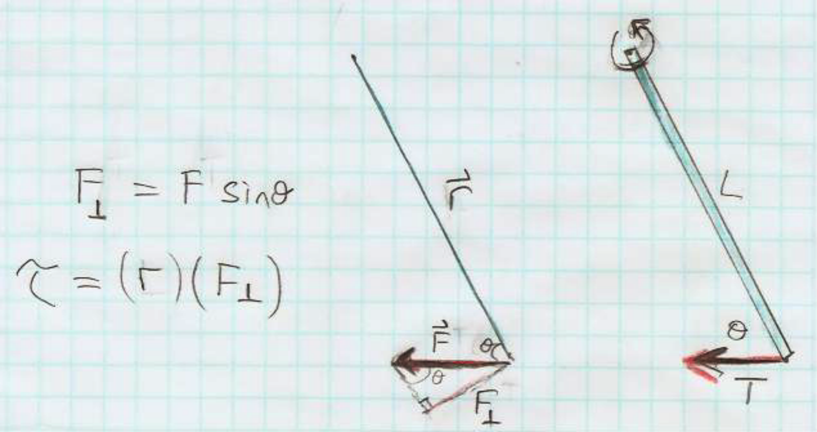
Let's start with torque (about top hinge) due to tension T.

- Usual convention: clockwise = negative, ccw = positive.
- Draw vector $\vec{r}$ from pivot to point where force is applied.
- Draw force vector $\vec{F}$, with its line-of-action passing through the point where the force is applied.
- Decompose $\vec{r}$ to find component $r_\perp$ that is perpendicular to $\vec{F}$. The component $r_\perp$ is called the "lever arm."
- Magnitude of torque is $|\tau| = (r_\perp)(F)$.



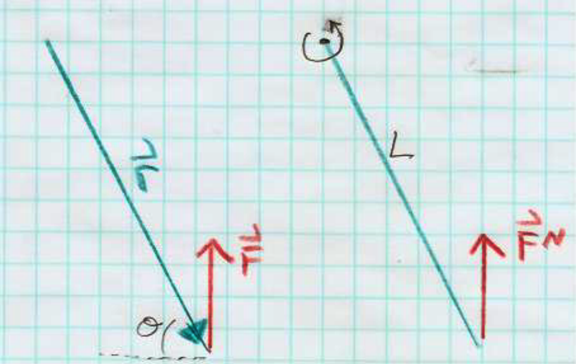
Alternative method: use $(r)(F_\perp)$ instead of $(r_\perp)(F)$.

- Draw vector $\vec{r}$ from pivot to point where force is applied.
- Draw force vector $\vec{F}$, with its line-of-action passing through the point where the force is applied.
- Decompose $\vec{F}$ to find component $F_\perp$ perpendicular to $\vec{r}$.
- Magnitude of torque is $|\tau| = (r)(F_\perp)$.



Now you try it for the normal force $\vec{F}^N$.

- Draw vector $\vec{r}$ from pivot to point where force is applied.
- Draw force vector $\vec{F}$, with its line-of-action passing through the point where the force is applied.
- Decompose $\vec{r}$ to find component $r_\perp$ that is perpendicular to $\vec{F}$. The component $r_\perp$ is called the "lever arm."
- Magnitude of torque is $|\tau| = r_\perp F$.



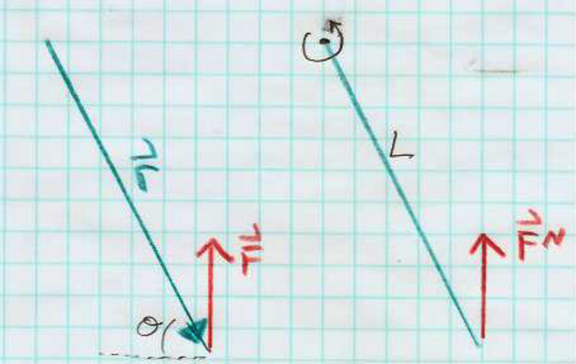
Which component of $\vec{r}$ is perpendicular to the normal force $\vec{F}^N$?

(A) horizontal component

(B) vertical component

Now you try it for the normal force $\vec{F}^N$.

- Draw vector $\vec{r}$ from pivot to point where force is applied.
- Draw force vector $\vec{F}$, with its line-of-action passing through the point where the force is applied.
- Decompose $\vec{r}$ to find component $r_\perp$ that is perpendicular to $\vec{F}$. The component $r_\perp$ is called the "lever arm."
- Magnitude of torque is $|\tau| = r_\perp F$.



How long is the horizontal component of $\vec{r}$ (i.e. the $\vec{r}$ component which is perpendicular to $\vec{F}$) ?

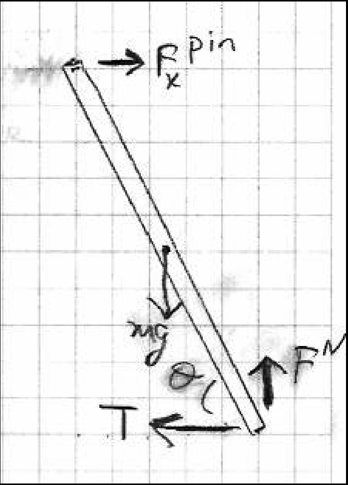
(A) $L\cos\theta$

(B) $L\sin\theta$

(C) $L\tan\theta$

OK, now back to the original question: Let's sum up the "moments" (a.k.a. torques) **about the top hinge.**

Which statement correctly expresses $\sum M_z = 0$ (a.k.a. $\sum \tau = 0$)? (Let the mass and length of each wooden board be L and m.)



(A) $-mg\left(\frac{L}{2}\right)\cos\theta + F^N L\cos\theta - TL\sin\theta = 0$

(B) $-mg\left(\frac{L}{2}\right)\cos\theta + F^N L\cos\theta + TL\sin\theta = 0$

(C) $-mg\left(\frac{L}{2}\right)\sin\theta + F^N L\sin\theta - TL\cos\theta = 0$

(D) $-mg\left(\frac{L}{2}\right)\sin\theta + F^N L\sin\theta + TL\cos\theta = 0$

(E) $+mg\left(\frac{L}{2}\right)\cos\theta + F^N L\cos\theta - TL\sin\theta = 0$

(F) $+mg\left(\frac{L}{2}\right)\cos\theta + F^N L\cos\theta + TL\sin\theta = 0$

(G) $+mg\left(\frac{L}{2}\right)\sin\theta + F^N L\sin\theta - TL\cos\theta = 0$

(H) $+mg\left(\frac{L}{2}\right)\sin\theta + F^N L\sin\theta + TL\cos\theta = 0$

Next: what about $\sum F_x = 0$ and $\sum F_y = 0$?

Next: what about $\sum F_x = 0$ and $\sum F_y = 0$?

$$-mg\left(\frac{L}{2}\right)\cos\theta + F^N L\cos\theta - TL\sin\theta = 0$$

$$\sum F_y = 0 \Rightarrow -mg + F^N = 0$$

$$-mg\frac{L}{2}\cos\theta + mgL\cos\theta - TL\sin\theta = 0$$

$$mg\frac{L}{2}\cos\theta = TL\sin\theta$$

$$T = \frac{mg\cos\theta}{2\sin\theta} = \boxed{\frac{mg}{2\tan\theta} = T}$$

We said $mg = 20\ \mathrm{N}$, so we expect the string tension to be

$$T = \frac{10\ \mathrm{N}}{\tan\theta}$$

How would this change if we suspended a weight Mg from the hinge? (By symmetry, each side of arch carries **half** of this Mg.)

Another equilibrium problem!

The top end of a ladder of inertia m rests against a smooth (i.e. slippery) wall, and the bottom end rests on the ground. The coefficient of static friction between the ground and the ladder is μ_s. What is the minimum angle between the ground and the ladder such that the ladder does not slip?

Let's start by drawing an EFBD for the ladder.

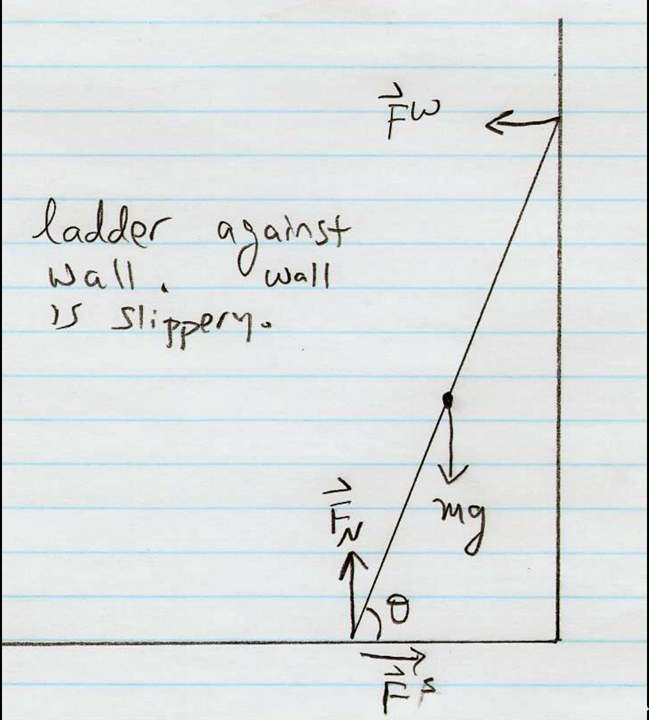
Why must we say the wall is slippery?

Is the slippery wall more like a pin or a roller support?

What plays the role here that string tension played in the previous problem?

Does the combination of two forces at the bottom act more like a pin or a roller support?

Which forces would an engineer call "reaction" forces?

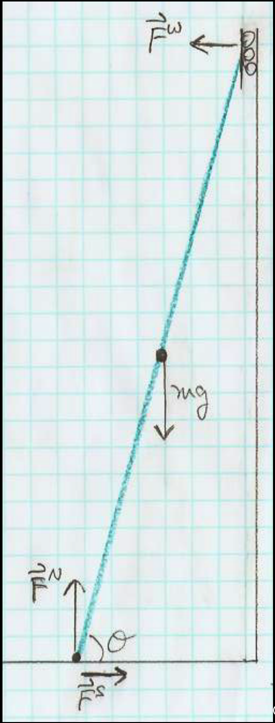


Which choice of pivot axis will give us the simplest equation for $\sum M_z = 0$? (We'll get an equation involving only two forces if we choose this axis.)

(A) Use bottom of ladder as pivot axis.

(B) Use center of ladder as pivot axis.

(C) Use top of ladder as pivot axis.



How would I write $\sum M_z = 0$ about the bottom end of the ladder? (Take length of ladder to be L.)

(A) $F^W L\cos\theta + mgL\sin\theta = 0$

(B) $F^W L\cos\theta + mg\frac{L}{2}\sin\theta = 0$

(C) $F^W L\cos\theta - mgL\sin\theta = 0$

(D) $F^W L\cos\theta - mg\frac{L}{2}\sin\theta = 0$

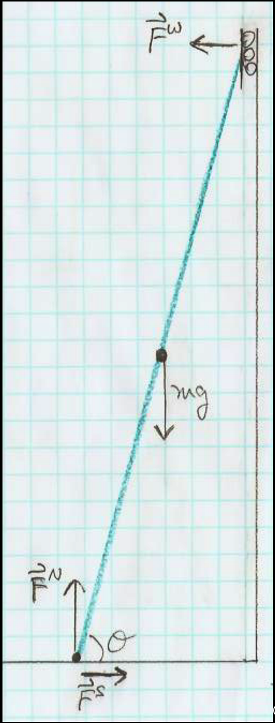
(E) $F^W L\sin\theta + mgL\cos\theta = 0$

(F) $F^W L\sin\theta + mg\frac{L}{2}\cos\theta = 0$

(G) $F^W L\sin\theta - mgL\cos\theta = 0$

(H) $F^W L\sin\theta - mg\frac{L}{2}\cos\theta = 0$

What do we learn from $\sum F_x = 0$ and $\sum F_y = 0$?



What do we learn from $\sum F_x = 0$ and $\sum F_y = 0$?

Let's answer the original question:

What is the minimum angle between the ground and the ladder such that the ladder does not slip?

Suppose we add to this picture a woman of mass M who has climbed up a distance d along the length of the ladder. Now how do we write the moment equation $\sum M_z = 0$?

(A) $F^W L \sin\theta - mg\frac{L}{2}\cos\theta + Mg\frac{d}{2}\cos\theta = 0$

(B) $F^W L \sin\theta - mg\frac{L}{2}\cos\theta + Mg\frac{d}{2}\sin\theta = 0$

(C) $F^W L \sin\theta - mg\frac{L}{2}\cos\theta + Mgd\cos\theta = 0$

(D) $F^W L \sin\theta - mg\frac{L}{2}\cos\theta + Mgd\sin\theta = 0$

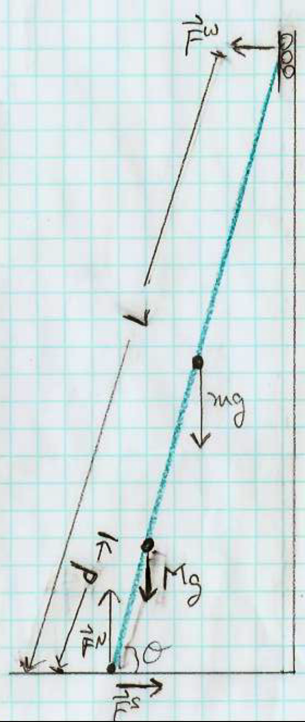
(E) $F^W L \sin\theta - mg\frac{L}{2}\cos\theta - Mg\frac{d}{2}\cos\theta = 0$

(F) $F^W L \sin\theta - mg\frac{L}{2}\cos\theta - Mg\frac{d}{2}\sin\theta = 0$

(G) $F^W L \sin\theta - mg\frac{L}{2}\cos\theta - Mgd\cos\theta = 0$

(H) $F^W L \sin\theta - mg\frac{L}{2}\cos\theta - Mgd\sin\theta = 0$

What do we learn from $\sum F_x = 0$ and $\sum F_y = 0$?

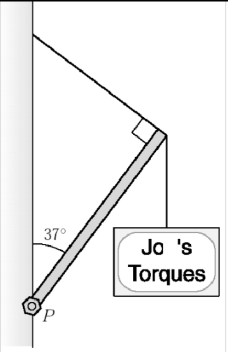


What do we learn from $\sum F_x = 0$ and $\sum F_y = 0$?

For a given θ, how far up can she climb before the ladder slips?

Here's a trickier equilibrium problem:

4*. You want to hang a 22 kg sign (shown at right) that advertises your new business. To do this, you attach a 7.0 kg beam of length 1.0 m to a wall at its base by a pivot P. You then attach a thin cable to the beam and to the wall in such a way that the cable and beam are perpendicular to each other. The beam makes an angle of 37° with the vertical. You hang the sign from the end of the beam to which the cable is attached. (a) What must be the minimum tensile strength of the cable (the amount of tension it can sustain) if it is not to snap? (b) Determine the horizontal and vertical components of the force the pivot exerts on the beam.



What forces act on the beam?

What 3 equations can we write for the beam?

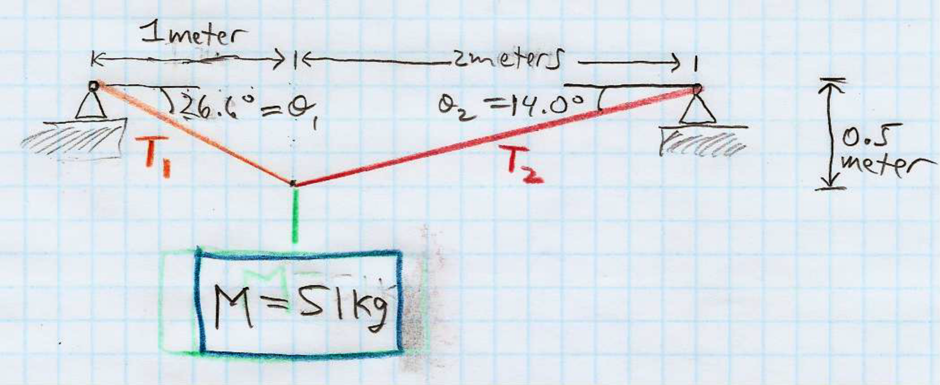
A tightly stretched "high wire" has length $L = 50\ \mathrm{m}$. It sags by $d = 1.0\ \mathrm{m}$ when a tightrope walker of mass $M = 51\ \mathrm{kg}$ stands at the center of the wire.

What is the tension in the wire?

Is it possible to increase the tension in the wire so that there is no sag at all (i.e. so that $d = 0$)?

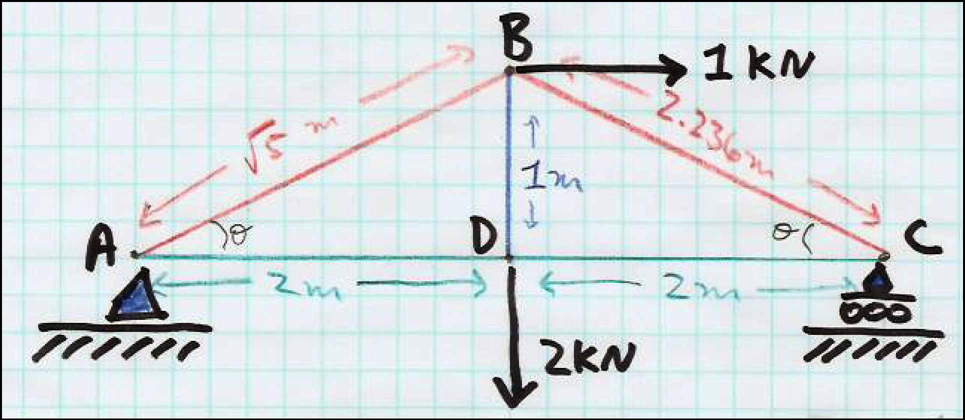
What happens to the tension as we make the sag smaller and smaller?

Now suppose a 51 kg sign is suspended from a cable (but not at the center), as shown below.



How would you find the tensions T_1 and T_2?

Once you know T_1 and T_2, what are the horizontal and vertical forces exerted by the two supports on the cable?



How many equations does the “method of joints” allow us to write down for this truss? (Consider how many joints the truss has.)

(A) 4 (B) 8 (C) 12 (D) 15

Physics 8 — Wednesday, October 30, 2019

- This week, you're reading Ch2 (statics) and Ch3 (determinate systems: equilibrium, trusses, arches) of Onouye/Kane. Feel free to buy one of my $10 used copies if you wish. At the end of the term, you can keep it, or sell it back to me for $10.
- HW8 due this Friday. HW help: Wed 4–6 (4–7?) [Bill] 3C4, Thu 6–8pm [Grace] 2C4.

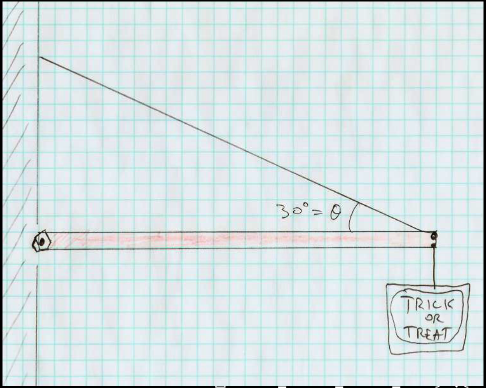

NO CAMPING

Physics 8 — Friday, November 1, 2019

- Turn in HW8. Pick up HW9 handout in back corner of room.
- This week, you read Ch2 (statics) and Ch3 (determinate systems: equilibrium, trusses, arches) of Onouye/Kane. Next week, you'll skim Ch4 (load tracing) and read Ch5 (strength of materials). Feel free to buy one of my \$10 used copies if you wish. At the end of the term, you can keep it, or sell it back to me for \$10.

A beam of mass $M = 20\ \mathrm{kg}$ and length $L = 2\ \mathrm{m}$ is attached to a wall by a hinge. A sign of mass $m = 10\ \mathrm{kg}$ hangs from the end of the beam. The end of the beam is supported by a cable (at $\theta = 30^\circ$ angle w.r.t. horizontal beam), which is anchored to the wall above the hinge.

Let's start by drawing an EFBD for the beam, showing each force acting on the beam and its line of action.



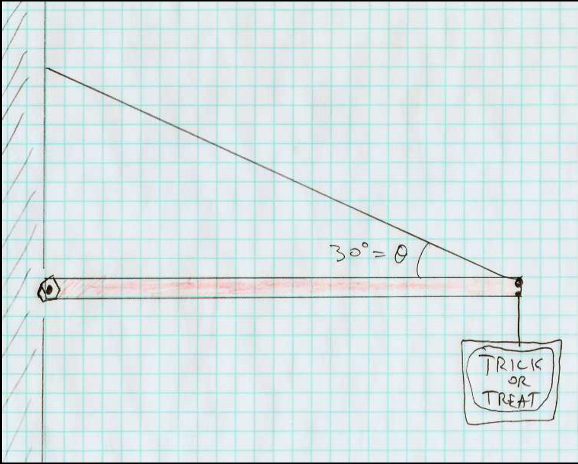
A beam of mass $M = 20$ kg and length $L = 2$ m is attached to a wall by a hinge. A sign of mass $m = 10$ kg hangs from the end of the beam. The end of the beam is supported by a cable (at $\theta = 30^\circ$ angle w.r.t. horizontal beam), which is anchored to the wall above the hinge.

What forces act on the beam? (Draw EFBD.)

Find the cable tension T.

Find the "reaction" forces F_x and F_y exerted by the hinge on the beam.

What 3 equations can we write for the beam? (Next few slides.)



(Redraw this on the board.)

Beam mass M, length L. Sign mass m. Cable angle θ from horizontal. Hinge exerts forces F_x, F_y on beam.

How do we write "sum of horizontal forces (on beam) $= 0$" ?

(A) $F_x + T\cos\theta = 0$

(B) $F_x + T\sin\theta = 0$

(C) $F_x - T\cos\theta = 0$

(D) $F_x - T\sin\theta = 0$

Beam mass M, length L. Sign mass m. Cable angle θ from horizontal. Hinge exerts forces F_x, F_y on beam.

How do we write "sum of vertical forces (on beam) = 0" ?

(A) $F_y + T\cos\theta + (M+m)g = 0$

(B) $F_y + T\cos\theta - (M+m)g = 0$

(C) $F_y + T\cos\theta - (M+m)g = 0$

(D) $F_y + T\sin\theta + (M+m)g = 0$

(E) $F_y + T\cos\theta - (M+m)g = 0$

(F) $F_y + T\sin\theta - (M+m)g = 0$

(G) $F_y - T\cos\theta - (M+m)g = 0$

(H) $F_y - T\sin\theta - (M+m)g = 0$

Beam mass M, length L. Sign mass m. Cable angle θ from horizontal. Hinge exerts forces F_x, F_y on beam.

How do we write "sum of torques (about hinge) = 0" ?

(A) $+\frac{L}{2}Mg + Lmg + LT\cos\theta = 0$

(B) $+\frac{L}{2}Mg + Lmg + LT\sin\theta = 0$

(C) $-\frac{L}{2}Mg + Lmg + LT\cos\theta = 0$

(D) $-\frac{L}{2}Mg + Lmg + LT\sin\theta = 0$

(E) $-\frac{L}{2}Mg - Lmg + LT\cos\theta = 0$

(F) $-\frac{L}{2}Mg - Lmg + LT\sin\theta = 0$

The 3 equations for static equilibrium in the *xy* plane

sum of horizontal forces = 0:

$$F_x - T\cos\theta = 0$$

sum of vertical forces = 0:

$$F_y + T\sin\theta - (M+m)g = 0$$

sum of torques (a.k.a. moments) about hinge = 0:

$$-\frac{L}{2}Mg - Lmg + LT\sin\theta = 0$$

Here's my solution

$$F_x - T\cos\theta = 0$$

$$F_y - Mg - mg + T\sin\theta = 0$$

$$-\left(\frac{L}{2}\right)(Mg) - (L)(mg) + (L)(T\sin\theta) = 0$$

$$LT\sin\theta = Lmg + \frac{1}{2}LMg = L\left(m + \frac{M}{2}\right)g$$

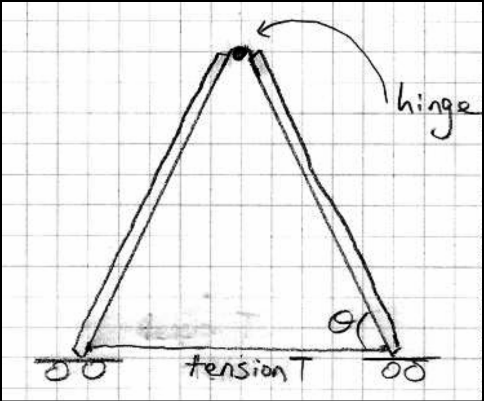
$$T = \frac{\left(m + \frac{M}{2}\right)g}{\sin\theta}$$

$$F_x = T\cos\theta = \frac{\left(m + \frac{M}{2}\right)g}{\tan\theta}$$

$$F_y = (M+m)g - T\sin\theta = (M+m)g - \left(m + \frac{M}{2}\right)g = \frac{Mg}{2}$$

Let's build & measure a simplified arch

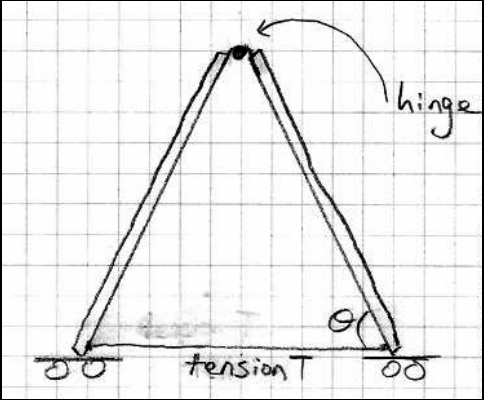
Often the essence of physics is to reduce a complicated problem to a similar problem that is easier to analyze.



(Does this make the function of a "roller support" more obvious?!)

(We'll emphasize function over form here . . .)

Often the essence of physics is to reduce a complicated problem to a similar problem that is easier to analyze. Use a cable to hold bottom together so that we can use scale to measure tension.

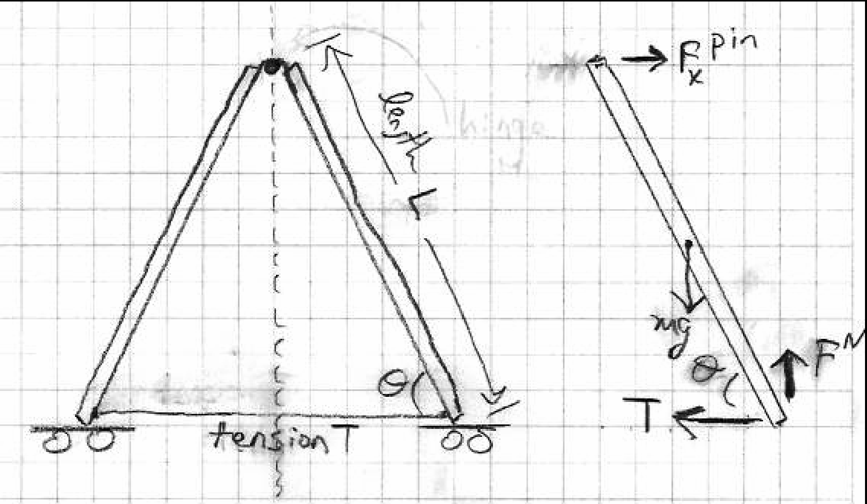


Weight (mg) of each side is 20 N.

We'll exploit mirror symmetry and analyze just one side of arch.

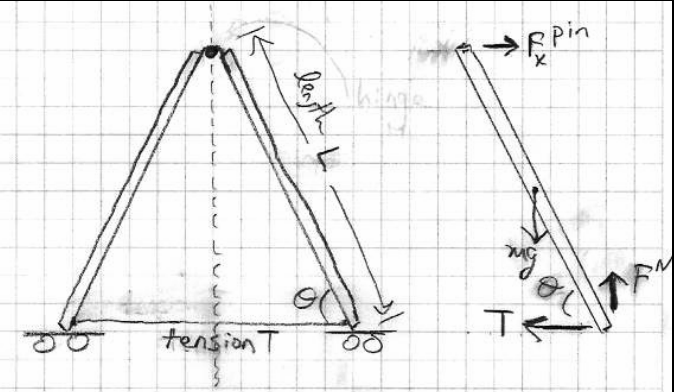
What forces act (and where) on the r.h.s. of the arch? **(Draw EFBD for the right-hand board.)**

Use a cable to hold bottom of "arch" together so that we can use scale to measure tension. Weight (mg) of each side is 20 N. We'll exploit mirror symmetry and analyze just one side of arch.



Right side shows EFBD for right-hand board.

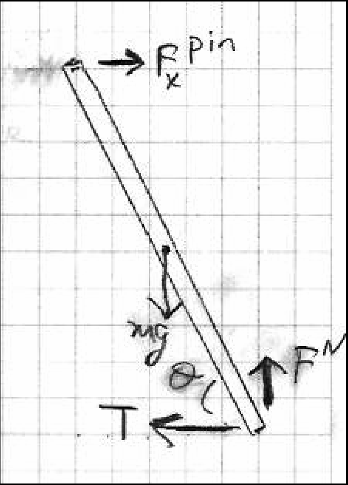
How many unknown variables is it possible to determine using the equations for static equilibrium in a plane?



(A) one
(B) two
(C) three
(D) four
(E) five

Static equilibrium lets us write down three equations for a given object: $\sum F_x = 0$, $\sum F_y = 0$, $\sum M_z = 0$. Let's first sum up the "moments" (a.k.a. torques) **about the top hinge.**

Which statement correctly expresses $\sum M_z = 0$ (a.k.a. $\sum \tau = 0$)? (Let the mass and length of each wooden board be L and m.)



(A) $-mg\left(\frac{L}{2}\right)\cos\theta + F^N L\cos\theta - TL\sin\theta = 0$

(B) $-mg\left(\frac{L}{2}\right)\cos\theta + F^N L\cos\theta + TL\sin\theta = 0$

(C) $-mg\left(\frac{L}{2}\right)\sin\theta + F^N L\sin\theta - TL\cos\theta = 0$

(D) $-mg\left(\frac{L}{2}\right)\sin\theta + F^N L\sin\theta + TL\cos\theta = 0$

(E) $+mg\left(\frac{L}{2}\right)\cos\theta + F^N L\cos\theta - TL\sin\theta = 0$

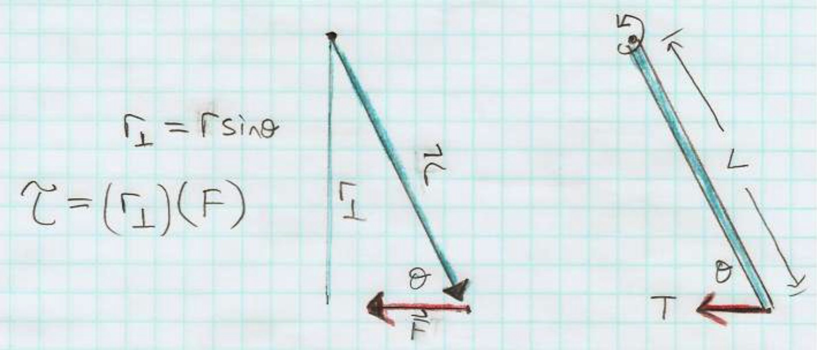
(F) $+mg\left(\frac{L}{2}\right)\cos\theta + F^N L\cos\theta + TL\sin\theta = 0$

(G) $+mg\left(\frac{L}{2}\right)\sin\theta + F^N L\sin\theta - TL\cos\theta = 0$

(H) $+mg\left(\frac{L}{2}\right)\sin\theta + F^N L\sin\theta + TL\cos\theta = 0$

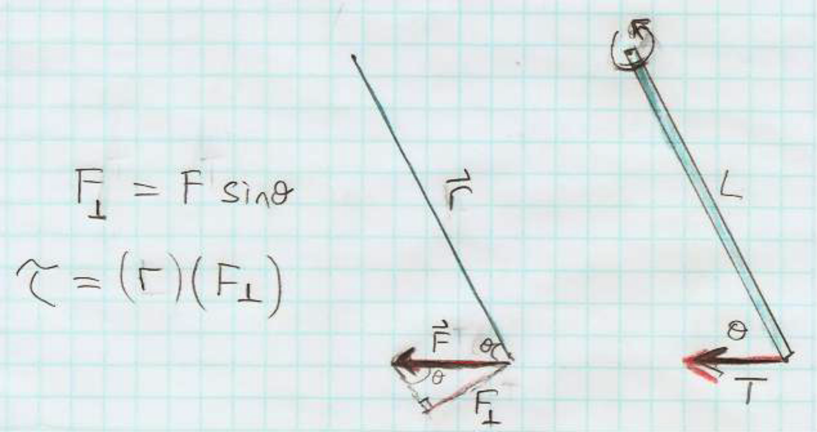
Let's start with torque (about top hinge) due to tension T.

- Usual convention: clockwise = negative, ccw = positive.
- Draw vector $\vec{r}$ from pivot to point where force is applied.
- Draw force vector $\vec{F}$, with its line-of-action passing through the point where the force is applied.
- Decompose $\vec{r}$ to find component $r_\perp$ that is perpendicular to $\vec{F}$. The component $r_\perp$ is called the "lever arm."
- Magnitude of torque is $|\tau| = (r_\perp)(F)$.



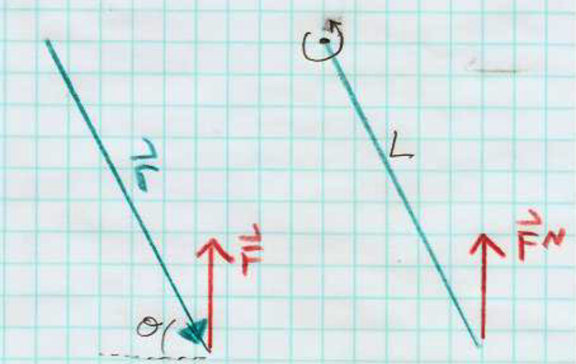
Alternative method: use $(r)(F_\perp)$ instead of $(r_\perp)(F)$.

- Draw vector $\vec{r}$ from pivot to point where force is applied.
- Draw force vector $\vec{F}$, with its line-of-action passing through the point where the force is applied.
- Decompose $\vec{F}$ to find component $F_\perp$ perpendicular to $\vec{r}$.
- Magnitude of torque is $|\tau| = (r)(F_\perp)$.



Now you try it for the normal force $\vec{F}^N$.

- Draw vector $\vec{r}$ from pivot to point where force is applied.
- Draw force vector $\vec{F}$, with its line-of-action passing through the point where the force is applied.
- Decompose $\vec{r}$ to find component $r_\perp$ that is perpendicular to $\vec{F}$. The component $r_\perp$ is called the "lever arm."
- Magnitude of torque is $|\tau| = r_\perp F$.



Which component of $\vec{r}$ is perpendicular to the normal force $\vec{F}^N$?

(A) horizontal component

(B) vertical component

Now you try it for the normal force $\vec{F}^N$.

- Draw vector $\vec{r}$ from pivot to point where force is applied.
- Draw force vector $\vec{F}$, with its line-of-action passing through the point where the force is applied.
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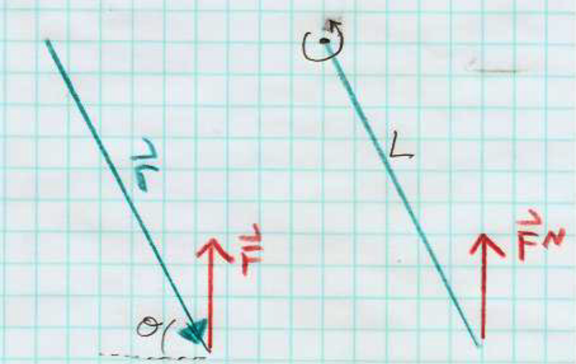


How long is the horizontal component of $\vec{r}$ (i.e. the $\vec{r}$ component which is perpendicular to $\vec{F}$) ?

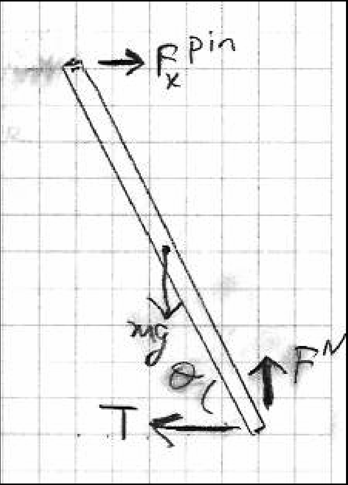
(A) $L \cos\theta$

(B) $L \sin\theta$

(C) $L \tan\theta$

OK, now back to the original question: Let's sum up the "moments" (a.k.a. torques) **about the top hinge.**

Which statement correctly expresses $\sum M_z = 0$ (a.k.a. $\sum \tau = 0$)? (Let the mass and length of each wooden board be L and m.)



(A) $-mg\left(\frac{L}{2}\right)\cos\theta + F^N L\cos\theta - TL\sin\theta = 0$

(B) $-mg\left(\frac{L}{2}\right)\cos\theta + F^N L\cos\theta + TL\sin\theta = 0$

(C) $-mg\left(\frac{L}{2}\right)\sin\theta + F^N L\sin\theta - TL\cos\theta = 0$

(D) $-mg\left(\frac{L}{2}\right)\sin\theta + F^N L\sin\theta + TL\cos\theta = 0$

(E) $+mg\left(\frac{L}{2}\right)\cos\theta + F^N L\cos\theta - TL\sin\theta = 0$

(F) $+mg\left(\frac{L}{2}\right)\cos\theta + F^N L\cos\theta + TL\sin\theta = 0$

(G) $+mg\left(\frac{L}{2}\right)\sin\theta + F^N L\sin\theta - TL\cos\theta = 0$

(H) $+mg\left(\frac{L}{2}\right)\sin\theta + F^N L\sin\theta + TL\cos\theta = 0$

Next: what about $\sum F_x = 0$ and $\sum F_y = 0$?

Next: what about $\sum F_x = 0$ and $\sum F_y = 0$?

$$-mg\left(\frac{L}{2}\right)\cos\theta + F^N L\cos\theta - TL\sin\theta = 0$$

$$\sum F_y = 0 \Rightarrow -mg + F^N = 0$$

$$-mg\frac{L}{2}\cos\theta + mgL\cos\theta - TL\sin\theta = 0$$

$$mg\frac{L}{2}\cos\theta = TL\sin\theta$$

$$T = \frac{mg\cos\theta}{2\sin\theta} = \boxed{\frac{mg}{2\tan\theta} = T}$$

We said $mg = 20\ \mathrm{N}$, so we expect the string tension to be

$$T = \frac{10\ \mathrm{N}}{\tan\theta}$$

How would this change if we suspended a weight Mg from the hinge? (By symmetry, each side of arch carries **half** of this Mg.)

Let's use forces and torques to analyze the big red wheel that we first saw on Monday. The wheel has rotational inertia I.

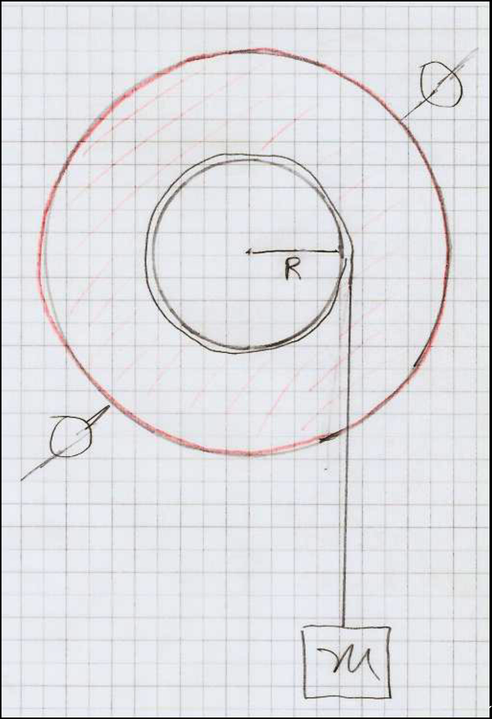
The string is wrapped at radius R, with an object of mass m dangling on the string.

For the dangling object, write

$$ma_y = \sum F_y$$

For the cylinder, write

$$I\alpha = \sum \tau$$



After some math, I get

$$\alpha = \frac{mgR}{I_{\text{wheel}} + mR^2} \approx \frac{mgR}{I_{\text{wheel}}}$$

(The approximation is for the limit where the object falls at $a \ll g$, so the string tension is $T = (mg - ma) \approx mg$.)

$$I\alpha = \tau = RT \Rightarrow T = \frac{I\alpha}{R}$$

$$ma = mg - T \Rightarrow m(\alpha R) = mg - \left(\frac{I\alpha}{R}\right)$$

$$\alpha = \frac{g}{R} - \left(\frac{I}{mR^2}\right)\alpha \Rightarrow \alpha\left(1 + \frac{I}{mR^2}\right) = \frac{g}{R}$$

$$\alpha = \frac{g}{R\left(1 + \frac{I}{mR^2}\right)} = \frac{mgR}{I + mR^2}$$

Why did increasing the dangling mass m **increase** the wheel's rotational acceleration α ?

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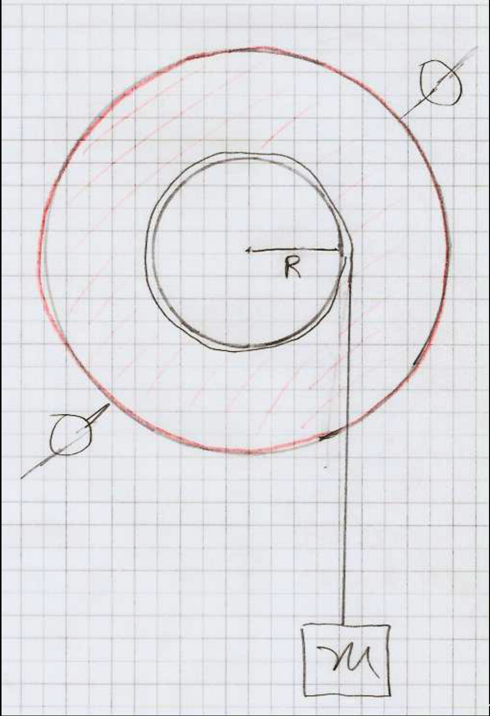

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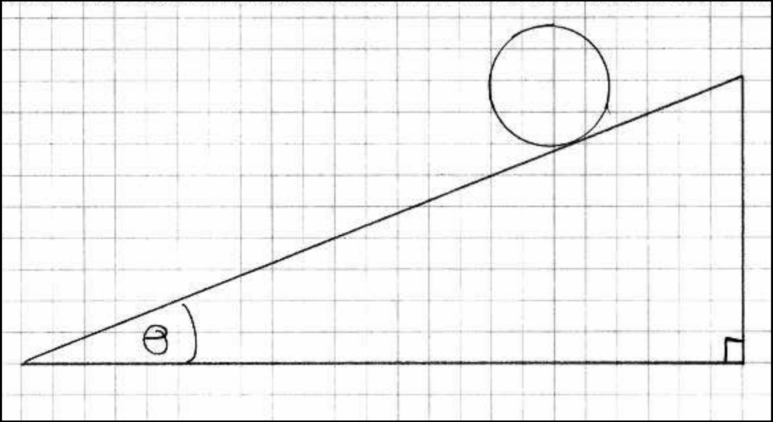

What 3 forces act on the cylinder? What is the rotation axis? Draw FBD and extended FBD. What are the torque(s) about this axis? How are α and a related? Write $\vec{F} = m\vec{a}$ and $\tau = I\alpha$.

$$I\alpha = \sum \tau$$

$$I\left(\frac{a_x}{R}\right) = RF^s$$

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Remember that the object with the larger “shape factor” $I/(mR^2)$ rolls downhill more slowly.

While we’re here, let’s revisit the “center-of-mass chalkline” demonstration from a few weeks ago.

Now that we know about torque, we can see why the CoM always winds up directly beneath the pivot, once we understand that the line-of-action for gravity passes through the CoM.

Another equilibrium problem!

The top end of a ladder of inertia m rests against a smooth (i.e. slippery) wall, and the bottom end rests on the ground. The coefficient of static friction between the ground and the ladder is μ_s. What is the minimum angle between the ground and the ladder such that the ladder does not slip?

Let's start by drawing an EFBD for the ladder.

Why must we say the wall is slippery?

Is the slippery wall more like a pin or a roller support?

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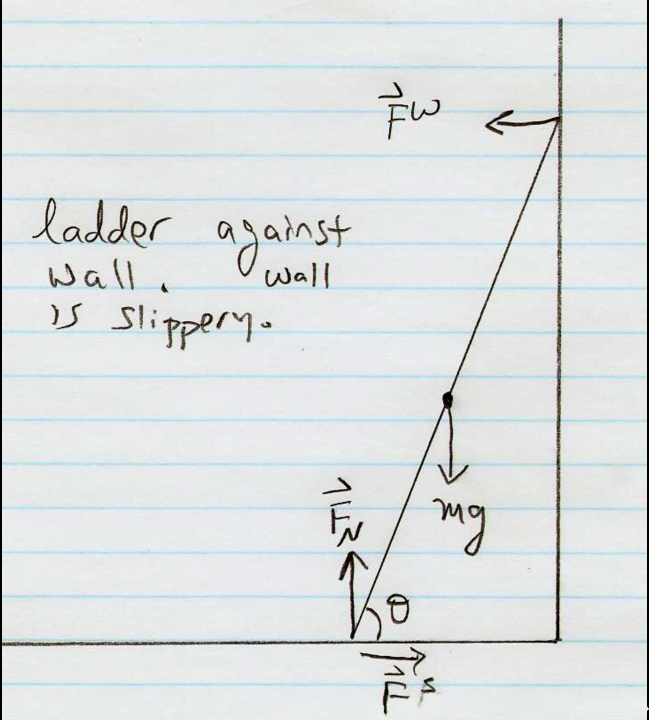

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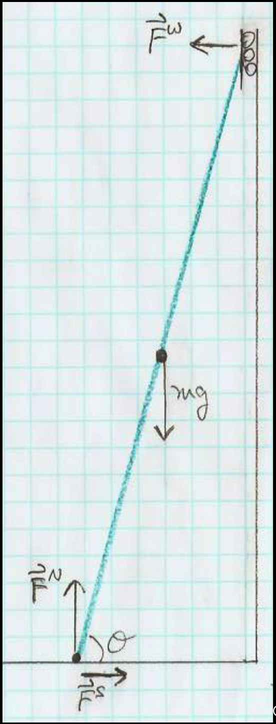

How would I write $\sum M_z = 0$ about the bottom end of the ladder? (Take length of ladder to be L.)

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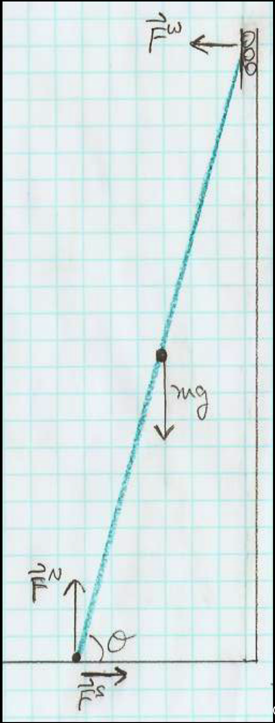

What do we learn from $\sum F_x = 0$ and $\sum F_y = 0$?

Let's answer the original question:

What is the minimum angle between the ground and the ladder such that the ladder does not slip?

Suppose we add to this picture a woman of mass M who has climbed up a distance d along the length of the ladder. Now how do we write the moment equation $\sum M_z = 0$?

(A) $F^W L \sin\theta - mg\frac{L}{2}\cos\theta + Mg\frac{d}{2}\cos\theta = 0$

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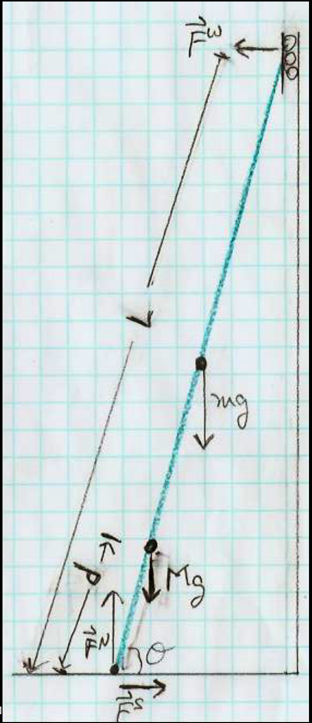
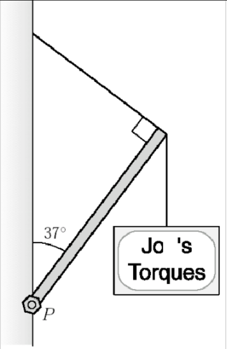


What do we learn from $\sum F_x = 0$ and $\sum F_y = 0$?

For a given θ, how far up can she climb before the ladder slips?

Here's a trickier equilibrium problem:

4*. You want to hang a 22 kg sign (shown at right) that advertises your new business. To do this, you attach a 7.0 kg beam of length 1.0 m to a wall at its base by a pivot P. You then attach a thin cable to the beam and to the wall in such a way that the cable and beam are perpendicular to each other. The beam makes an angle of 37° with the vertical. You hang the sign from the end of the beam to which the cable is attached. (a) What must be the minimum tensile strength of the cable (the amount of tension it can sustain) if it is not to snap? (b) Determine the horizontal and vertical components of the force the pivot exerts on the beam.



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What 3 equations can we write for the beam?

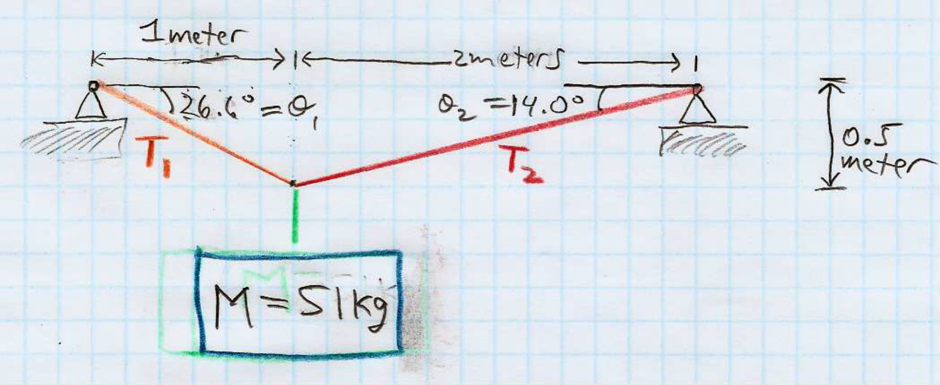
A tightly stretched “high wire” has length $L = 50\ \mathrm{m}$. It sags by $d = 1.0\ \mathrm{m}$ when a tightrope walker of mass $M = 51\ \mathrm{kg}$ stands at the center of the wire.

What is the tension in the wire?

Is it possible to increase the tension in the wire so that there is no sag at all (i.e. so that $d = 0$)?

What happens to the tension as we make the sag smaller and smaller?

Now suppose a 51 kg sign is suspended from a cable (but not at the center), as shown below.



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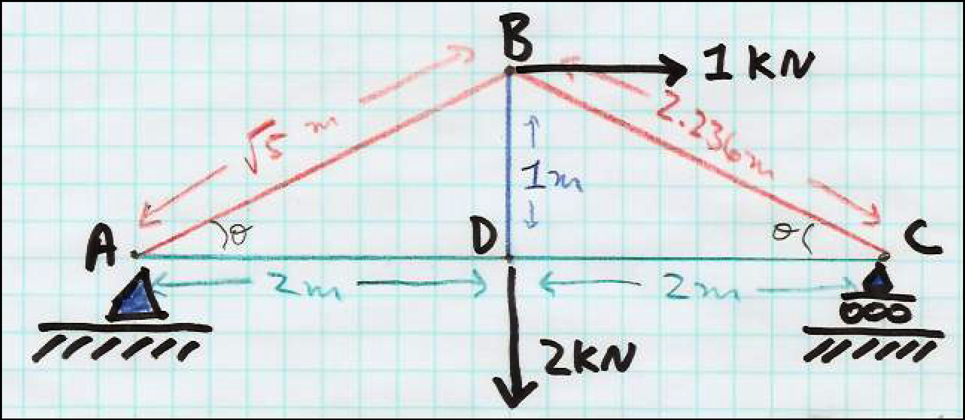
Once you know T_1 and T_2, what are the horizontal and vertical forces exerted by the two supports on the cable?



How many equations does the "method of joints" allow us to write down for this truss? (Consider how many joints the truss has.)

(A) 4 (B) 8 (C) 12 (D) 15

Physics 8 — Friday, November 1, 2019

- Turn in HW8. Pick up HW9 handout in back corner of room.
- This week, you read Ch2 (statics) and Ch3 (determinate systems: equilibrium, trusses, arches) of Onouye/Kane. Next week, you'll skim Ch4 (load tracing) and read Ch5 (strength of materials). Feel free to buy one of my $10 used copies if you wish. At the end of the term, you can keep it, or sell it back to me for $10.

Physics 8 — Monday, November 4, 2019

- I finally added summaries of key results from Onouye/Kane ch1-ch7 to the "equation sheet." I'm working on ch8, and I may do ch9 as well (though ch9 will be XC for you).
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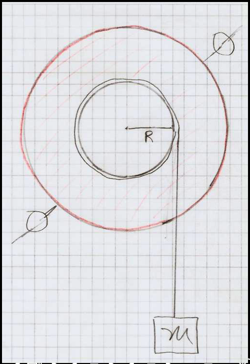
Let's use forces and torques to analyze the big red wheel that we first saw last Monday. The wheel has rotational inertia I. The string is wrapped at radius R, with an object of mass m dangling on the string. For the dangling object, write

$$ma_y = \sum F_y$$

For the cylinder, write

$$I\alpha = \sum \tau$$



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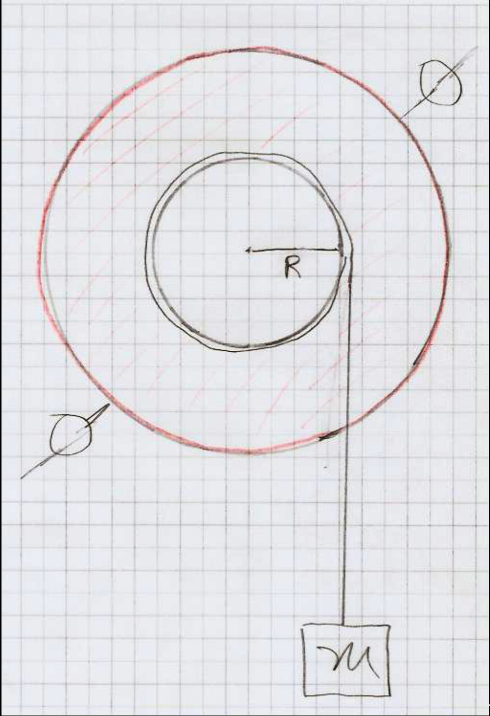
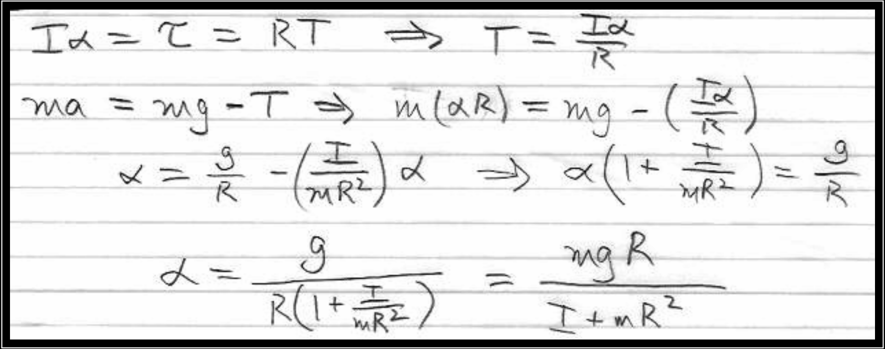


After some math, I get

$$\alpha = \frac{mgR}{I_{\rm wheel} + mR^2} \approx \frac{mgR}{I_{\rm wheel}}$$

(The approximation is for the limit where the object falls at $a \ll g$, so the string tension is $T = (mg - ma) \approx mg$.)



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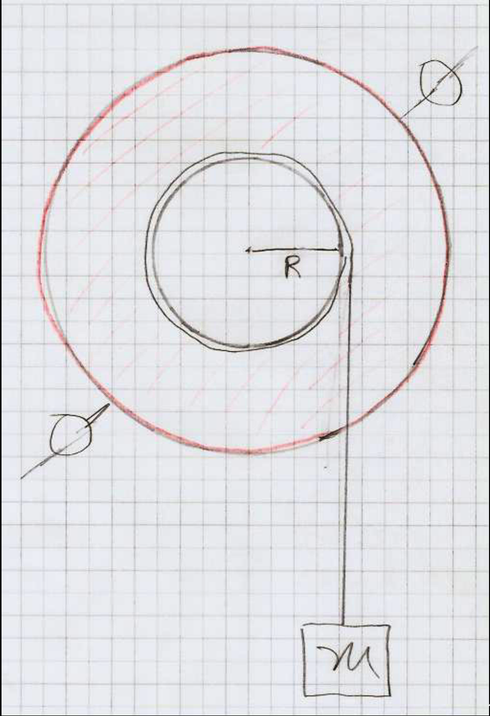

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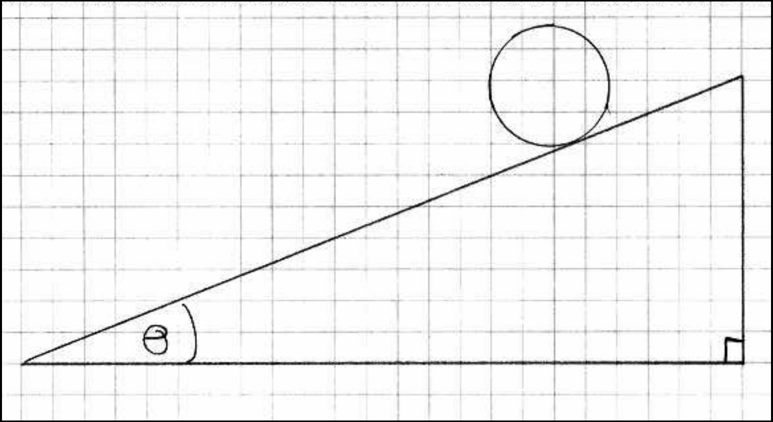

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We stopped after this.

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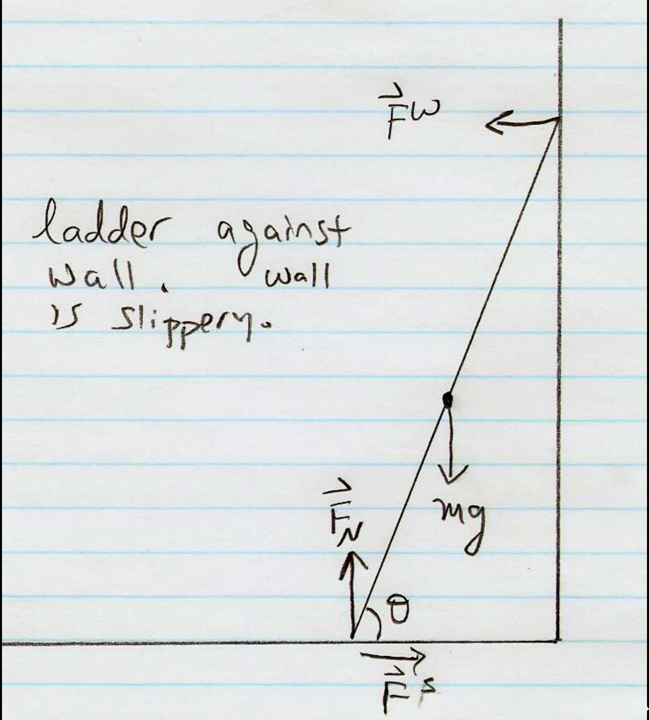

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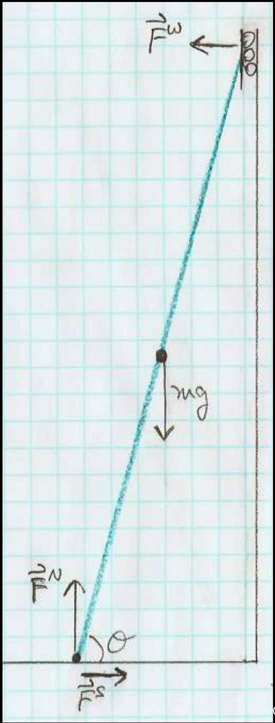

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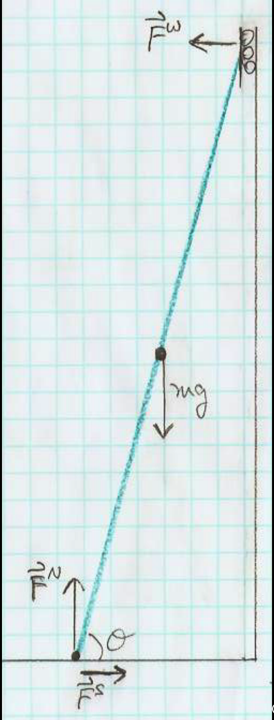

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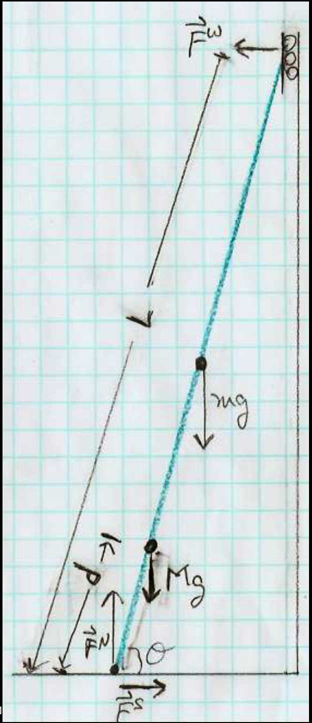

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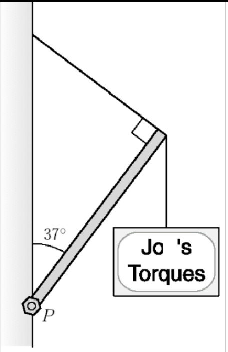

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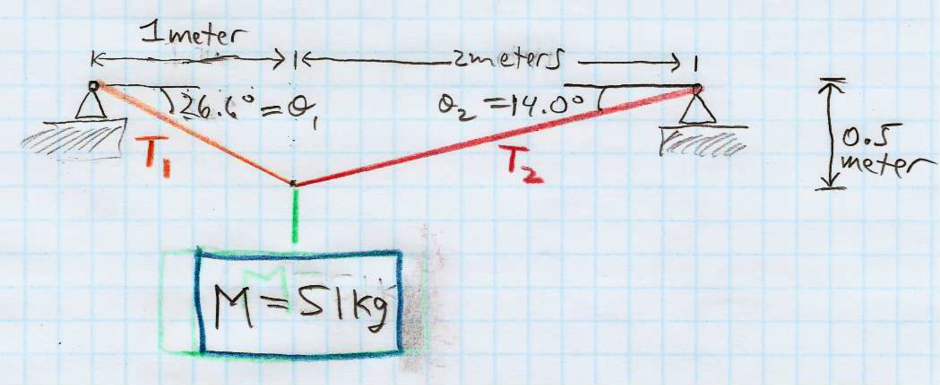

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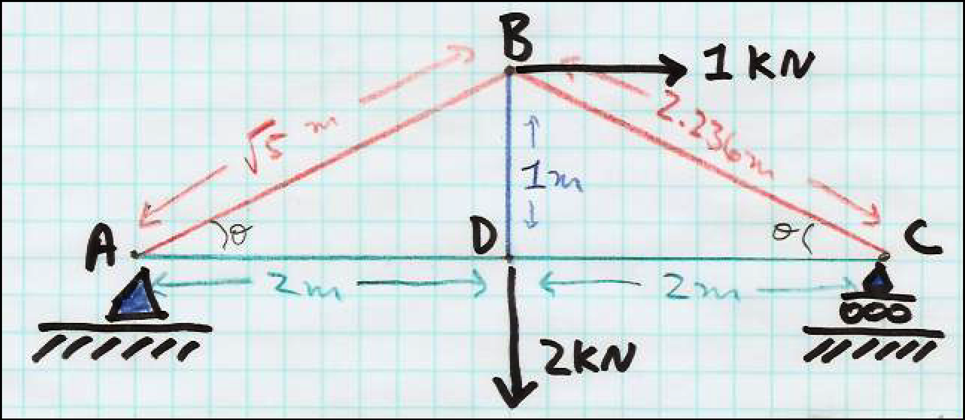
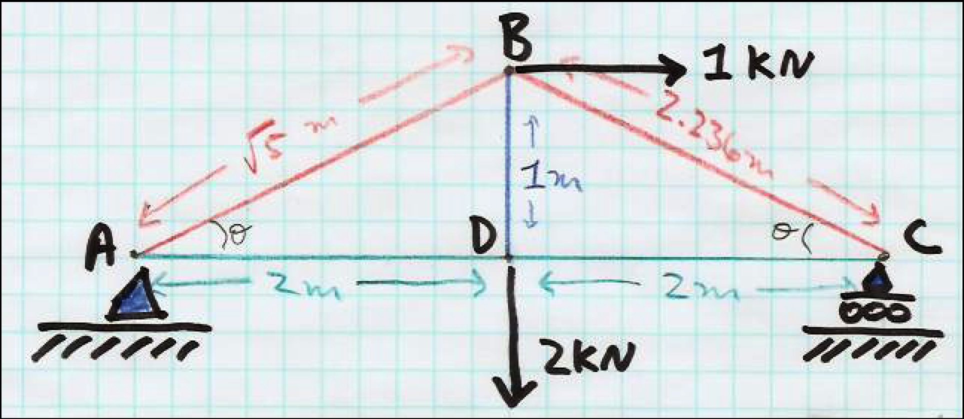


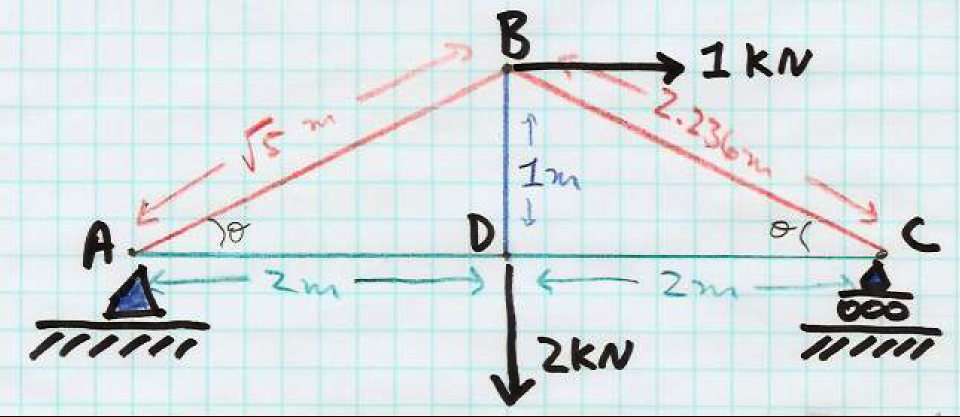
How many equations does the "method of joints" allow us to write down for this truss? (Consider how many joints the truss has.)

(A) 4 (B) 8 (C) 12 (D) 15



How many unknown internal forces (tensions or compressions) do we need to determine when we "solve" this truss?

(A) 4 (B) 5 (C) 6 (D) 7



This is a "simply supported" truss. How many independent "reaction forces" do the two supports exert on the truss? (If there are independent horizontal and vertical components, count them as separate forces.)

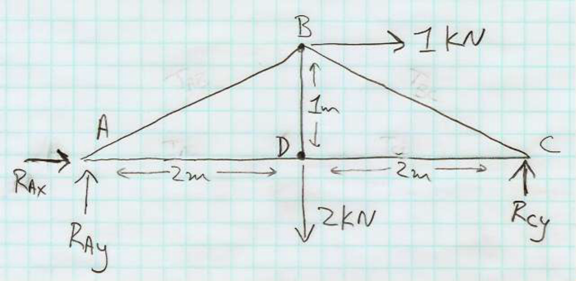
(A) 2 (B) 3 (C) 4 (D) 6

Notice that $8 = 5 + 3$.

For a planar truss that is stable and that you can solve using the equations of static equilibrium,

$$2N_{\text{joints}} = N_{\text{bars}} + 3$$

You get two force equations per joint. You need to solve for one unknown tension/compression per bar plus three support "reaction" forces.



What do we learn by writing $\sum F_x = 0$, $\sum F_y = 0$, $\sum M_z = 0$ for the truss as a whole? (Use joint **A** as pivot.)

(I write R_{Ax}, R_{Ay}, R_{Cy} for the 3 "reaction forces" exerted by the supports on the truss.)

(A) $R_{Ay} - 2\text{ kN} + R_{Cy} = 0,$
$R_{Ax} + 1\text{ kN} = 0,$
$-(2\text{ kN})(2\text{ m}) - (1\text{ kN})(2\text{ m}) + (R_{Cy})(2\text{ m}) = 0$

(B) $R_{Ay} - 2\text{ kN} + R_{Cy} = 0,$
$R_{Ax} + 1\text{ kN} = 0,$
$-(2\text{ kN})(2\text{ m}) - (1\text{ kN})(1\text{ m}) + (R_{Cy})(4\text{ m}) = 0$

(C) $R_{Ay} - 2\text{ kN} + R_{Cy} = 0,$
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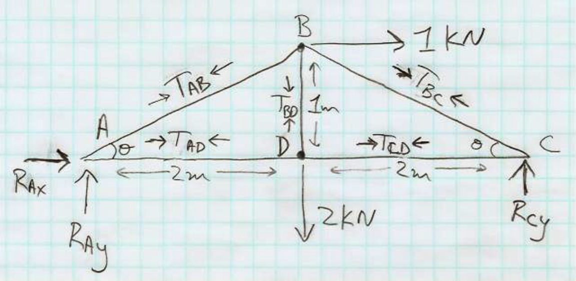

What two equations does the "method of joints" let us write for joint **C** ?

(Let the tension in member $i \leftrightarrow j$ be T_{ij}. For compression members, we will find $T_{ij} < 0$.)

(A) $T_{CD} - T_{BC}\cos\theta = 0$
$R_{Cy} - T_{BC}\sin\theta = 0$

(B) $T_{CD} - T_{BC}\sin\theta = 0$
$R_{Cy} - T_{BC}\cos\theta = 0$

(C) $T_{CD} + T_{BC}\cos\theta = 0$
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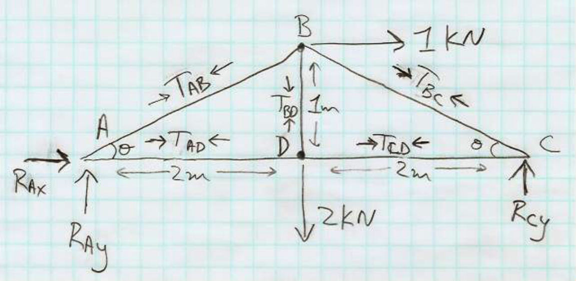

What two equations does the "method of joints" let us write for joint **A** ?

(Let the tension in member $i \leftrightarrow j$ be T_{ij}. For compression members, we will find $T_{ij} < 0$.)

(A) $R_{Ax} - T_{AD} - T_{AB}\cos\theta = 0$
$R_{Ay} - T_{AB}\sin\theta = 0$

(B) $R_{Ax} - T_{AD} - T_{AB}\sin\theta = 0$
$R_{Ay} - T_{AB}\cos\theta = 0$

(C) $R_{Ax} + T_{AD} + T_{AB}\cos\theta = 0$
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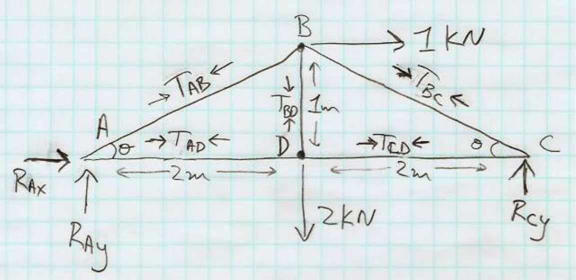
(D) $R_{Ax} + T_{AD} + T_{AB}\sin\theta = 0$
$R_{Ay} + T_{AB}\cos\theta = 0$



What two equations does the "method of joints" let us write for joint **D** ?

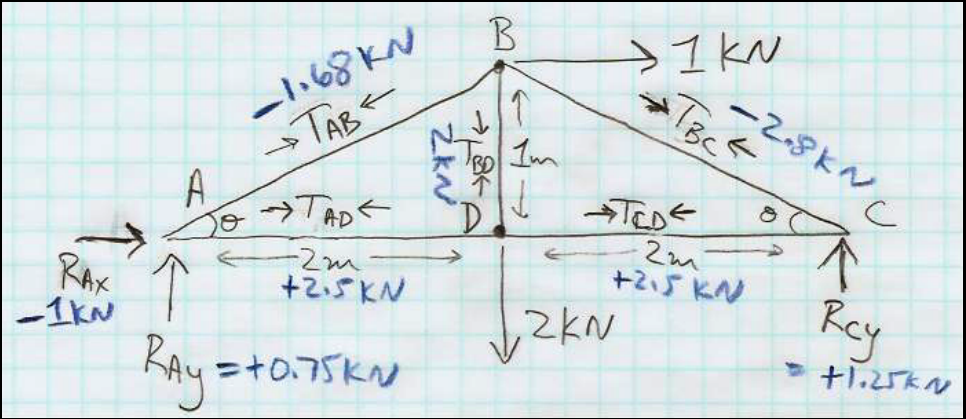
(Let the tension in member $i \leftrightarrow j$ be T_{ij}. For compression members, we will find $T_{ij} < 0$.)

(A) $-2\ \mathrm{kN} + T_{BD} = 0$ and $-T_{AD} + T_{CD} = 0$

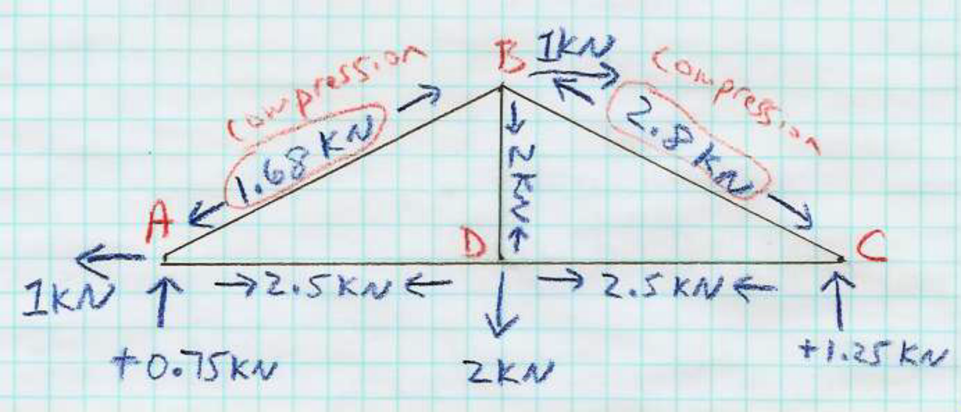
(B) $-2\ \mathrm{kN} + T_{BD} = 0$ and $-T_{AD} - T_{CD} = 0$

(C) $-2\ \mathrm{kN} - T_{BD} = 0$ and $-T_{AD} + T_{CD} = 0$

(D) $-2\ \mathrm{kN} - T_{BD} = 0$ and $-T_{AD} - T_{CD} = 0$



I named each member force T_{ij} (for "tension") and let $T_{ij} > 0$ mean tension and $T_{ij} < 0$ mean compression. Once you've solved the truss, it's best to draw the arrows with the correct signs for clarity. (Next page.)



Forces redrawn with arrows in correct directions, now that we know the sign of each force. Members **AB** and **BC** are in compression. All other members are in tension.

Another option is to write down all $2J$ equations at once and to type them into **Mathematica**, Maple, Wolfram Alpha, etc.

```
In[92] eq := {

RAx + TAB*cos + TAD == 0,
RAy + TAB*sin == 0,
-TAB*cos+TBC*cos+1 == 0,
-TBD-TAB*sin-TBC*sin == 0,
-TAD+TCD == 0,
-2 + TBD == 0,
-TCD - TBC*cos == 0,
RCy + TBC*sin == 0,

sin==1.0/Sqrt[5.0],
cos==2.0/Sqrt[5.0]

}
```

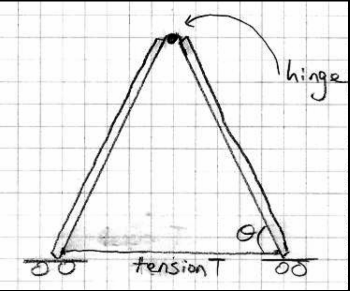
```
In[93] Solve[eq]

Out[93] {

RAx → -1.,
RAy → 0.75,
RCy → 1.25,
TAB → -1.67705,
TAD → 2.5,
TBC → -2.79508,
TBD → 2.,
TCD → 2.5,

cos → 0.894427,
sin → 0.447214

}
```

Physics 8 — Monday, November 4, 2019

- I finally added summaries of key results from Onouye/Kane ch1-ch7 to the "equation sheet." I'm working on ch8, and I may do ch9 as well (though ch9 will be XC for you).
- This week, you'll skim Ch4 (load tracing) and read Ch5 (strength of materials) of Onouye/Kane. Feel free to buy one of my $10 used copies if you wish. At the end of the term, you can keep it, or sell it back to me for $10.

Physics 8 — Wednesday, November 6, 2019

- I finally added summaries of key results from Onouye/Kane ch1-ch7 to the "equation sheet." I'm working on ch8, and I may do ch9 as well (though ch9 will be XC for you).
- This week, you skimmed Ch4 (load tracing) and read Ch5 (strength of materials) of Onouye/Kane.
- HW9 due Friday. HW help: (Bill) Wed 4-6:30pm DRL 3C4, (Grace/Brooke) Thu 6-8pm DRL 2C4.
- I should have paused, after we worked out the hinged arch last Friday, to talk about the torques (moments) due to the vertical vs. horizontal forces and how they vary vs. θ.



Another equilibrium problem!

The top end of a ladder of inertia m rests against a smooth (i.e. slippery) wall, and the bottom end rests on the ground. The coefficient of static friction between the ground and the ladder is μ_s. What is the minimum angle between the ground and the ladder such that the ladder does not slip?

Let's start by drawing an EFBD for the ladder.

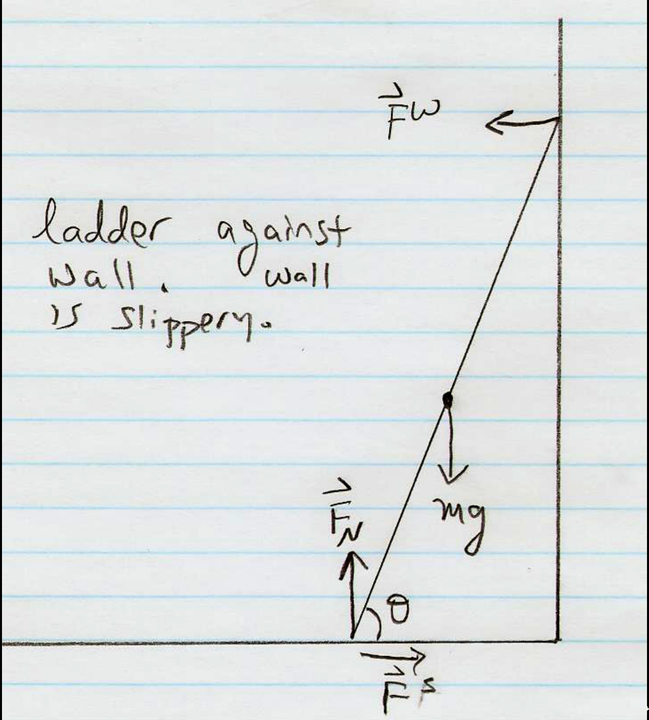
Why must we say the wall is slippery?

Is the slippery wall more like a pin or a roller support?

What plays the role here that string tension played in the previous problem?

Does the combination of two forces at the bottom act more like a pin or a roller support?

Which forces would an engineer call “reaction” forces?

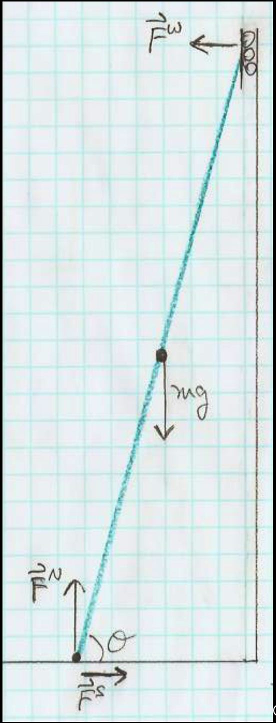


Which choice of pivot axis will give us the simplest equation for $\sum M_z = 0$? (We'll get an equation involving only two forces if we choose this axis.)

(A) Use bottom of ladder as pivot axis.

(B) Use center of ladder as pivot axis.

(C) Use top of ladder as pivot axis.



How would I write $\sum M_z = 0$ about the bottom end of the ladder? (Take length of ladder to be L.)

(A) $F^W L\cos\theta + mgL\sin\theta = 0$

(B) $F^W L\cos\theta + mg\frac{L}{2}\sin\theta = 0$

(C) $F^W L\cos\theta - mgL\sin\theta = 0$

(D) $F^W L\cos\theta - mg\frac{L}{2}\sin\theta = 0$

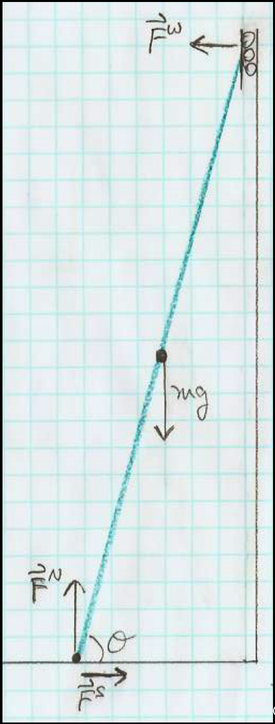
(E) $F^W L\sin\theta + mgL\cos\theta = 0$

(F) $F^W L\sin\theta + mg\frac{L}{2}\cos\theta = 0$

(G) $F^W L\sin\theta - mgL\cos\theta = 0$

(H) $F^W L\sin\theta - mg\frac{L}{2}\cos\theta = 0$

What do we learn from $\sum F_x = 0$ and $\sum F_y = 0$?



What do we learn from $\sum F_x = 0$ and $\sum F_y = 0$?

Let's answer the original question:

What is the minimum angle between the ground and the ladder such that the ladder does not slip?

Suppose we add to this picture a woman of mass M who has climbed up a distance d along the length of the ladder. Now how do we write the moment equation $\sum M_z = 0$?

(A) $F^W L \sin\theta - mg\frac{L}{2}\cos\theta + Mg\frac{d}{2}\cos\theta = 0$

(B) $F^W L \sin\theta - mg\frac{L}{2}\cos\theta + Mg\frac{d}{2}\sin\theta = 0$

(C) $F^W L \sin\theta - mg\frac{L}{2}\cos\theta + Mgd\cos\theta = 0$

(D) $F^W L \sin\theta - mg\frac{L}{2}\cos\theta + Mgd\sin\theta = 0$

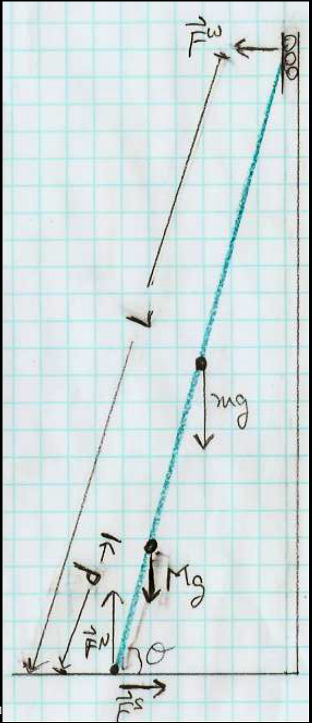
(E) $F^W L \sin\theta - mg\frac{L}{2}\cos\theta - Mg\frac{d}{2}\cos\theta = 0$

(F) $F^W L \sin\theta - mg\frac{L}{2}\cos\theta - Mg\frac{d}{2}\sin\theta = 0$

(G) $F^W L \sin\theta - mg\frac{L}{2}\cos\theta - Mgd\cos\theta = 0$

(H) $F^W L \sin\theta - mg\frac{L}{2}\cos\theta - Mgd\sin\theta = 0$

What do we learn from $\sum F_x = 0$ and $\sum F_y = 0$?

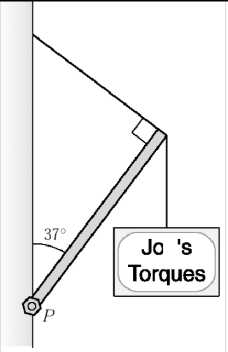


What do we learn from $\sum F_x = 0$ and $\sum F_y = 0$?

For a given θ, how far up can she climb before the ladder slips?

Here's a trickier equilibrium problem:

4*. You want to hang a 22 kg sign (shown at right) that advertises your new business. To do this, you attach a 7.0 kg beam of length 1.0 m to a wall at its base by a pivot P. You then attach a thin cable to the beam and to the wall in such a way that the cable and beam are perpendicular to each other. The beam makes an angle of 37° with the vertical. You hang the sign from the end of the beam to which the cable is attached. (a) What must be the minimum tensile strength of the cable (the amount of tension it can sustain) if it is not to snap? (b) Determine the horizontal and vertical components of the force the pivot exerts on the beam.



What forces act on the beam?

What 3 equations can we write for the beam?

(We stopped after this.)

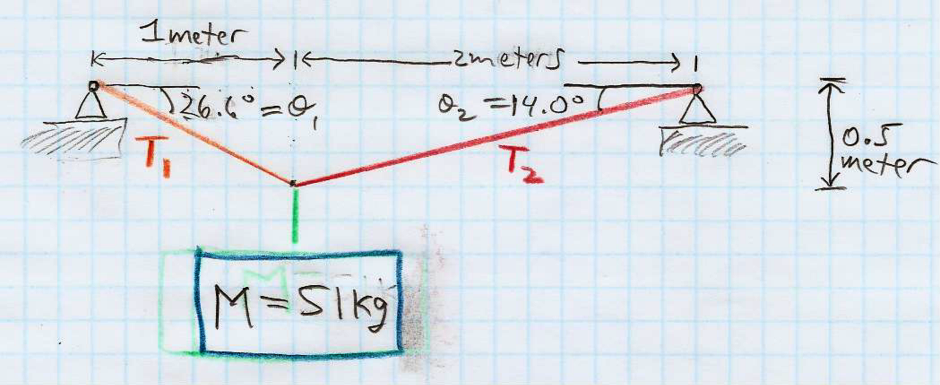
A tightly stretched “high wire” has length $L = 50\ \mathrm{m}$. It sags by $d = 1.0\ \mathrm{m}$ when a tightrope walker of mass $M = 51\ \mathrm{kg}$ stands at the center of the wire.

What is the tension in the wire?

Is it possible to increase the tension in the wire so that there is no sag at all (i.e. so that $d = 0$)?

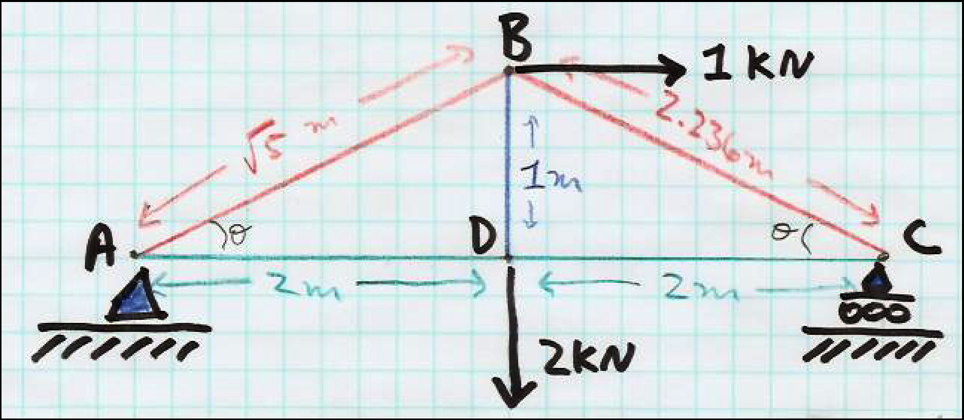
What happens to the tension as we make the sag smaller and smaller?

Now suppose a 51 kg sign is suspended from a cable (but not at the center), as shown below.



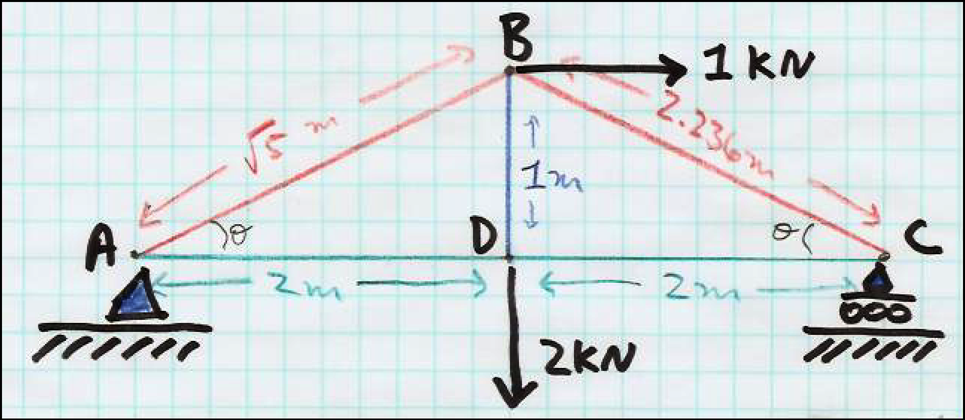
How would you find the tensions T_1 and T_2?

Once you know T_1 and T_2, what are the horizontal and vertical forces exerted by the two supports on the cable?



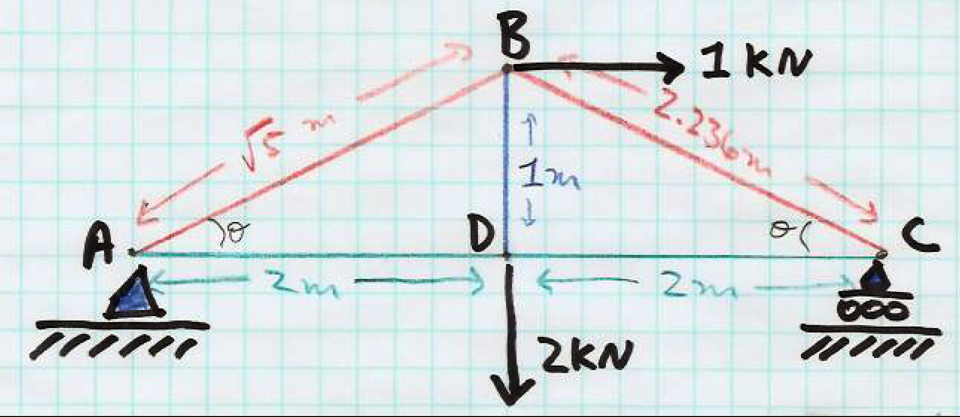
How many equations does the "method of joints" allow us to write down for this truss? (Consider how many joints the truss has.)

(A) 4 (B) 8 (C) 12 (D) 15



How many unknown internal forces (tensions or compressions) do we need to determine when we "solve" this truss?

(A) 4 (B) 5 (C) 6 (D) 7



This is a "simply supported" truss. How many independent "reaction forces" do the two supports exert on the truss? (If there are independent horizontal and vertical components, count them as separate forces.)

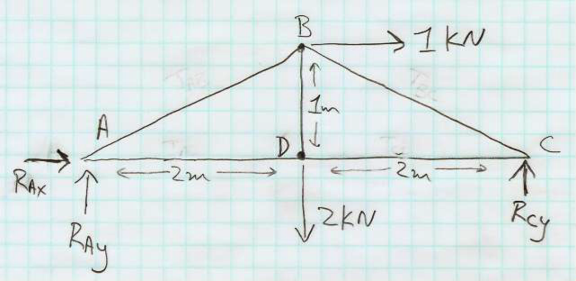
(A) 2 (B) 3 (C) 4 (D) 6

Notice that $8 = 5 + 3$.

For a planar truss that is stable and that you can solve using the equations of static equilibrium,

$$2N_{\text{joints}} = N_{\text{bars}} + 3$$

You get two force equations per joint. You need to solve for one unknown tension/compression per bar plus three support "reaction" forces.



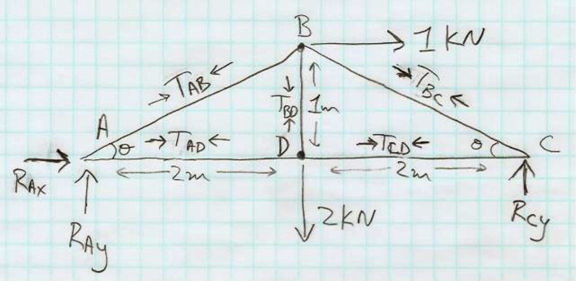
What do we learn by writing $\sum F_x = 0$, $\sum F_y = 0$, $\sum M_z = 0$ for the truss as a whole? (Use joint **A** as pivot.)

(I write R_{Ax}, R_{Ay}, R_{Cy} for the 3 "reaction forces" exerted by the supports on the truss.)

(A)
$$R_{Ay} - 2\ \mathrm{kN} + R_{Cy} = 0,$$
$$R_{Ax} + 1\ \mathrm{kN} = 0,$$
$$-(2\ \mathrm{kN})(2\ \mathrm{m}) - (1\ \mathrm{kN})(2\ \mathrm{m}) + (R_{Cy})(2\ \mathrm{m}) = 0$$

(B)
$$R_{Ay} - 2\ \mathrm{kN} + R_{Cy} = 0,$$
$$R_{Ax} + 1\ \mathrm{kN} = 0,$$
$$-(2\ \mathrm{kN})(2\ \mathrm{m}) - (1\ \mathrm{kN})(1\ \mathrm{m}) + (R_{Cy})(4\ \mathrm{m}) = 0$$

(C)
$$R_{Ay} - 2\ \mathrm{kN} + R_{Cy} = 0,$$
$$R_{Ax} + 1\ \mathrm{kN} = 0,$$
$$-(2\ \mathrm{kN})(2\ \mathrm{m}) - (1\ \mathrm{kN})(2\ \mathrm{m}) + (R_{Cy})(4\ \mathrm{m}) = 0$$



What two equations does the "method of joints" let us write for joint **C** ?

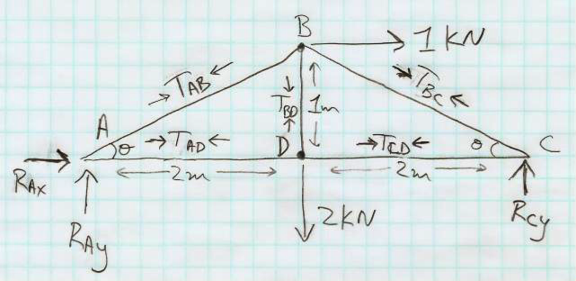
(Let the tension in member $i \leftrightarrow j$ be T_{ij}. For compression members, we will find $T_{ij} < 0$.)

(A) $T_{CD} - T_{BC}\cos\theta = 0$
$R_{Cy} - T_{BC}\sin\theta = 0$

(B) $T_{CD} - T_{BC}\sin\theta = 0$
$R_{Cy} - T_{BC}\cos\theta = 0$

(C) $T_{CD} + T_{BC}\cos\theta = 0$
$R_{Cy} + T_{BC}\sin\theta = 0$

(D) $T_{CD} + T_{BC}\sin\theta = 0$
$R_{Cy} + T_{BC}\cos\theta = 0$



What two equations does the "method of joints" let us write for joint **A** ?

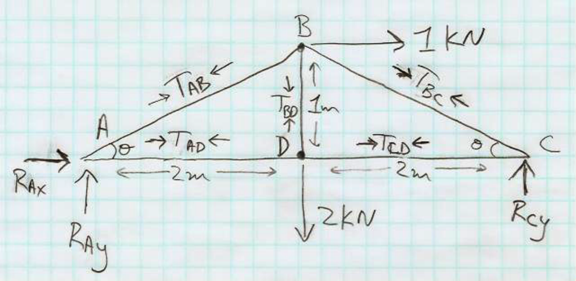
(Let the tension in member $i \leftrightarrow j$ be T_{ij}. For compression members, we will find $T_{ij} < 0$.)

(A) $R_{Ax} - T_{AD} - T_{AB}\cos\theta = 0$
$R_{Ay} - T_{AB}\sin\theta = 0$

(B) $R_{Ax} - T_{AD} - T_{AB}\sin\theta = 0$
$R_{Ay} - T_{AB}\cos\theta = 0$

(C) $R_{Ax} + T_{AD} + T_{AB}\cos\theta = 0$
$R_{Ay} + T_{AB}\sin\theta = 0$

(D) $R_{Ax} + T_{AD} + T_{AB}\sin\theta = 0$
$R_{Ay} + T_{AB}\cos\theta = 0$



What two equations does the "method of joints" let us write for joint **D** ?

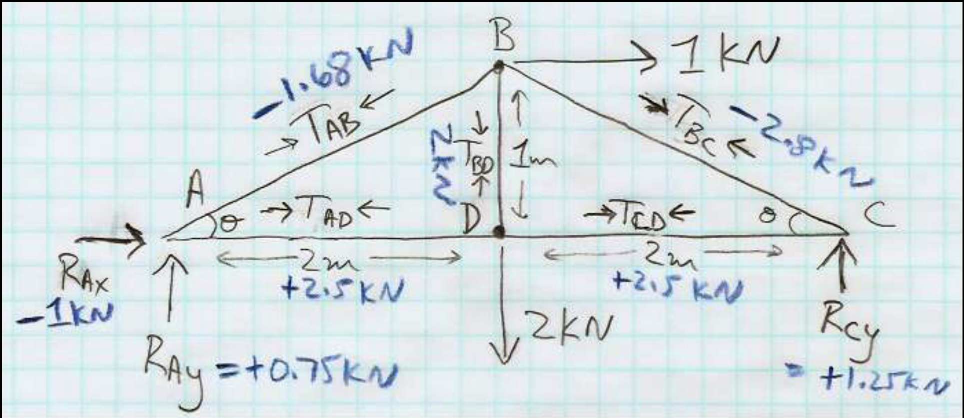
(Let the tension in member $i \leftrightarrow j$ be T_{ij}. For compression members, we will find $T_{ij} < 0$.)

(A) $-2\ \mathrm{kN} + T_{BD} = 0$ and $-T_{AD} + T_{CD} = 0$

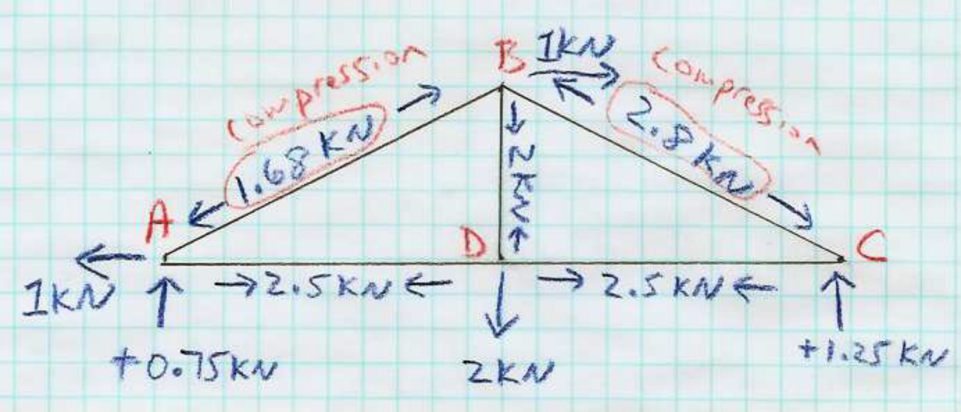
(B) $-2\ \mathrm{kN} + T_{BD} = 0$ and $-T_{AD} - T_{CD} = 0$

(C) $-2\ \mathrm{kN} - T_{BD} = 0$ and $-T_{AD} + T_{CD} = 0$

(D) $-2\ \mathrm{kN} - T_{BD} = 0$ and $-T_{AD} - T_{CD} = 0$



I named each member force T_{ij} (for “tension”) and let $T_{ij} > 0$ mean tension and $T_{ij} < 0$ mean compression. Once you've solved the truss, it's best to draw the arrows with the correct signs for clarity. (Next page.)

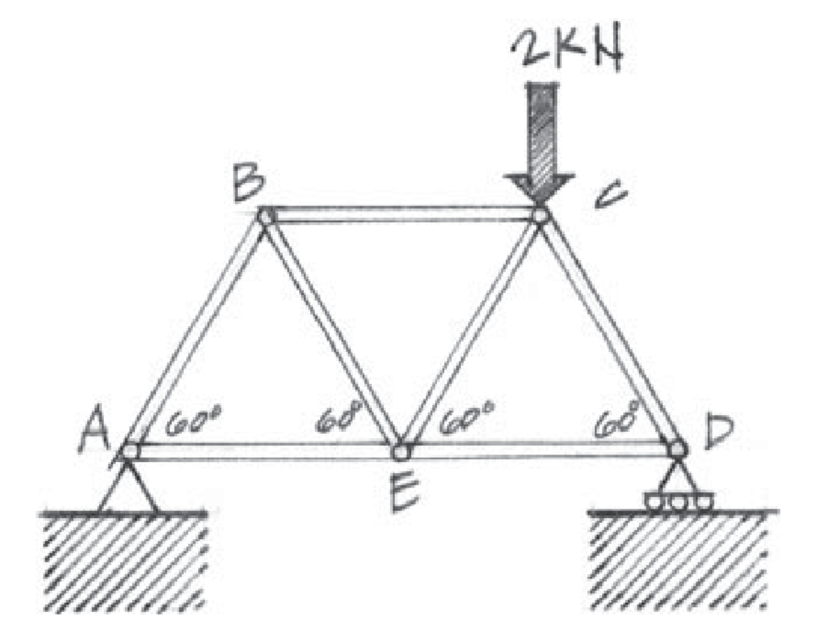


Forces redrawn with arrows in correct directions, now that we know the sign of each force. Members **AB** and **BC** are in compression. All other members are in tension.

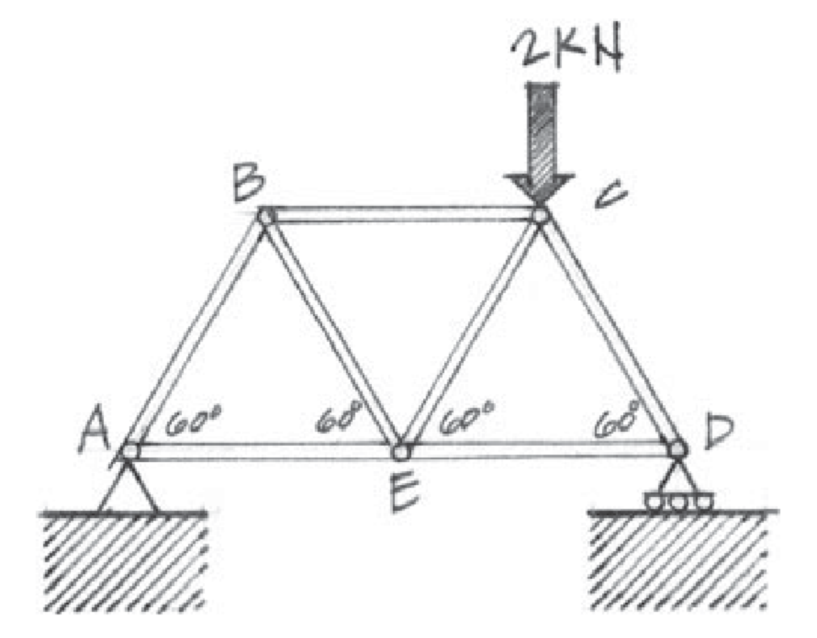
Another option is to write down all $2J$ equations at once and to type them into **Mathematica**, Maple, Wolfram Alpha, etc.

```
In[92] eq := {

RAx + TAB*cos + TAD == 0,
RAy + TAB*sin == 0,
-TAB*cos+TBC*cos+1 == 0,
-TBD-TAB*sin-TBC*sin == 0,
-TAD+TCD == 0,
-2 + TBD == 0,
-TCD - TBC*cos == 0,
RCy + TBC*sin == 0,

sin==1.0/Sqrt[5.0],
cos==2.0/Sqrt[5.0]

}
```

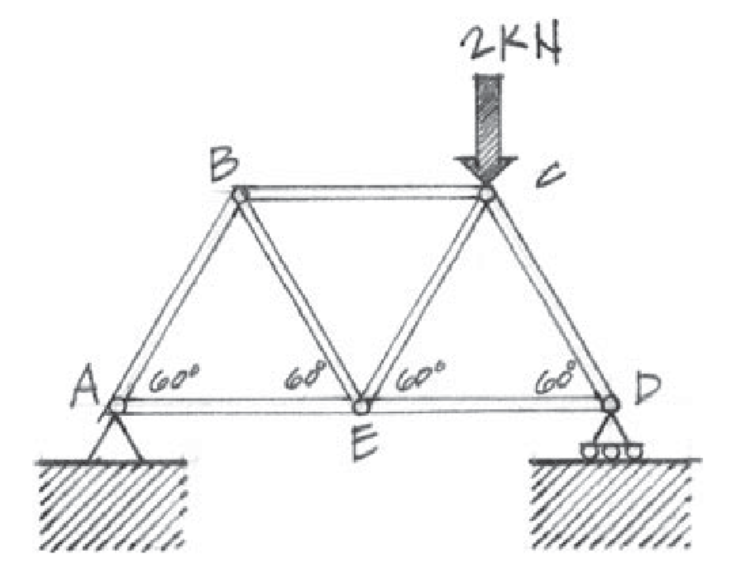
```
In[93] Solve[eq]

Out[93] {

RAx → -1.,
RAy → 0.75,
RCy → 1.25,
TAB → -1.67705,
TAD → 2.5,
TBC → -2.79508,
TBD → 2.,
TCD → 2.5,

cos → 0.894427,
sin → 0.447214

}
```



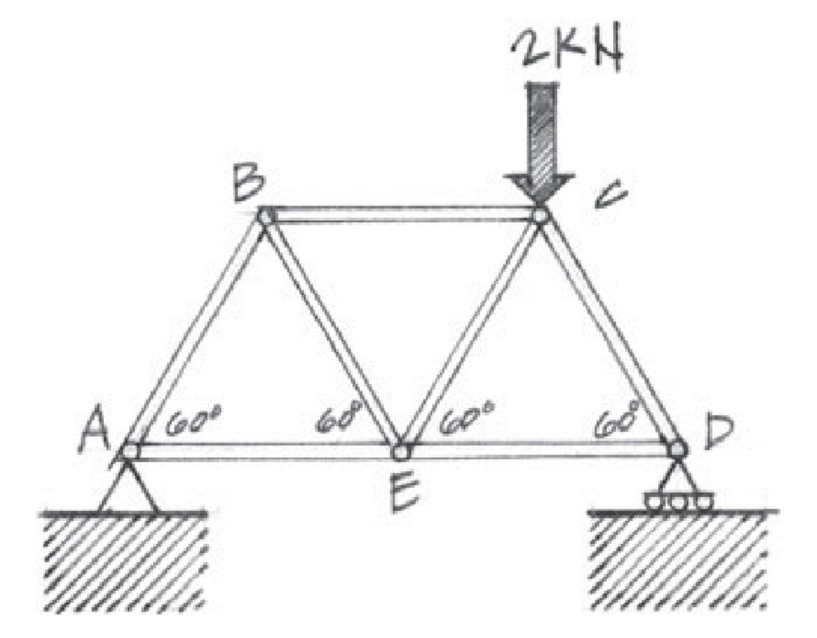
How many "reaction forces" are exerted by the supports (i.e. exerted on the truss by the supports)?



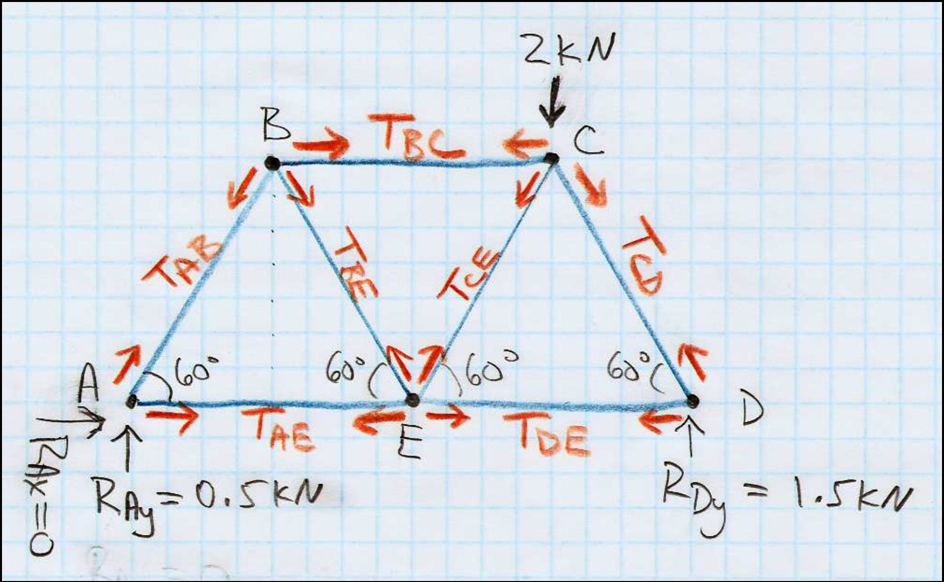
How many internal forces (tensions or compressions in the members) do we need to solve for to "solve" this truss?



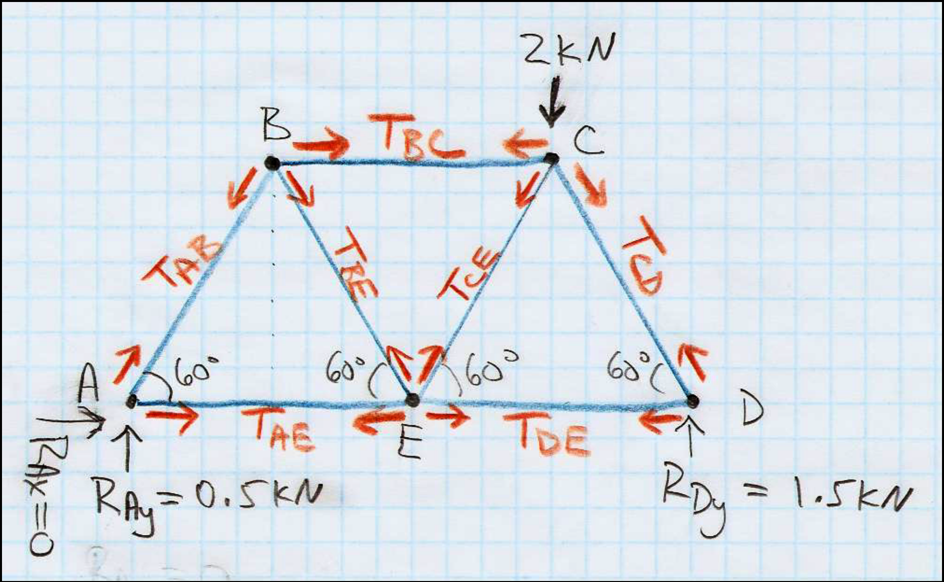
Do you see any joint at which there are ≤ 2 unknown forces? If so, we can start there. If not, we need to start with an EFBD for the truss as a whole.



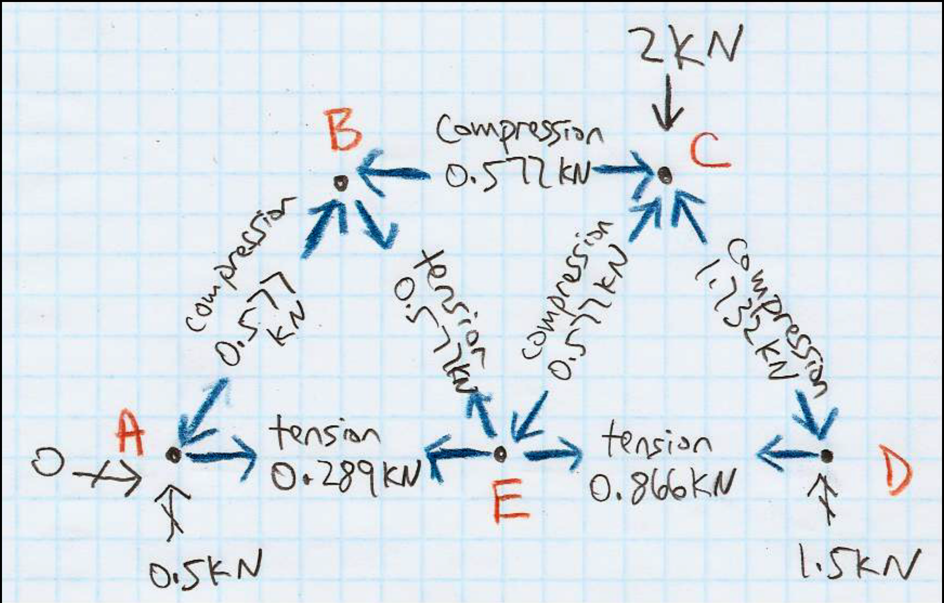
Try to guess $R_{A,x}$, $R_{A,y}$, and $R_{D,y}$ by inspection. Then let's check with the usual equations.



Now start from a joint having ≤ 2 unknown forces. In this case, I just went through the joints alphabetically. You can make your life easier by seeking out equations having just 1 unknown.



$T_{AB} = -0.577$ kN, $T_{AE} = +0.289$ kN, $T_{BE} = +0.577$ kN,
$T_{BC} = -0.577$ kN, $T_{CE} = -0.577$ kN, $T_{CD} = -1.732$ kN,
$T_{DE} = +0.866$ kN. My notation: tension > 0, compression < 0.


2KN
B
Compression
0.577 kN
C
compression
0.577 kN
tension
0.577 kN
compression
0.577 kN
compression
1.732 kN
A
0
tension
0.289 kN
E
tension
0.866 kN
D
0.5KN
1.5KN

$\cos 60° = 0.5$, $\sin 60° = 0.866$

Joint A: $0 + T_{AE} + T_{AB}\cos 60° = 0$

$0.5\,kN + T_{AB}\sin 60° = 0 \Rightarrow$ $\boxed{T_{AB} = -0.577\,kN}$

$\boxed{T_{AE} = +0.289\,kN}$

Joint B: $-T_{AB}\cos 60° + T_{BE}\cos 60° + T_{BC} = 0$

$-T_{AB}\sin 60° - T_{BE}\sin 60° = 0 \Rightarrow$ $\boxed{T_{BE} = +0.577\,kN}$

$\boxed{T_{BC} = -0.577\,kN}$

Joint C: $-T_{BC} - T_{CE}\cos 60° + T_{CD}\cos 60° = 0 \Rightarrow T_{CD} = T_{CE} - 1.155\,kN$

(could do C, D, or E at this point)

$-2\,kN - T_{CE}\sin 60° - T_{CD}\sin 60° = 0$

$-2\,kN - T_{CE}\sin 60° - (T_{CE} - 1.155\,kN)\sin 60° = 0$

$2T_{CE}\sin 60° = -2\,kN + 1.0\,kN = -1\,kN$

$\boxed{T_{CE} = -0.577\,kN}$

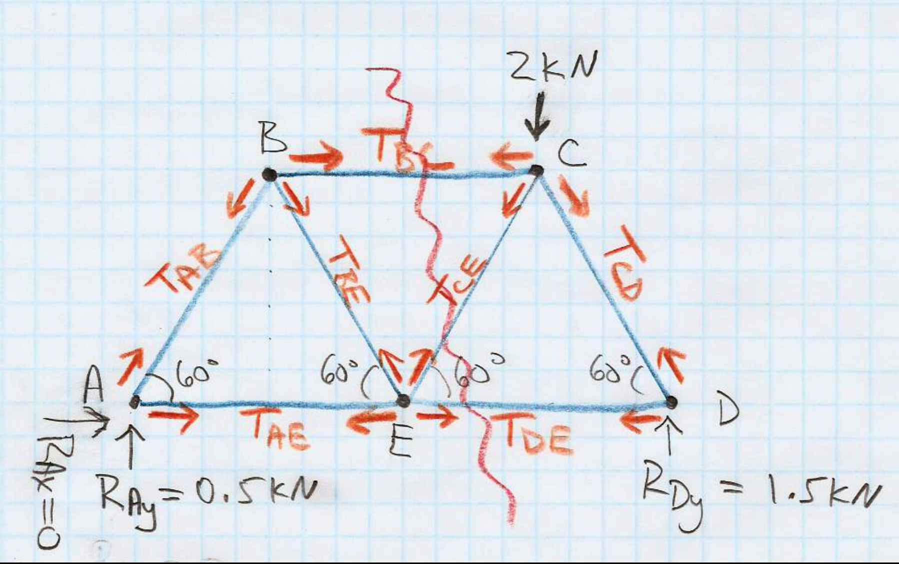
$T_{CD} = T_{CE} - 1.155\,kN =$ $\boxed{-1.732\,kN = T_{CD}}$

Joint D: $-T_{DE} - T_{CD}\cos 60° = 0 \Rightarrow$ $\boxed{T_{DE} = +0.866\,kN}$

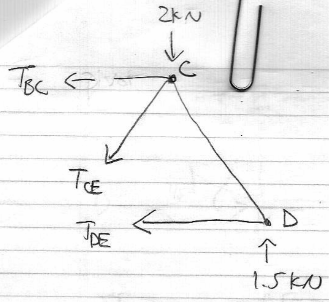
(check): $1.5\,kN + T_{CD}\sin 60° = 0$ ✓

(check Joint E): $-T_{AE} + T_{DE} - T_{BE}\cos 60° + T_{CE}\cos 60° = 0$ ✓

$+T_{BE}\sin 60° + T_{CE}\sin 60° = 0$ ✓



Let's try drawing an EFBD for the **right** side of the cut ("section").



$$\sum F_x = 0 \Rightarrow -T_{BC} - T_{CE}\cos 60^\circ - T_{DE} = 0$$

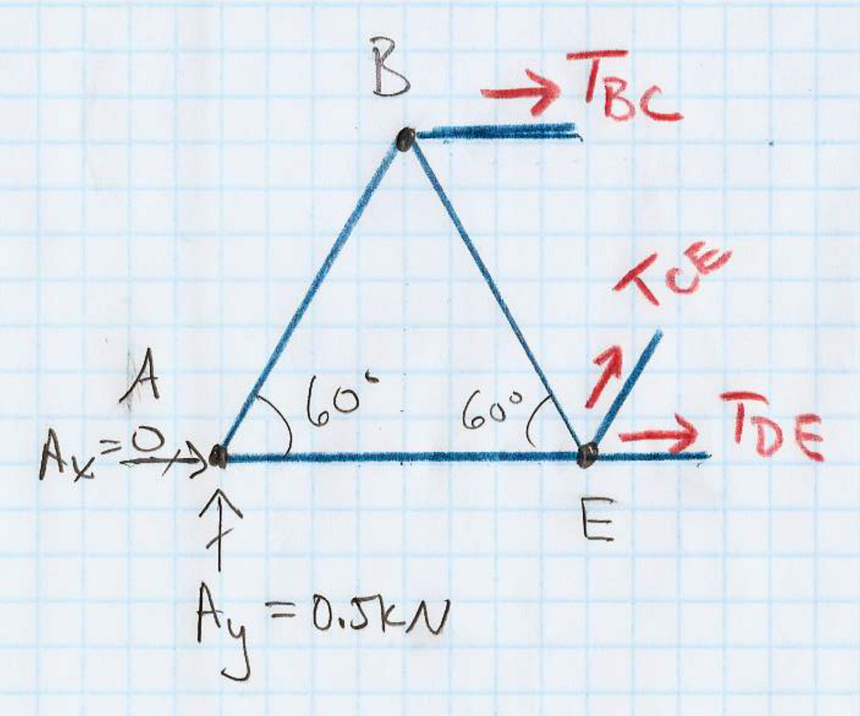
$$\sum F_y = 0 \Rightarrow -2kN + 1.5kN - T_{CE}\sin 60^\circ = 0$$

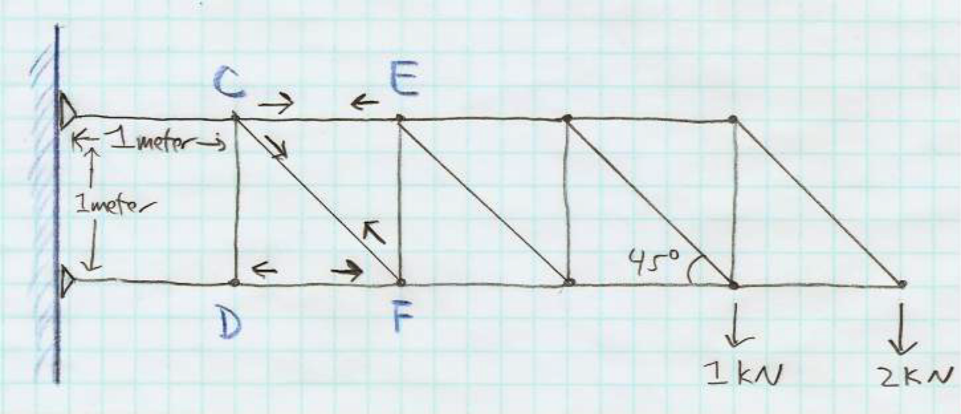
$$\sum M_C = 0 \Rightarrow +(1.5kN)\left(\frac{L}{2}\right) - T_{DE}\, L\sin 60^\circ = 0$$

$$\rightarrow T_{DE} = \frac{1.5kN}{2\sin 60^\circ} = +0.866kN$$

$$\rightarrow T_{CE} = \frac{-0.5kN}{\sin 60^\circ} = -0.577kN$$

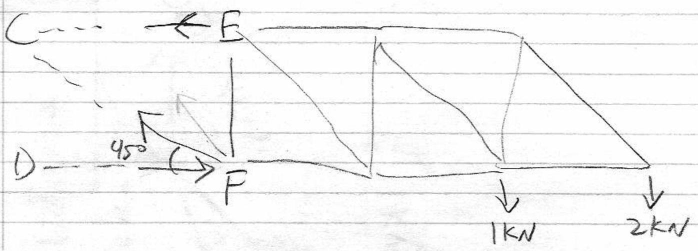
$$\rightarrow T_{BC} = -(T_{CE}\cos 60^\circ + T_{DE}) = -0.577$$


B
TBC
TCE
A
60°
60°
TDE
Ax=0
E
Ay = 0.5kN



Here's another truss problem that you can solve using the Method of Sections. Find forces in members **CE**, **CF**, and **DF**, with assumed force directions as shown.

- What happens if an assumed force direction is backwards?
- Where should we "section" the truss?
- Then what do we do next?



$$\Sigma F_x = 0 = -CE - CF\cos 45° + DF = 0$$

$$\Sigma F_y = 0 = CF\sin 45° = 3\,kN \Rightarrow \boxed{CF = 3\,kN\sqrt{2}}$$

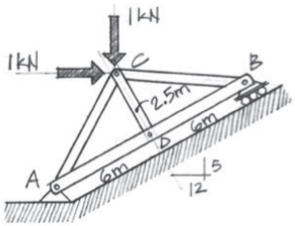
$$\Sigma M_{DF} = 0 = CE(1) - (1\,kN)(2) - (2\,kN)(3)$$

$$\Rightarrow \boxed{CE = 8\,kN}$$

$$DF = CE + CF\cos 45° = 8\,kN + 3\,kN$$

$$\boxed{DF = 11\,kN}$$

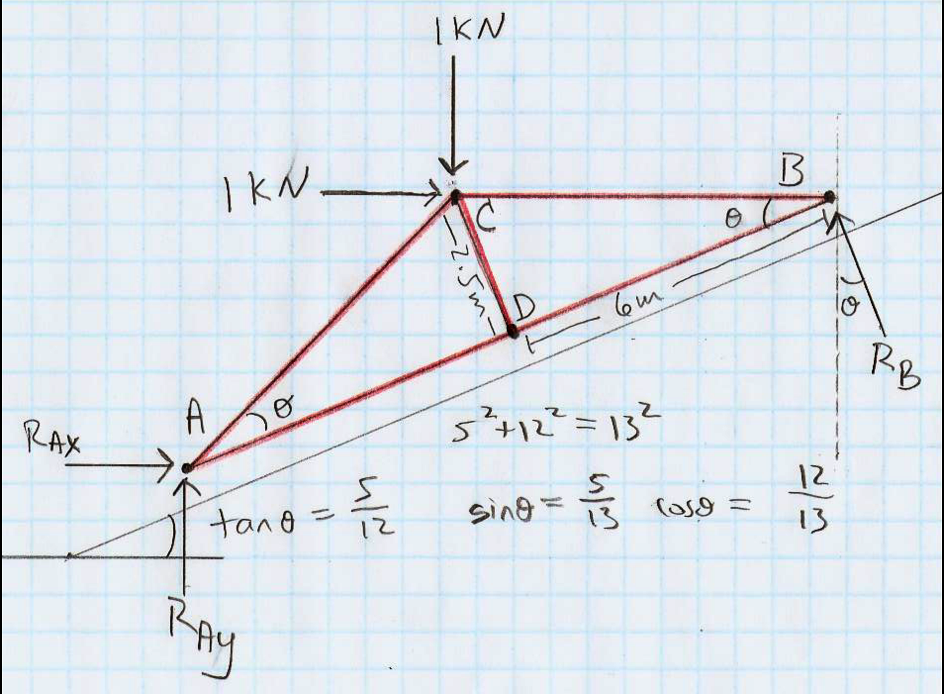
2.38 An inclined king-post truss supports a vertical and horizontal force at C. Determine the support reactions developed at A and B.



This is not really a "truss problem," since we're not asked to solve for the internal forces in the truss, but it is an example of a pretty tricky equilibrium problem.

Let's try working through this together in class.

Notice, from the given dimensions: the angle of the incline is the same as the interior angle at joints A and B of the truss. Also notice pin/hinge support at A and roller support at B.



1 KN
1 KN
C
B
θ
2.5m
D
6m
A
θ
R_{AX}
R_B
θ
R_{Ay}
$5^2+12^2=13^2$
$\tan\theta = \frac{5}{12}$
$\sin\theta = \frac{5}{13}$
$\cos\theta = \frac{12}{13}$

① $0 = \sum_{\text{on truss}} F_x = R_{Ax} + 1\,kN - R_B \sin\theta$

$$\Rightarrow R_{Ax} = R_B \sin\theta - 1\,kN$$

② $0 = \sum_{\text{on truss}} F_y = R_{Ay} - 1\,kN + R_B \cos\theta$

$$\Rightarrow R_{Ay} = 1\,kN - R_B \cos\theta$$

③ $0 = \sum_{\circlearrowleft} M_A = R_B(12m) - 1\,kN(12m \sin\theta) - 1\,kN(6.5m \cos 2\theta)$

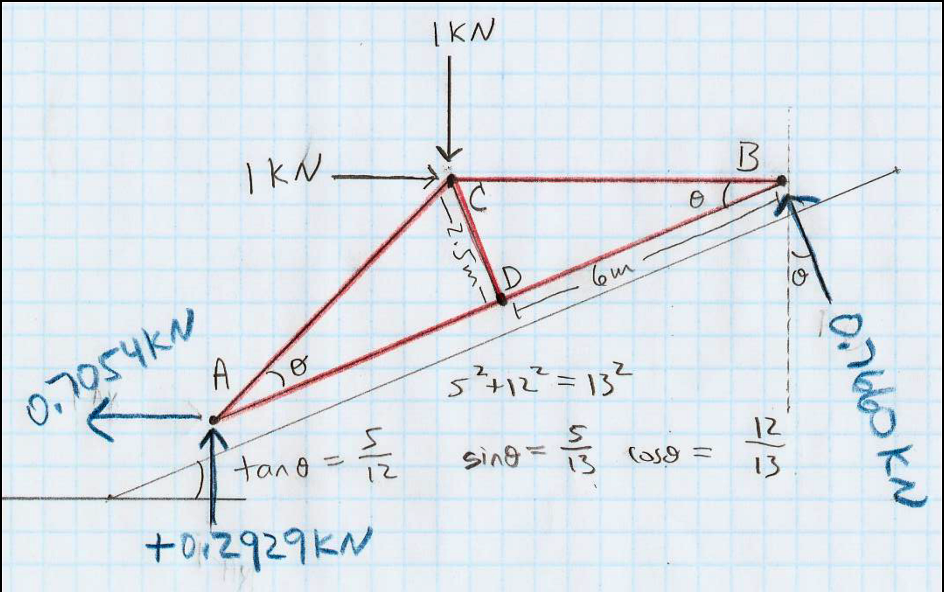
$$\Rightarrow R_B = 1\,kN(\sin\theta) + \frac{6.5}{12}\,kN(\cos 2\theta) = 0.7660\,kN$$

$$(R_B)_x = -R_B \sin\theta = -0.2946\,kN$$

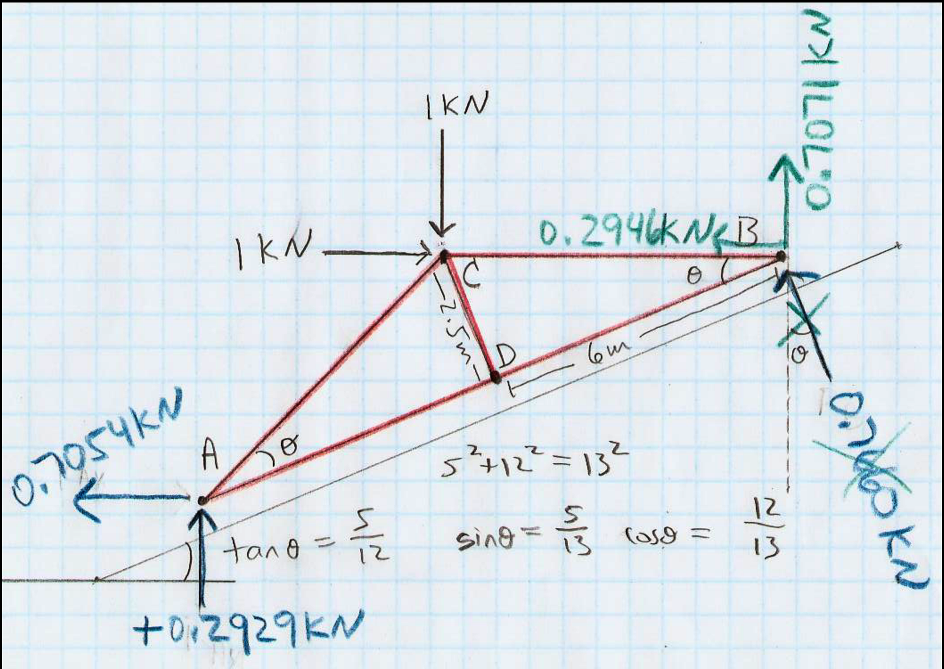
$$(R_B)_y = R_B \cos\theta = 0.7071\,kN$$

$$R_{Ax} = (0.2946 - 1)\,kN = -0.7054\,kN$$

$$R_{Ay} = (1 - 0.7071)\,kN = 0.2929\,kN$$

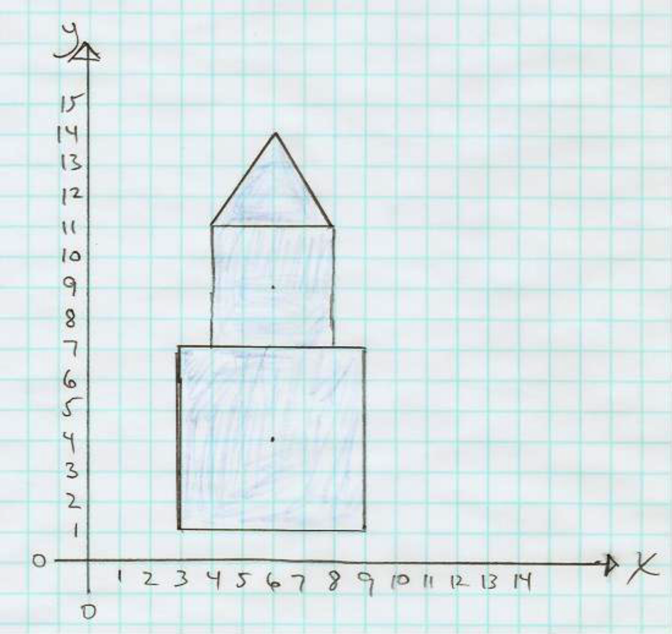


1KN
1KN
B
C
θ
2.5m
D
6m
θ
0.7054KN
A
θ
$5^2+12^2=13^2$
0.7660KN
$\tan\theta = \frac{5}{12}$
$\sin\theta = \frac{5}{13}$
$\cos\theta = \frac{12}{13}$
+0.2929KN


1KN
1KN
C
0.2946kN
B
0.7071 kN
θ
2.5m
D
6m
θ
A
θ
0.7054kN
$5^2+12^2=13^2$
$\tan\theta = \frac{5}{12}$
$\sin\theta = \frac{5}{13}$
$\cos\theta = \frac{12}{13}$
+0.2929kN

- The idea of computing centroids of simple and composite shapes is very, very briefly introduced in O/K ch3 (in the context of "distributed loads"), and will be discussed in much more detail in O/K ch6 (for next Monday).
- Let's go through one example using rectangles and triangles. It will help you in cases when you need to solve for the "reaction forces" on a beam that carries distributed loads. (Example coming up next.)

What is X_{centroid} for the shaded area?



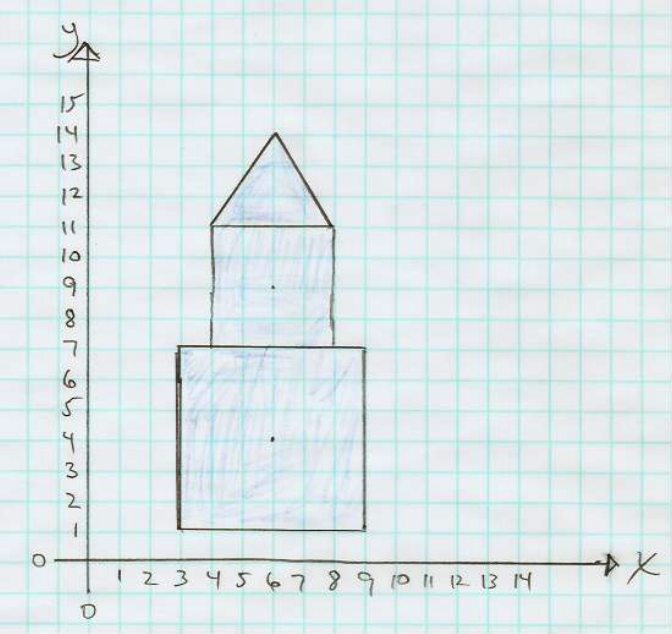
(A) 0

(B) 3

(C) 6

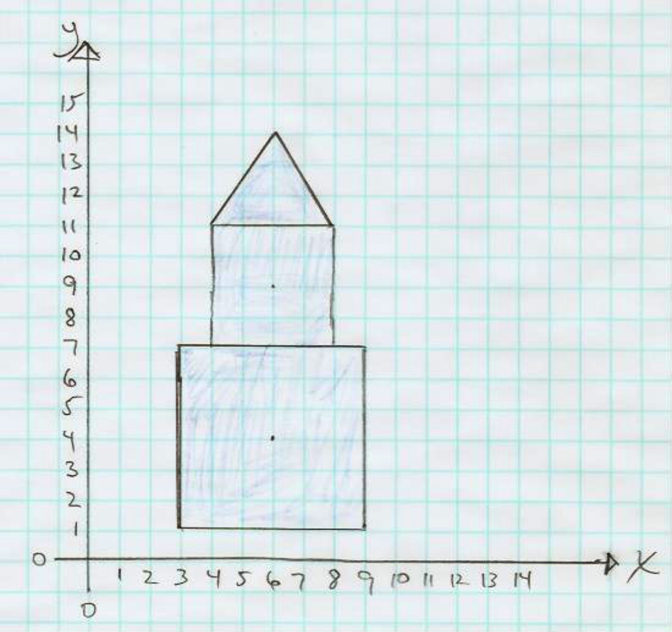
(D) 9

What are the areas of the three individual polygons?



(A) 36, 16, 16
(B) 36, 16, 12
(C) 36, 16, 8
(D) 36, 16, 6

What are the Y_{centroid} values of the three individual polygons?



(A) 4, 9, 11

(B) 4, 9, 11.667

(C) 4, 9, 12

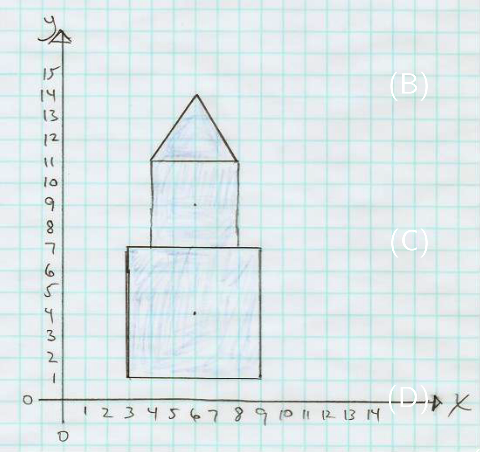
(D) 4, 9, 12.333

(E) 4, 9, 12.5

(F) 4, 9, 13

(G) 4, 9, 14

What is Y_{centroid} for the whole shaded area?



(A)

$$\frac{4+9+12}{3} = 8.33$$

(B)

$$\frac{(4)(36)+(9)(16)+(12)(6)}{36+16+6} = 6.21$$

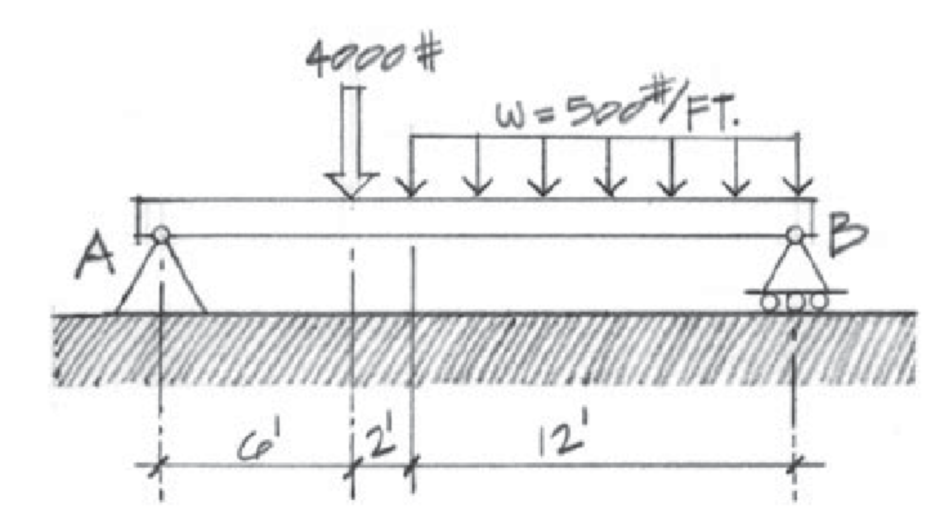
(C)

$$\frac{(4)(36)+(9)(16)+(12)(6)}{4+9+12} = 14.4$$

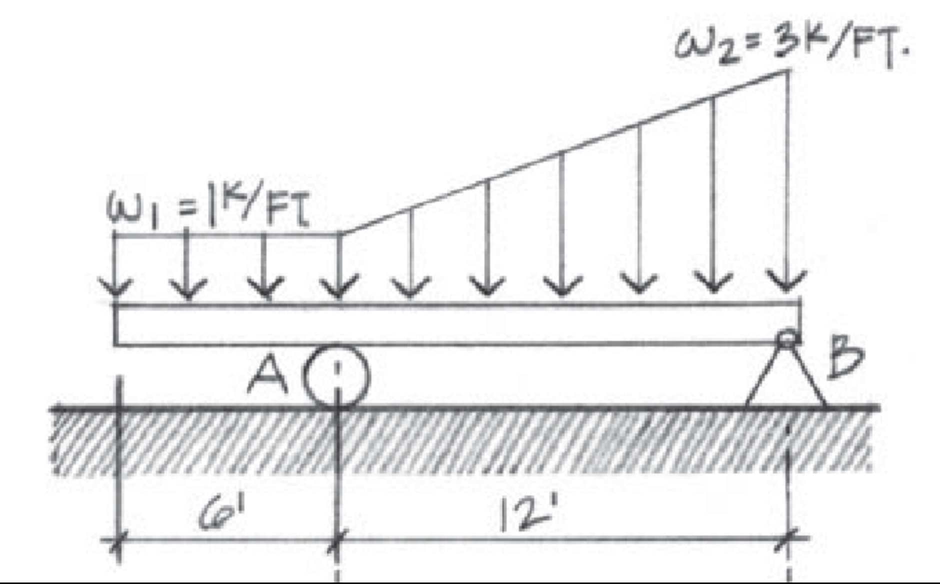
(D)

$$\frac{(4^2)(36)+(9^2)(16)+(12^2)(6)}{36+16+6} = 47.2$$

There's one problem similar to this (but using metric units) on HW(?): Determine the support reactions at A and B.



This one is harder than your homework, because the distributed load is non-uniform: Determine the support reactions at A and B.



Physics 8 — Wednesday, November 6, 2019

- I finally added summaries of key results from Onouye/Kane ch1-ch7 to the “equation sheet.” I’m working on ch8, and I may do ch9 as well (though ch9 will be XC for you).
- This week, you skimmed Ch4 (load tracing) and read Ch5 (strength of materials) of Onouye/Kane.
- HW9 due Friday. HW help: (Bill) Wed 4-6:30pm DRL 3C4, (Grace/Brooke) Thu 6-8pm DRL 2C4.

Physics 8 — Friday, November 8, 2019

- Turn in HW9. Pick up HW10 handout in back corner.
- I finally added summaries of key results from Onouye/Kane ch1-ch7 to the "equation sheet." I'm working on ch8, and I may do ch9 as well (though ch9 will be XC for you).
- This week, you skimmed Ch4 (load tracing) and read Ch5 (strength of materials) of Onouye/Kane. Next week, you'll read Ch6 (cross-sectional properties) and Ch7 (simple beams).

A tightly stretched "high wire" has length $L = 50\ \mathrm{m}$. It sags by $d = 1.0\ \mathrm{m}$ when a tightrope walker of mass $M = 51\ \mathrm{kg}$ stands at the center of the wire.

What is the tension in the wire? Is it possible to increase the tension in the wire so that there is no sag at all (i.e. so that $d = 0$)? What happens to the tension as we make the sag smaller and smaller?

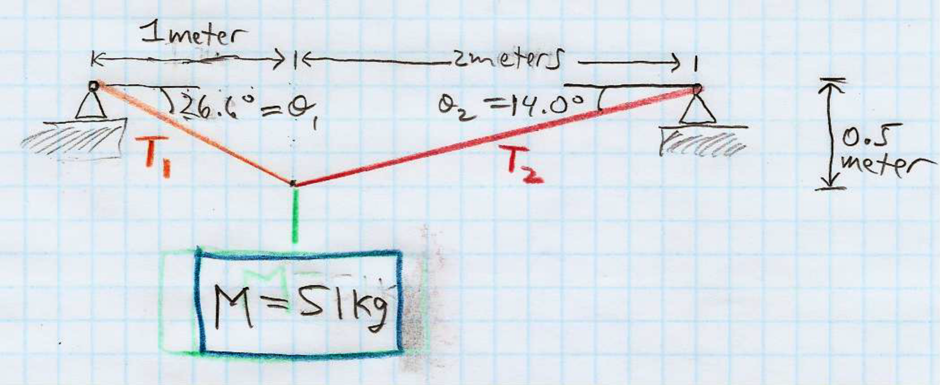
A tightly stretched "high wire" has length $L = 50\ \mathrm{m}$. It sags by $d = 1.0\ \mathrm{m}$ when a tightrope walker of mass $M = 51\ \mathrm{kg}$ stands at the center of the wire.

What is the tension in the wire?

Is it possible to increase the tension in the wire so that there is no sag at all (i.e. so that $d = 0$)?

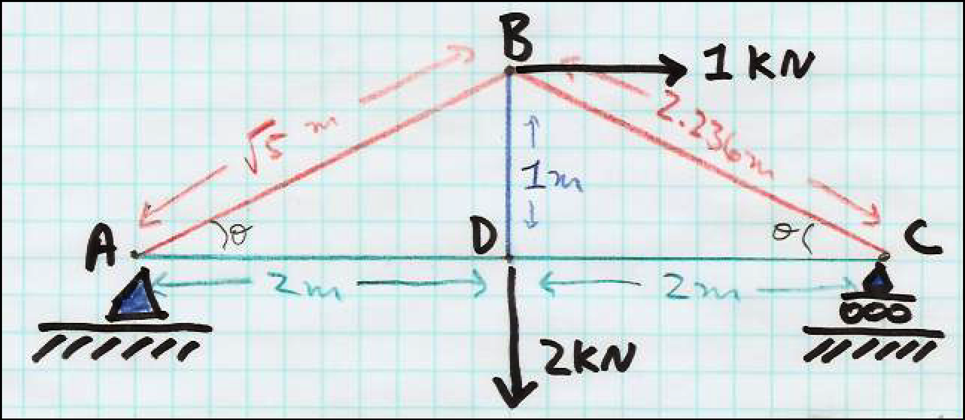
What happens to the tension as we make the sag smaller and smaller?

Now suppose a 51 kg sign is suspended from a cable (but not at the center), as shown below.



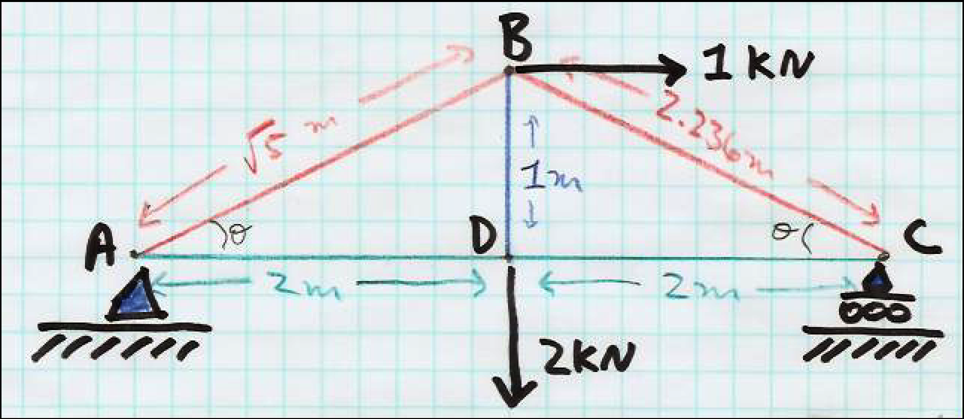
How would you find the tensions T_1 and T_2?

Once you know T_1 and T_2, what are the horizontal and vertical forces exerted by the two supports on the cable?



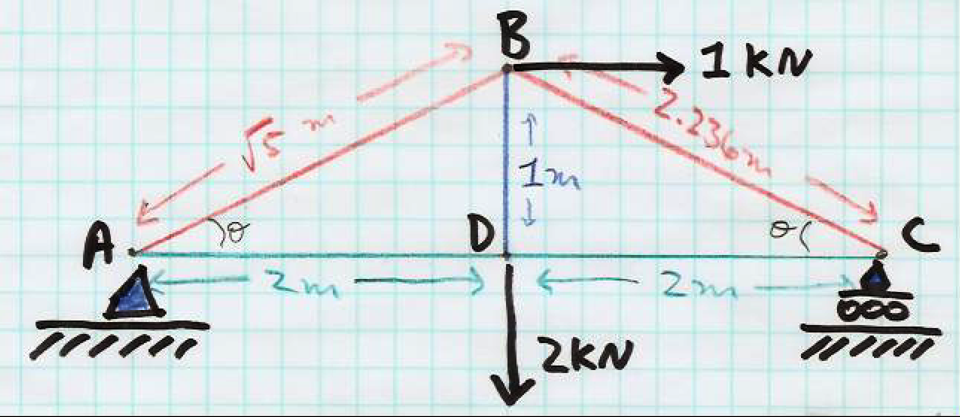
How many equations does the "method of joints" allow us to write down for this truss? (Consider how many joints the truss has.)

(A) 4 (B) 8 (C) 12 (D) 15



How many unknown internal forces (tensions or compressions) do we need to determine when we "solve" this truss?

(A) 4 (B) 5 (C) 6 (D) 7



This is a "simply supported" truss. How many independent "reaction forces" do the two supports exert on the truss? (If there are independent horizontal and vertical components, count them as separate forces.)

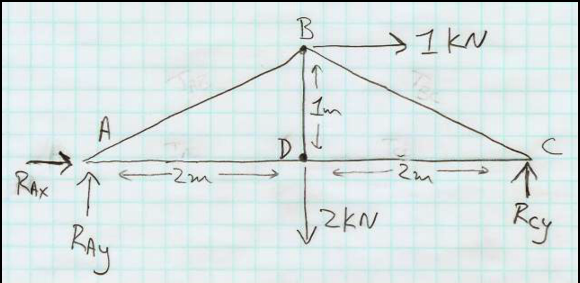
(A) 2 (B) 3 (C) 4 (D) 6

Notice that $8 = 5 + 3$.

For a planar truss that is stable and that you can solve using the equations of static equilibrium,

$$2N_{\text{joints}} = N_{\text{bars}} + 3$$

You get two force equations per joint. You need to solve for one unknown tension/compression per bar plus three support "reaction" forces.



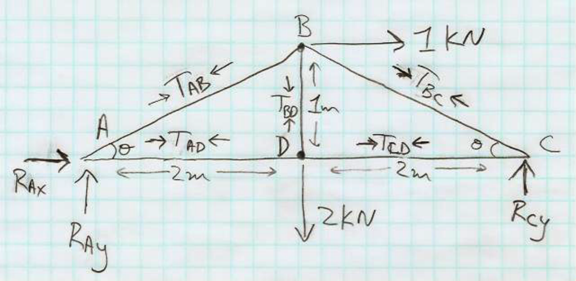
What do we learn by writing $\sum F_x = 0$, $\sum F_y = 0$, $\sum M_z = 0$ for the truss as a whole? (Use joint **A** as pivot.)

(I write R_{Ax}, R_{Ay}, R_{Cy} for the 3 "reaction forces" exerted by the supports on the truss.)

(A) $R_{Ay} - 2\ \mathrm{kN} + R_{Cy} = 0,$
$R_{Ax} + 1\ \mathrm{kN} = 0,$
$-(2\ \mathrm{kN})(2\ \mathrm{m}) - (1\ \mathrm{kN})(2\ \mathrm{m}) + (R_{Cy})(2\ \mathrm{m}) = 0$

(B) $R_{Ay} - 2\ \mathrm{kN} + R_{Cy} = 0,$
$R_{Ax} + 1\ \mathrm{kN} = 0,$
$-(2\ \mathrm{kN})(2\ \mathrm{m}) - (1\ \mathrm{kN})(1\ \mathrm{m}) + (R_{Cy})(4\ \mathrm{m}) = 0$

(C) $R_{Ay} - 2\ \mathrm{kN} + R_{Cy} = 0,$
$R_{Ax} + 1\ \mathrm{kN} = 0,$
$-(2\ \mathrm{kN})(2\ \mathrm{m}) - (1\ \mathrm{kN})(2\ \mathrm{m}) + (R_{Cy})(4\ \mathrm{m}) = 0$



What two equations does the "method of joints" let us write for joint **C** ?

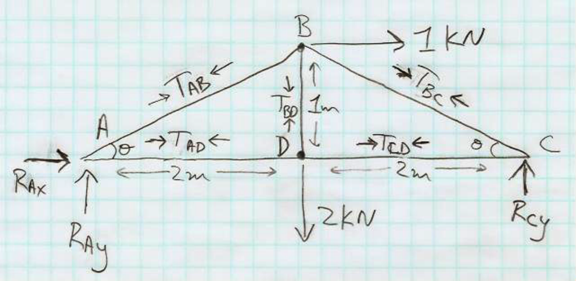
(Let the tension in member $i \leftrightarrow j$ be T_{ij}. For compression members, we will find $T_{ij} < 0$.)

(A) $T_{CD} - T_{BC}\cos\theta = 0$
$R_{Cy} - T_{BC}\sin\theta = 0$

(B) $T_{CD} - T_{BC}\sin\theta = 0$
$R_{Cy} - T_{BC}\cos\theta = 0$

(C) $T_{CD} + T_{BC}\cos\theta = 0$
$R_{Cy} + T_{BC}\sin\theta = 0$

(D) $T_{CD} + T_{BC}\sin\theta = 0$
$R_{Cy} + T_{BC}\cos\theta = 0$



What two equations does the "method of joints" let us write for joint **A** ?

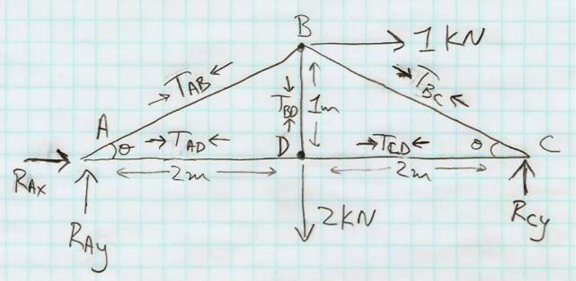
(Let the tension in member $i \leftrightarrow j$ be T_{ij}. For compression members, we will find $T_{ij} < 0$.)

(A) $R_{Ax} - T_{AD} - T_{AB} \cos\theta = 0$
$R_{Ay} - T_{AB} \sin\theta = 0$

(B) $R_{Ax} - T_{AD} - T_{AB} \sin\theta = 0$
$R_{Ay} - T_{AB} \cos\theta = 0$

(C) $R_{Ax} + T_{AD} + T_{AB} \cos\theta = 0$
$R_{Ay} + T_{AB} \sin\theta = 0$

(D) $R_{Ax} + T_{AD} + T_{AB} \sin\theta = 0$
$R_{Ay} + T_{AB} \cos\theta = 0$



What two equations does the "method of joints" let us write for joint **D** ?

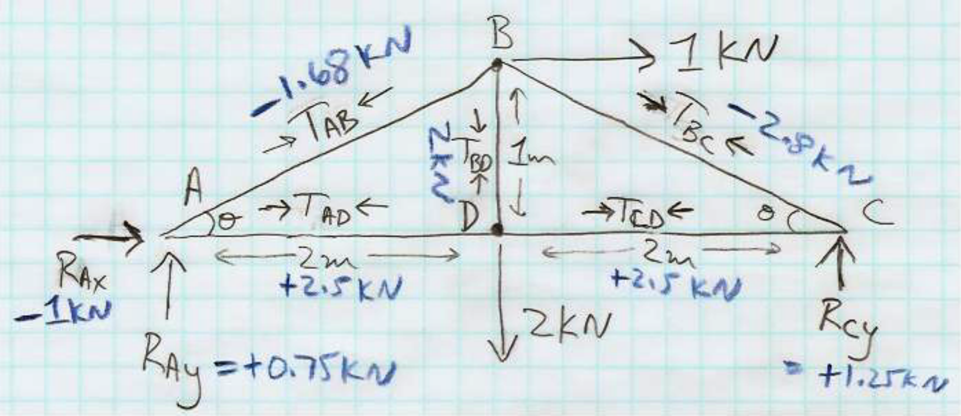
(Let the tension in member $i \leftrightarrow j$ be T_{ij}. For compression members, we will find $T_{ij} < 0$.)

(A) $-2\ \mathrm{kN} + T_{BD} = 0$ and $-T_{AD} + T_{CD} = 0$

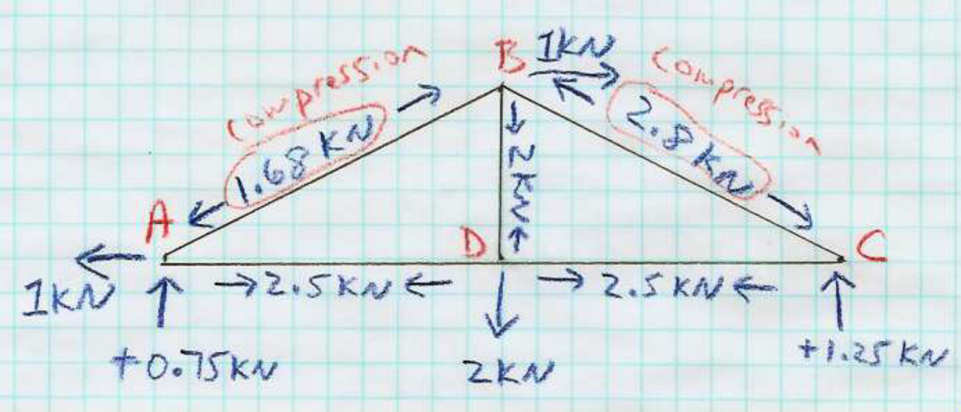
(B) $-2\ \mathrm{kN} + T_{BD} = 0$ and $-T_{AD} - T_{CD} = 0$

(C) $-2\ \mathrm{kN} - T_{BD} = 0$ and $-T_{AD} + T_{CD} = 0$

(D) $-2\ \mathrm{kN} - T_{BD} = 0$ and $-T_{AD} - T_{CD} = 0$



I named each member force T_{ij} (for "tension") and let $T_{ij} > 0$ mean tension and $T_{ij} < 0$ mean compression. Once you've solved the truss, it's best to draw the arrows with the correct signs for clarity. (Next page.)

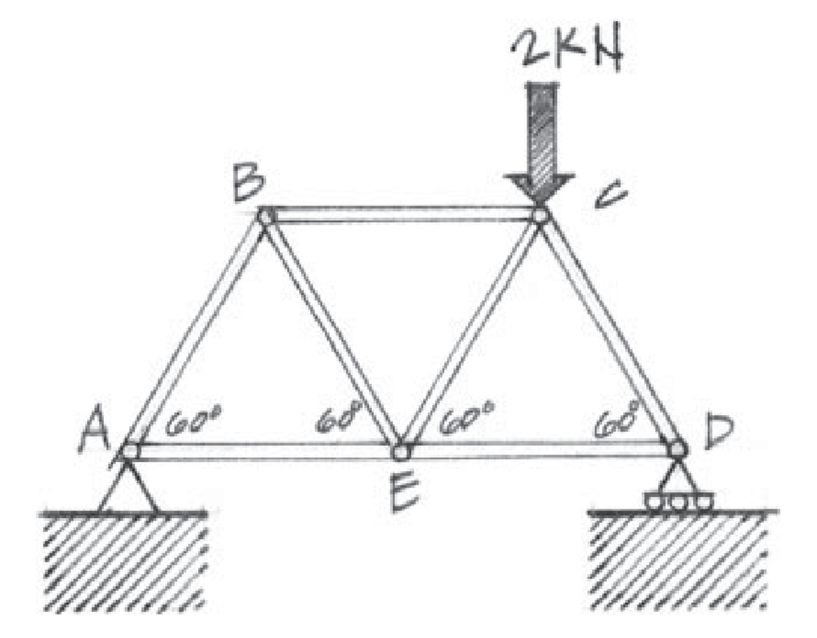


Forces redrawn with arrows in correct directions, now that we know the sign of each force. Members **AB** and **BC** are in compression. All other members are in tension.

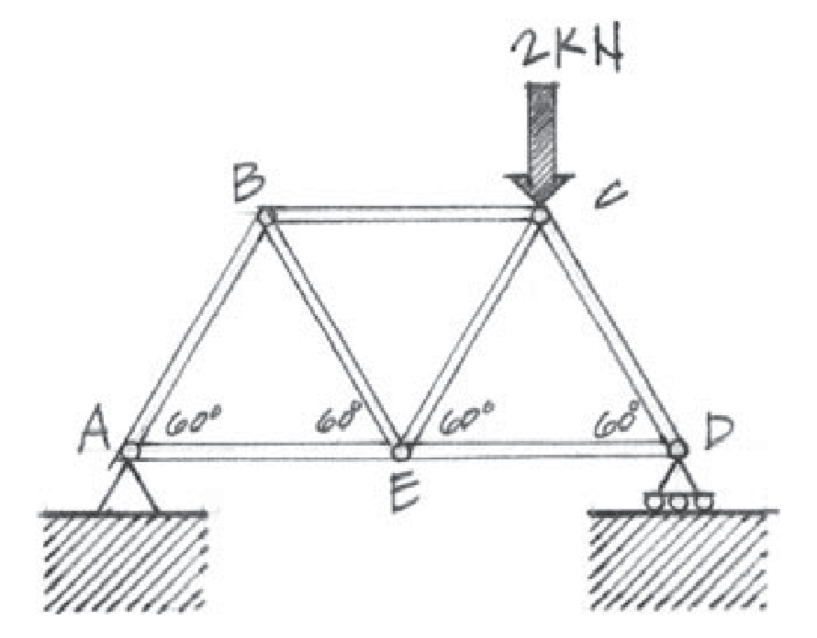
Another option is to write down all $2J$ equations at once and to type them into **Mathematica**, Maple, Wolfram Alpha, etc.

```
In[92] eq := {

RAx + TAB*cos + TAD == 0,
RAy + TAB*sin == 0,
-TAB*cos+TBC*cos+1 == 0,
-TBD-TAB*sin-TBC*sin == 0,
-TAD+TCD == 0,
-2 + TBD == 0,
-TCD - TBC*cos == 0,
RCy + TBC*sin == 0,

sin==1.0/Sqrt[5.0],
cos==2.0/Sqrt[5.0]

}
```

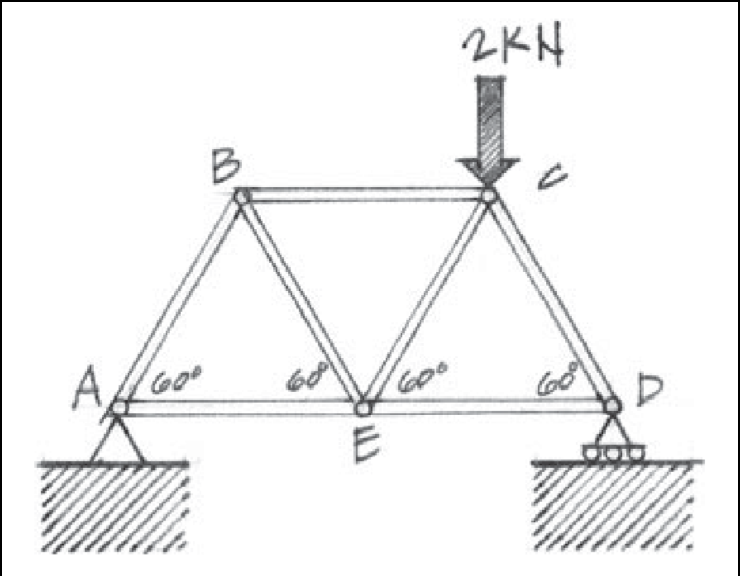
```
In[93] Solve[eq]

Out[93] {

RAx → -1.,
RAy → 0.75,
RCy → 1.25,
TAB → -1.67705,
TAD → 2.5,
TBC → -2.79508,
TBD → 2.,
TCD → 2.5,

cos → 0.894427,
sin → 0.447214

}
```



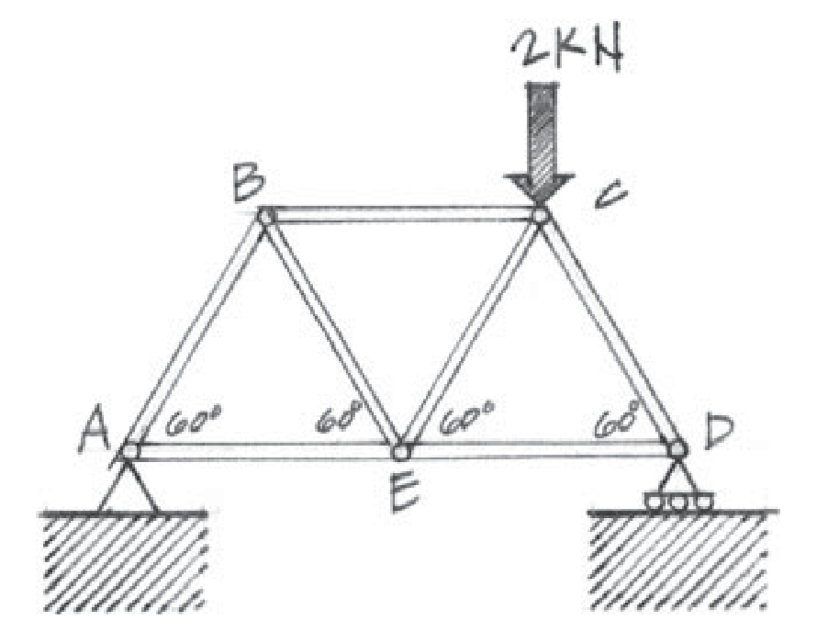
How many "reaction forces" are exerted by the supports (i.e. exerted on the truss by the supports)?



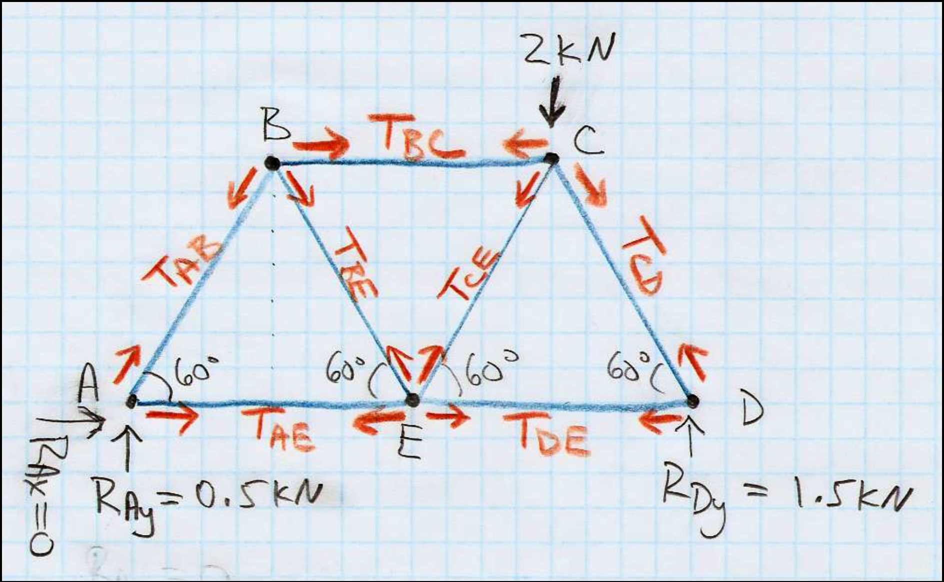
How many internal forces (tensions or compressions in the members) do we need to solve for to "solve" this truss?



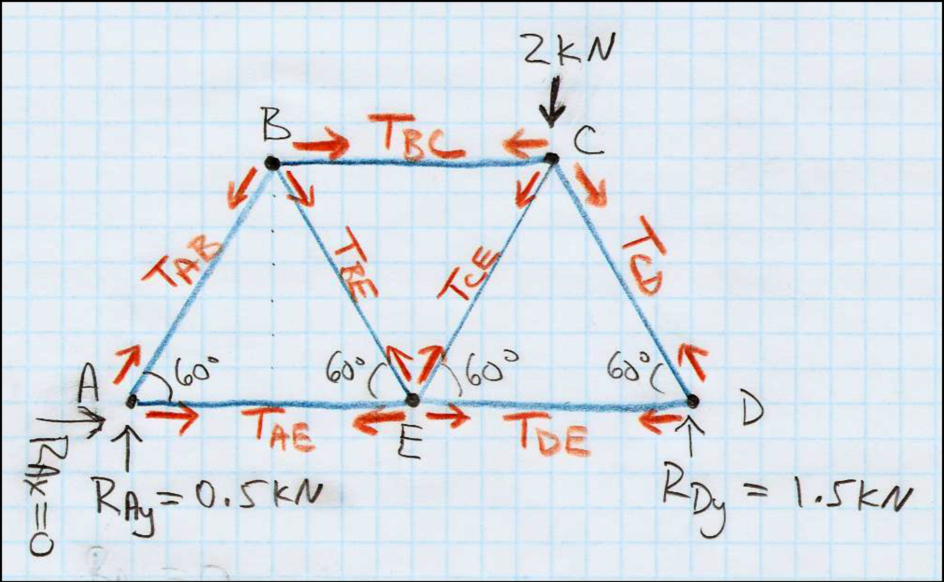
Do you see any joint at which there are ≤ 2 unknown forces? If so, we can start there. If not, we need to start with an EFBD for the truss as a whole.



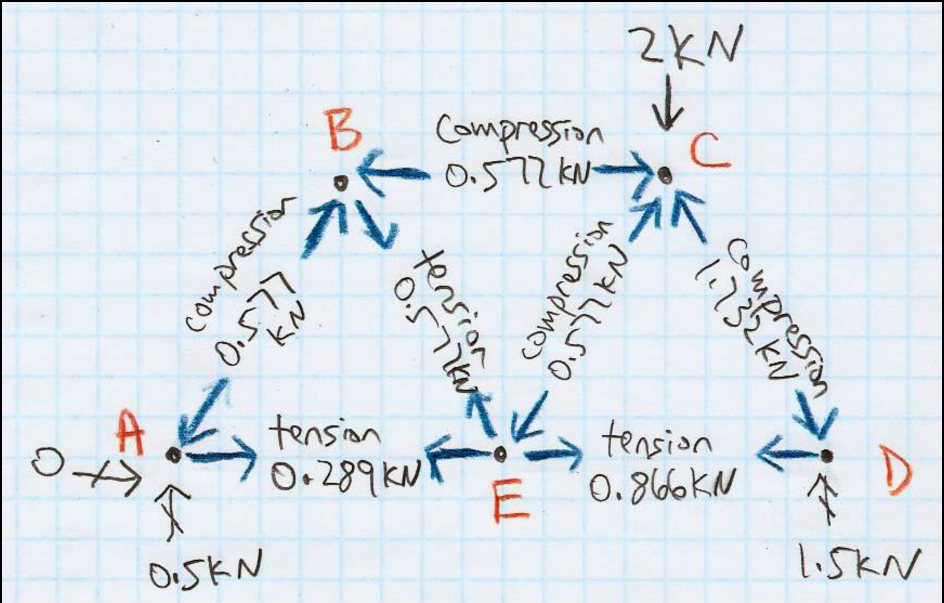
Try to guess $R_{A,x}$, $R_{A,y}$, and $R_{D,y}$ by inspection. Then let's check with the usual equations.



Now start from a joint having ≤ 2 unknown forces. In this case, I just went through the joints alphabetically. You can make your life easier by seeking out equations having just 1 unknown.



$T_{AB} = -0.577$ kN, $T_{AE} = +0.289$ kN, $T_{BE} = +0.577$ kN, $T_{BC} = -0.577$ kN, $T_{CE} = -0.577$ kN, $T_{CD} = -1.732$ kN, $T_{DE} = +0.866$ kN. My notation: tension > 0, compression < 0.



2KN
B
C
Compression
0.577 kN
compression
0.577 kN
tension
0.577 kN
compression
0.577kN
compression
1.732 kN
A
tension
0.289kN
E
tension
0.866kN
D
0
0.5KN
1.5KN

$\cos 60^\circ = 0.5, \quad \sin 60^\circ = 0.866$

Joint A: $0 + T_{AE} + T_{AB}\cos 60^\circ = 0$

$0.5\,kN + T_{AB}\sin 60^\circ = 0 \Rightarrow \boxed{T_{AB} = -0.577\,kN}$

$\boxed{T_{AE} = +0.289\,kN}$

Joint B: $-T_{AB}\cos 60^\circ + T_{BE}\cos 60^\circ + T_{BC} = 0$

$-T_{AB}\sin 60^\circ - T_{BE}\sin 60^\circ = 0 \Rightarrow \boxed{T_{BE} = +0.577\,kN}$

$\boxed{T_{BC} = -0.577\,kN}$

Joint C: $-T_{BC} - T_{CE}\cos 60^\circ + T_{CD}\cos 60^\circ = 0 \Rightarrow T_{CD} = T_{CE} - 1.155\,kN$

(could do C, D, or E at this point)

$-2\,kN - T_{CE}\sin 60^\circ - T_{CD}\sin 60^\circ = 0$

$-2\,kN - T_{CE}\sin 60^\circ - (T_{CE} - 1.155\,kN)\sin 60^\circ = 0$

$2T_{CE}\sin 60^\circ = -2\,kN + 1.0\,kN = -1\,kN$

$\boxed{T_{CE} = -0.577\,kN}$

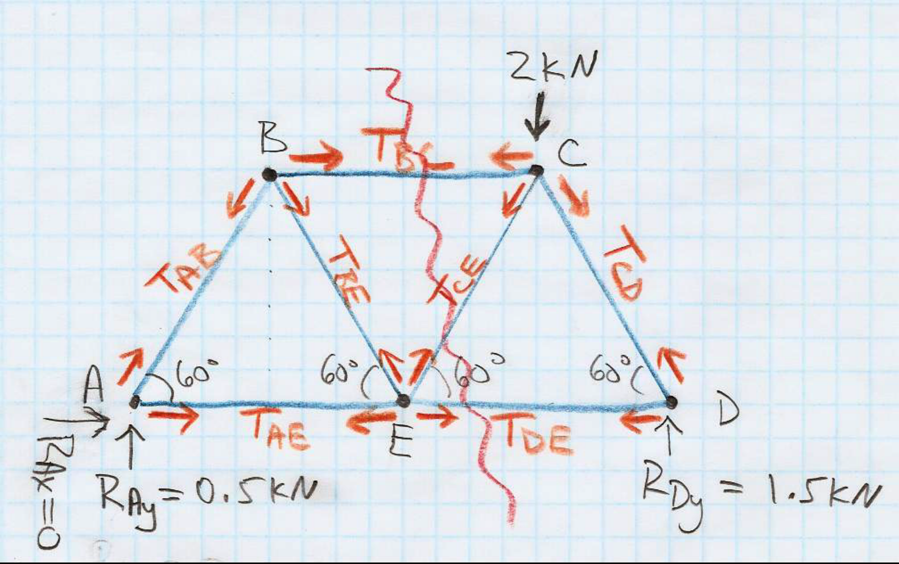
$T_{CD} = T_{CE} - 1.155\,kN = \boxed{-1.732\,kN = T_{CD}}$

Joint D: $-T_{DE} - T_{CD}\cos 60^\circ = 0 \Rightarrow \boxed{T_{DE} = +0.866\,kN}$

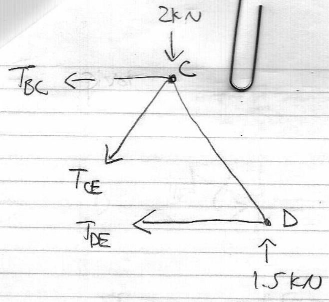
(check): $1.5\,kN + T_{CD}\sin 60^\circ = 0$ ✓

(check Joint E): $-T_{AE} + T_{DE} - T_{BE}\cos 60^\circ + T_{CE}\cos 60^\circ = 0$ ✓

$+T_{BE}\sin 60^\circ + T_{CE}\sin 60^\circ = 0$ ✓



Let's try drawing an EFBD for the **right** side of the cut ("section").



$$\sum F_x = 0 \Rightarrow -T_{BC} - T_{CE}\cos 60^\circ - T_{DE} = 0$$

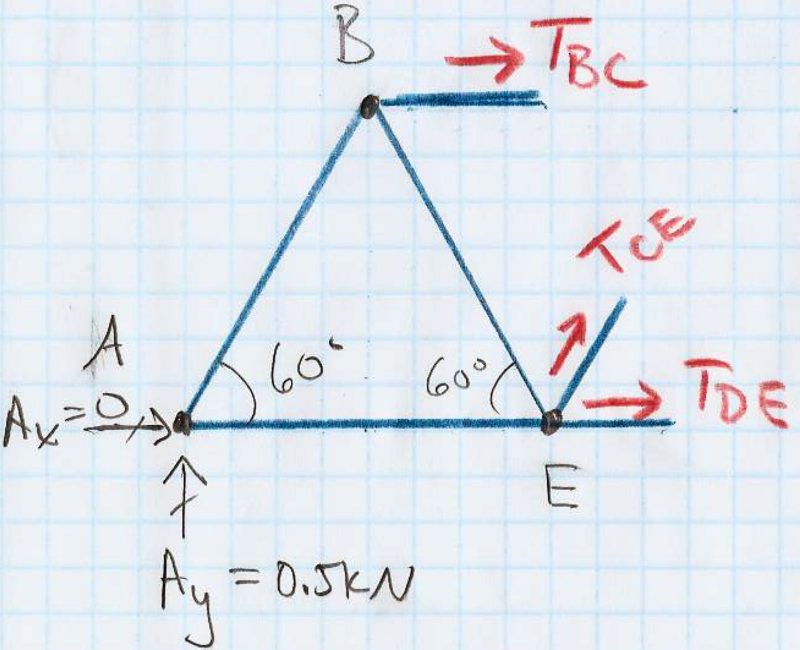
$$\sum F_y = 0 \Rightarrow -2kN + 1.5kN - T_{CE}\sin 60^\circ = 0$$

$$\sum M_c = 0 \Rightarrow +(1.5kN)\left(\frac{L}{2}\right) - T_{DE}\, L\sin 60^\circ = 0$$

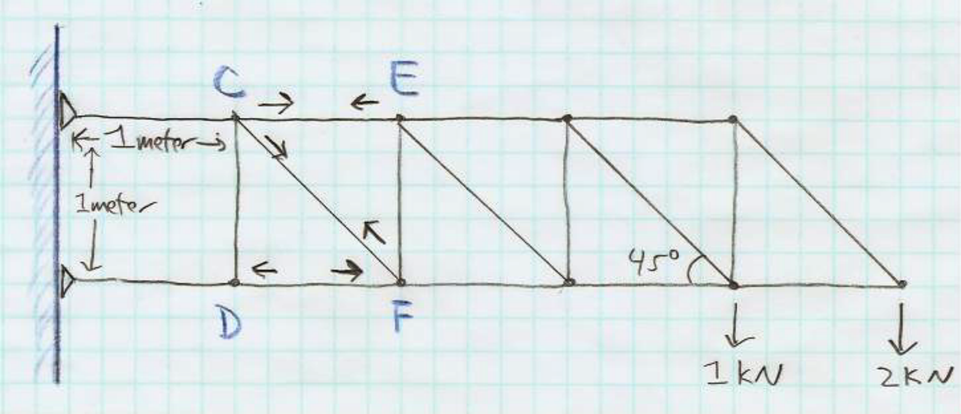
$$T_{DE} = \frac{1.5kN}{2\sin 60^\circ} = +0.866kN$$

$$T_{CE} = \frac{-0.5kN}{\sin 60^\circ} = -0.577kN$$

$$T_{BC} = -(T_{CE}\cos 60^\circ + T_{DE}) = -0.577$$



B
T_{BC}
T_{CE}
T_{DE}
A
60°
60°
$A_x = 0$
E
$A_y = 0.5kN$



Here's another truss problem that you can solve using the Method of Sections. Find forces in members **CE**, **CF**, and **DF**, with assumed force directions as shown.

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- Where should we "section" the truss?
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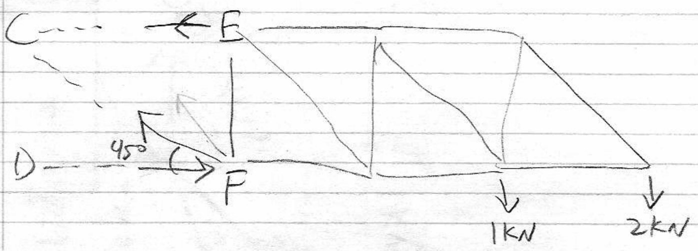


$$\sum F_x = 0 = -CE - CF\cos 45^\circ + DF = 0$$

$$\sum F_y = 0 = CF\sin 45^\circ = 3\,kN \Rightarrow \boxed{CF = 3\,kN\sqrt{2}}$$

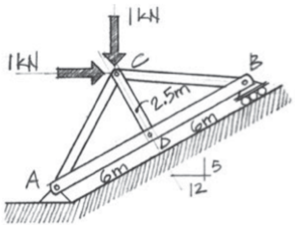
$$\sum M_{DF} = 0 = CE(1) - (1\,kN)(2) - (2\,kN)(3)$$

$$\Rightarrow \boxed{CE = 8\,kN}$$

$$DF = CE + CF\cos 45^\circ = 8\,kN + 3\,kN$$

$$\boxed{DF = 11\,kN}$$

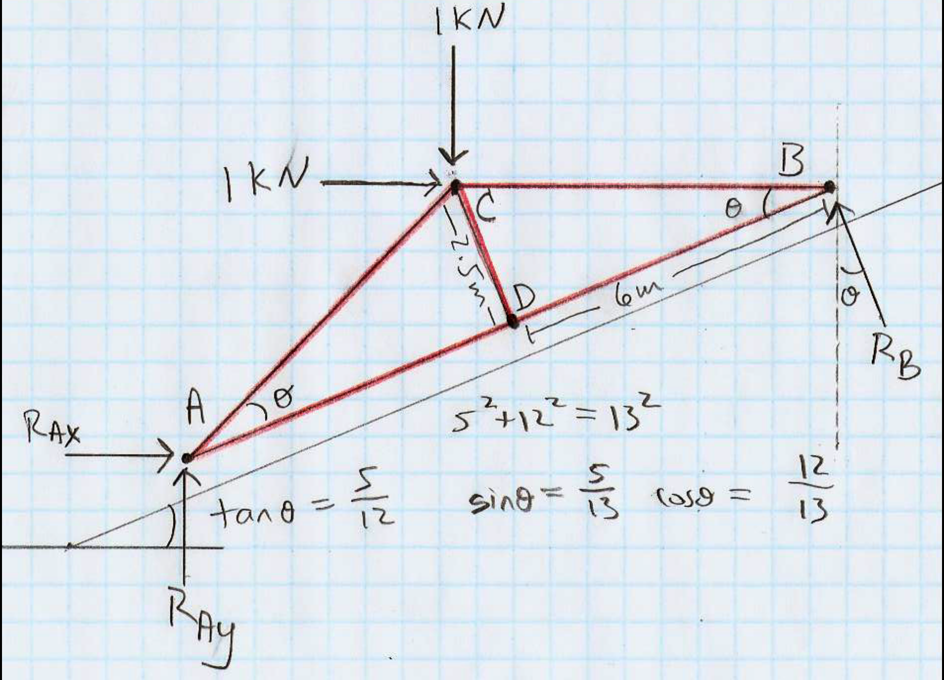
2.38 An inclined king-post truss supports a vertical and horizontal force at *C*. Determine the support reactions developed at *A* and *B*.



This is not really a "truss problem," since we're not asked to solve for the internal forces in the truss, but it is an example of a pretty tricky equilibrium problem.

Let's try working through this together in class.

Notice, from the given dimensions: the angle of the incline is the same as the interior angle at joints A and B of the truss. Also notice pin/hinge support at A and roller support at B.


1 KN
1 KN
A
B
C
D
θ
2.5m
6m
R_{Ax}
R_{Ay}
R_B
$5^2+12^2=13^2$
$\tan\theta = \frac{5}{12}$
$\sin\theta = \frac{5}{13}$
$\cos\theta = \frac{12}{13}$

① $0 = \sum_{\text{on truss}} F_x = R_{Ax} + 1\,kN - R_B \sin\theta$

$$\Rightarrow R_{Ax} = R_B \sin\theta - 1\,kN$$

② $0 = \sum_{\text{on truss}} F_y = R_{Ay} - 1\,kN + R_B \cos\theta$

$$\Rightarrow R_{Ay} = 1\,kN - R_B \cos\theta$$

③ $0 = \sum M_A = R_B(12m) - 1\,kN(12m \sin\theta) - 1\,kN(6.5m \cos 2\theta)$

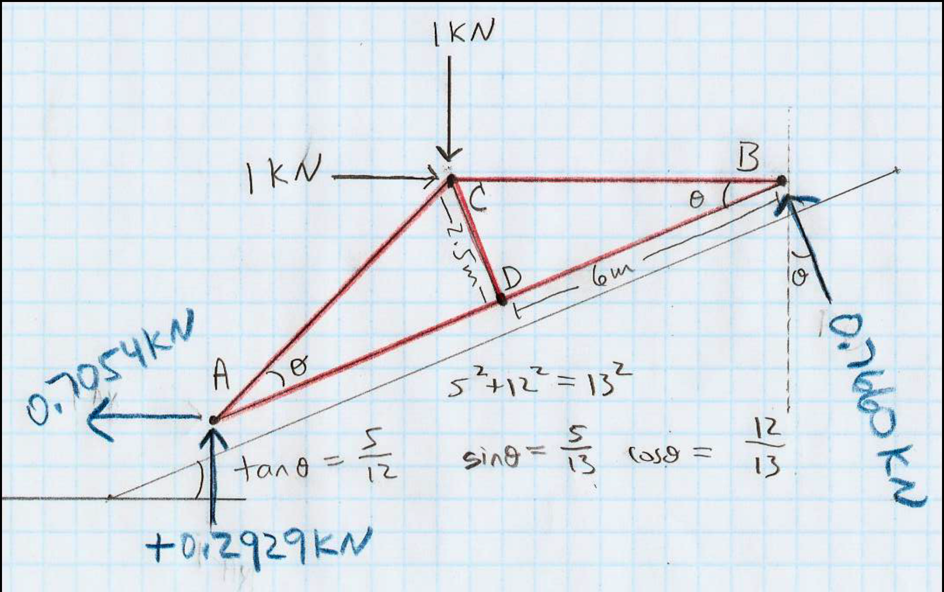
$$\Rightarrow R_B = 1\,kN(\sin\theta) + \frac{6.5}{12}\,kN(\cos 2\theta) = 0.7660\,kN$$

$$(R_B)_x = -R_B \sin\theta = -0.2946\,kN$$

$$(R_B)_y = R_B \cos\theta = 0.7071\,kN$$

$$R_{Ax} = (0.2946 - 1)\,kN = -0.7054\,kN$$

$$R_{Ay} = (1 - 0.7071)\,kN = 0.2929\,kN$$



1KN
1KN
B
C
θ
2.5m
D
6m
θ
0.7054KN
A
θ
$5^2+12^2=13^2$
0.7660 KN
$\tan\theta = \frac{5}{12}$
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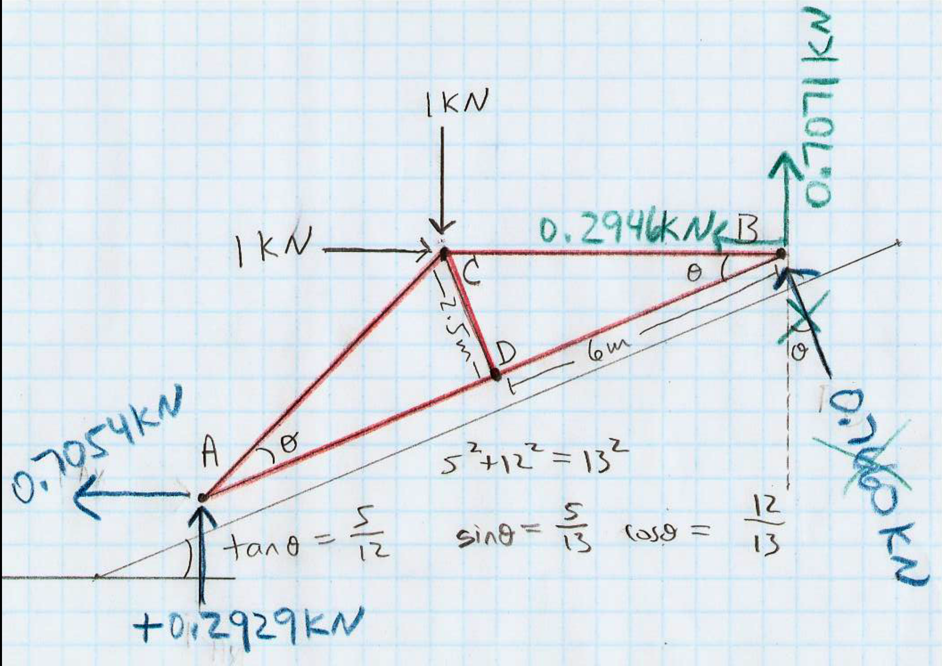
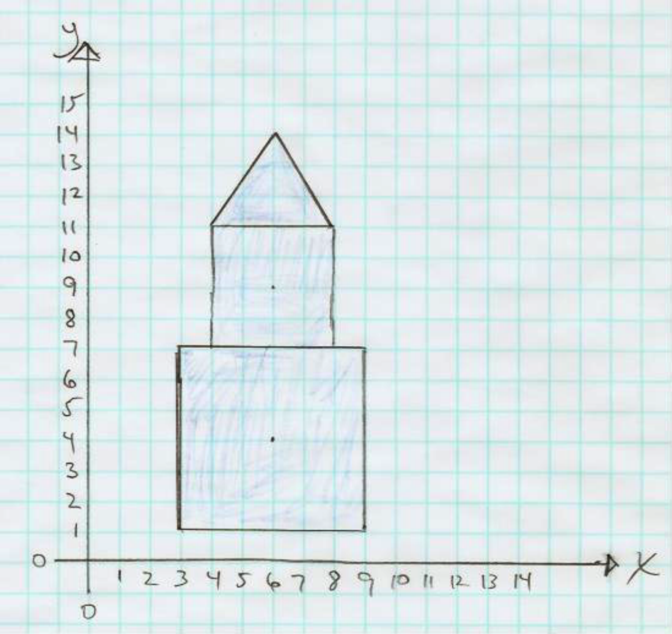
$\cos\theta = \frac{12}{13}$
+0.2929KN



1KN
1KN
0.2946KN
0.7071KN
B
C
θ
2.5m
D
6m
θ
A
θ
0.7054KN
$5^2+12^2=13^2$
$\tan\theta = \frac{5}{12}$
$\sin\theta = \frac{5}{13}$
$\cos\theta = \frac{12}{13}$
+0.2929KN
0.7660 KN

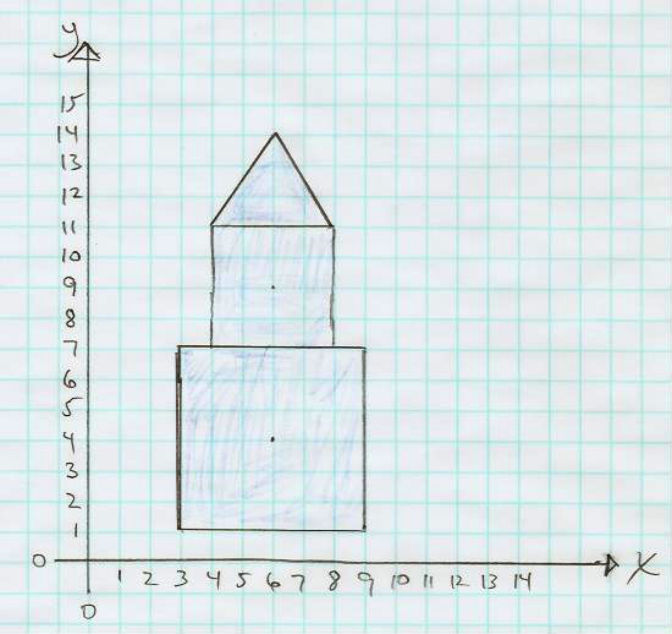
- The idea of computing centroids of simple and composite shapes is very, very briefly introduced in O/K ch3 (in the context of "distributed loads"), and will be discussed in much more detail in O/K ch6 (for Monday).
- Let's go through one example using rectangles and triangles. It will help you in cases when you need to solve for the "reaction forces" on a beam that carries distributed loads. (Example coming up next.)

What is X_{centroid} for the shaded area?



(A) 0
(B) 3
(C) 6
(D) 9

What are the areas of the three individual polygons?



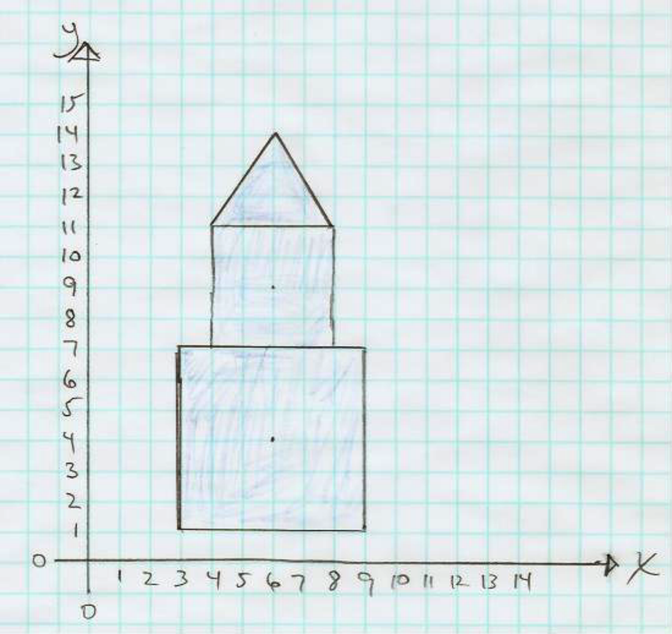
(A) 36, 16, 16

(B) 36, 16, 12

(C) 36, 16, 8

(D) 36, 16, 6

What are the Y_{centroid} values of the three individual polygons?



(A) 4, 9, 11

(B) 4, 9, 11.667

(C) 4, 9, 12

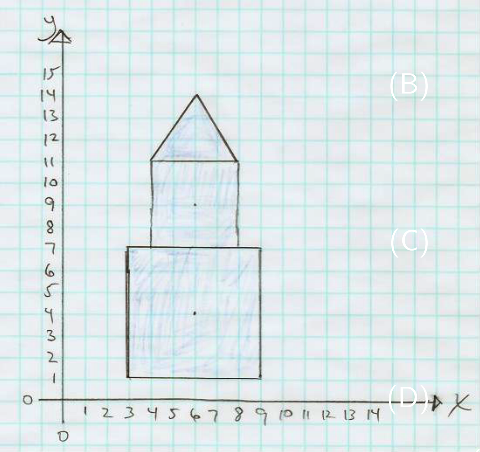
(D) 4, 9, 12.333

(E) 4, 9, 12.5

(F) 4, 9, 13

(G) 4, 9, 14

What is Y_{centroid} for the whole shaded area?



(A)

$$\frac{4+9+12}{3} = 8.33$$

(B)

$$\frac{(4)(36)+(9)(16)+(12)(6)}{36+16+6} = 6.21$$

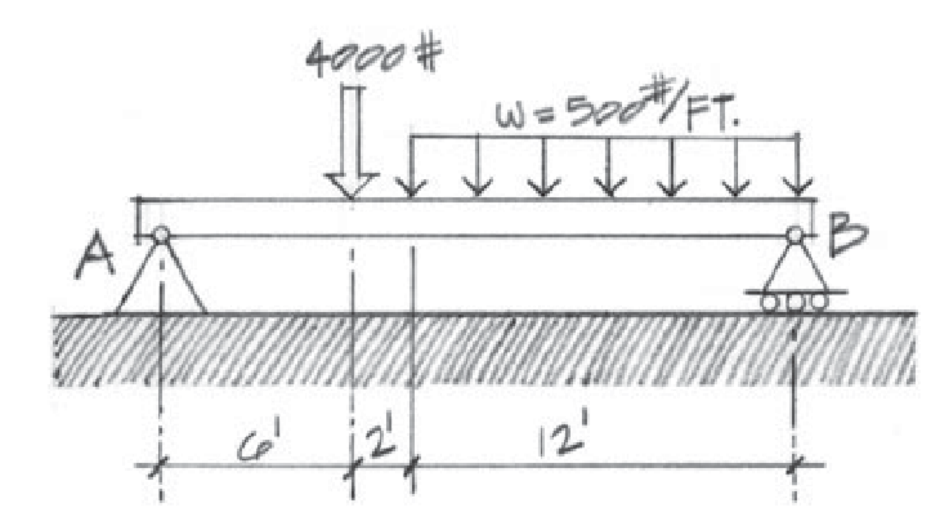
(C)

$$\frac{(4)(36)+(9)(16)+(12)(6)}{4+9+12} = 14.4$$

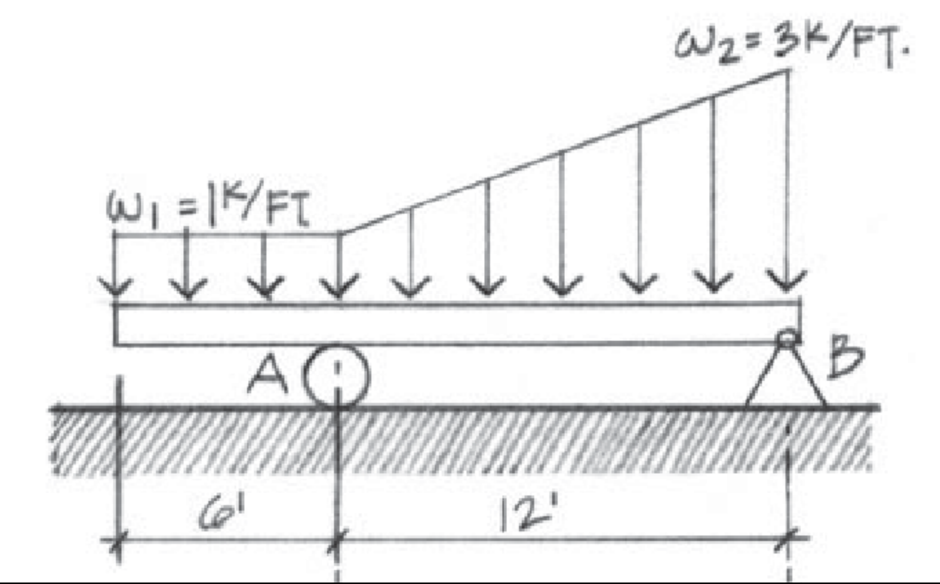
(D)

$$\frac{(4^2)(36)+(9^2)(16)+(12^2)(6)}{36+16+6} = 47.2$$

There's one problem similar to this (but using metric units) on HW(?): Determine the support reactions at A and B.



This one is harder than your homework, because the distributed load is non-uniform: Determine the support reactions at A and B.



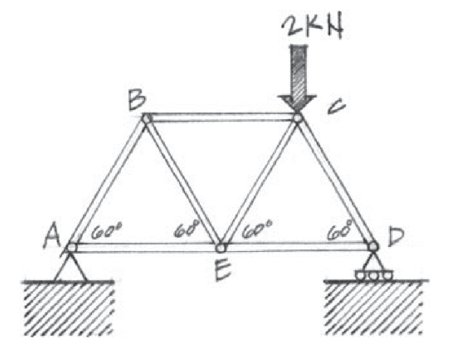
Physics 8 — Friday, November 8, 2019

- Turn in HW9. Pick up HW10 handout in back corner.
- I finally added summaries of key results from Onouye/Kane ch1-ch7 to the "equation sheet." I'm working on ch8, and I may do ch9 as well (though ch9 will be XC for you).
- This week, you skimmed Ch4 (load tracing) and read Ch5 (strength of materials) of Onouye/Kane. Next week, you'll read Ch6 (cross-sectional properties) and Ch7 (simple beams).

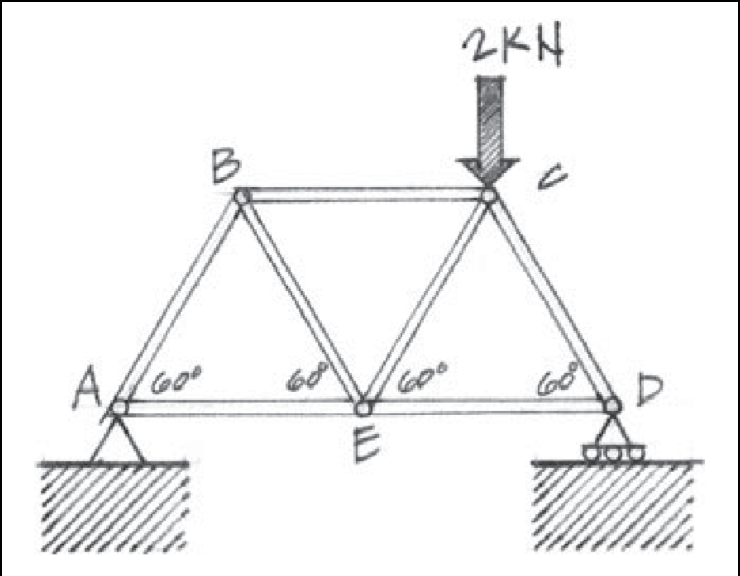
Physics 8 — Monday, November 11, 2019

- Last week, you skimmed Ch4 (load tracing) and read Ch5 (strength of materials) of Onouye/Kane. This week, you're reading Ch6 (cross-sectional properties) and Ch7 (simple beams).

How many "reaction forces" (or components thereof) are exerted by the supports (i.e. exerted on the truss by the supports)?

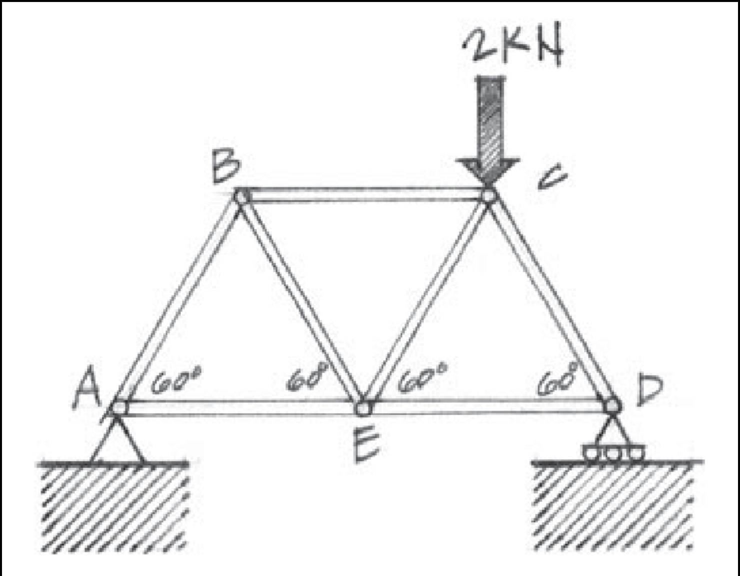


(A) 1 (B) 2 (C) 3 (D) 4 (E) 5 (F) 6 (G) 7 (H) 8



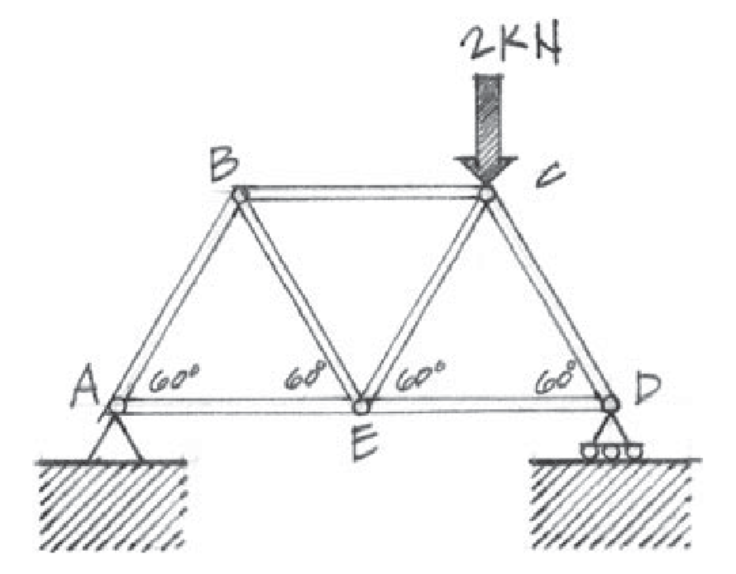
How many "reaction forces" are exerted by the supports (i.e. exerted on the truss by the supports)?

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5 (F) 6 (G) 7 (H) 8

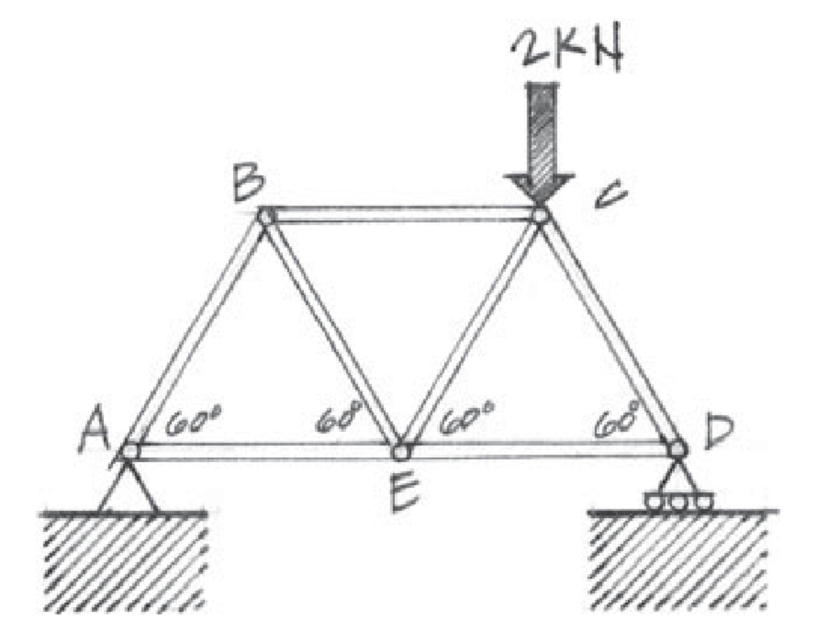


How many internal forces (tensions or compressions in the members) do we need to solve for to "solve" this truss?

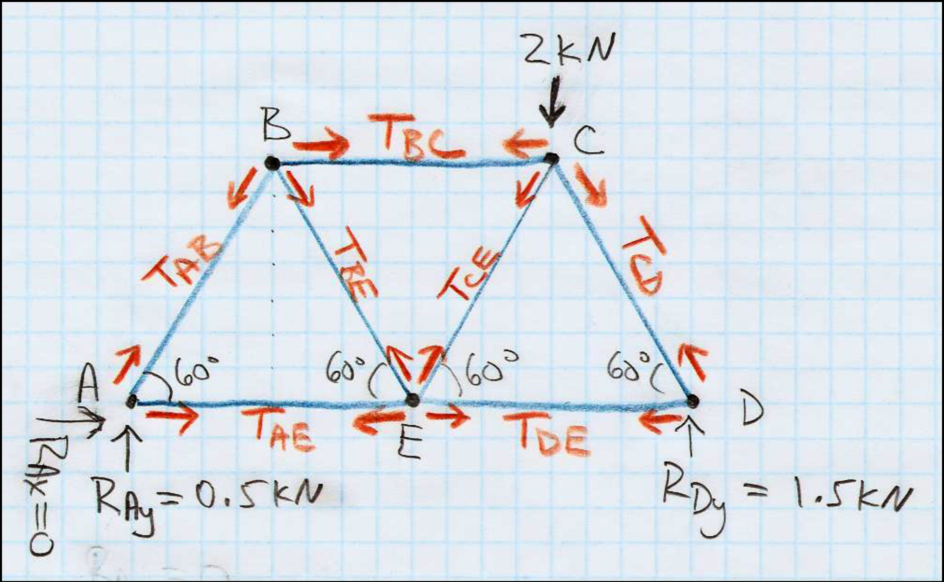
(A) 1 (B) 2 (C) 3 (D) 4 (E) 5 (F) 6 (G) 7 (H) 8



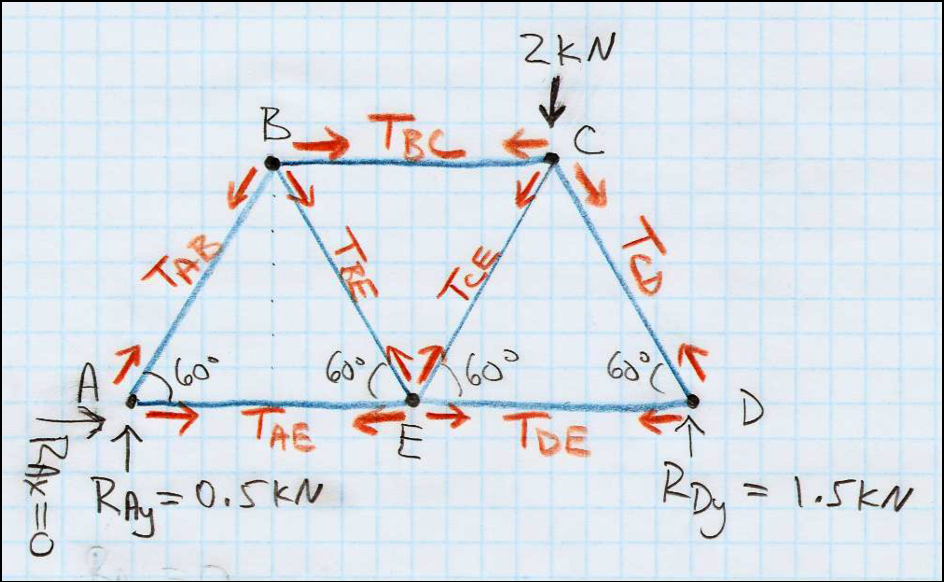
Do you see any joint at which there are ≤ 2 unknown forces (in this context, total, internal+external)? If so, we can start there. If not, we need to start with an EFBD for the truss as a whole.



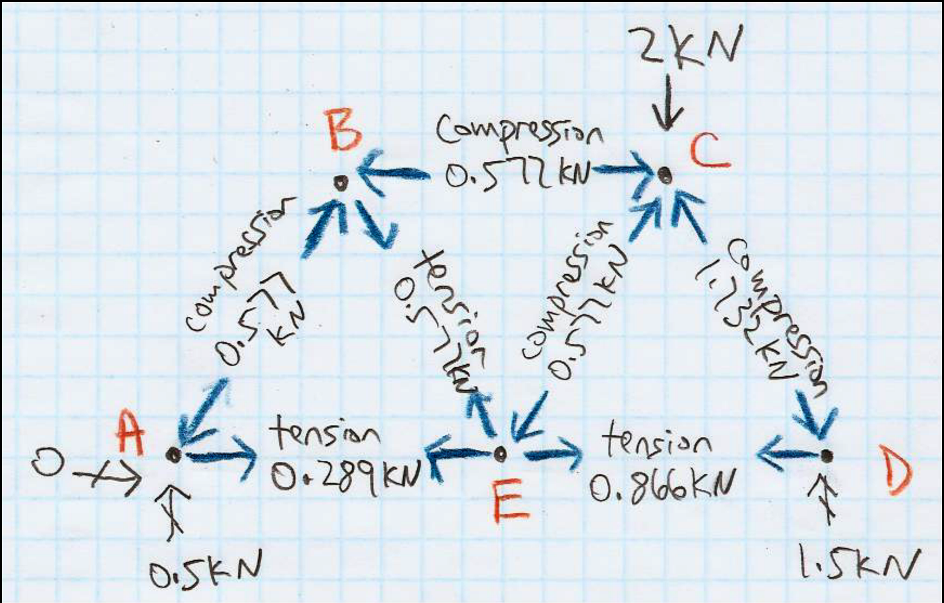
Try to guess $R_{A,x}$, $R_{A,y}$, and $R_{D,y}$ by inspection. Then let's check with the usual equations.



Now start from a joint having ≤ 2 unknown forces. In this case, I just went through the joints alphabetically. You can make your life easier by seeking out equations having just 1 unknown.



$T_{AB} = -0.577$ kN, $T_{AE} = +0.289$ kN, $T_{BE} = +0.577$ kN, $T_{BC} = -0.577$ kN, $T_{CE} = -0.577$ kN, $T_{CD} = -1.732$ kN, $T_{DE} = +0.866$ kN. My notation: tension > 0, compression < 0.



2KN
B
C
Compression
0.577kN
compression
0.577 kN
tension
0.577kN
compression
0.577kN
compression
1.732kN
A
0
tension
0.289kN
E
tension
0.866kN
D
0.5KN
1.5KN

$\cos 60^\circ = 0.5, \quad \sin 60^\circ = 0.866$

Joint A: $0 + T_{AE} + T_{AB}\cos 60^\circ = 0$

$0.5\,kN + T_{AB}\sin 60^\circ = 0 \Rightarrow \boxed{T_{AB} = -0.577\,kN}$

$\boxed{T_{AE} = +0.289\,kN}$

Joint B: $-T_{AB}\cos 60^\circ + T_{BE}\cos 60^\circ + T_{BC} = 0$

$-T_{AB}\sin 60^\circ - T_{BE}\sin 60^\circ = 0 \Rightarrow \boxed{T_{BE} = +0.577\,kN}$

$\boxed{T_{BC} = -0.577\,kN}$

Joint C: $-T_{BC} - T_{CE}\cos 60^\circ + T_{CD}\cos 60^\circ = 0 \Rightarrow T_{CD} = T_{CE} - 1.155\,kN$

(could do C, D, or E at this point)

$-2kN - T_{CE}\sin 60^\circ - T_{CD}\sin 60^\circ = 0$

$-2kN - T_{CE}\sin 60^\circ - (T_{CE} - 1.155\,kN)\sin 60^\circ = 0$

$2T_{CE}\sin 60^\circ = -2kN + 1.0\,kN = -1\,kN$

$\boxed{T_{CE} = -0.577\,kN}$

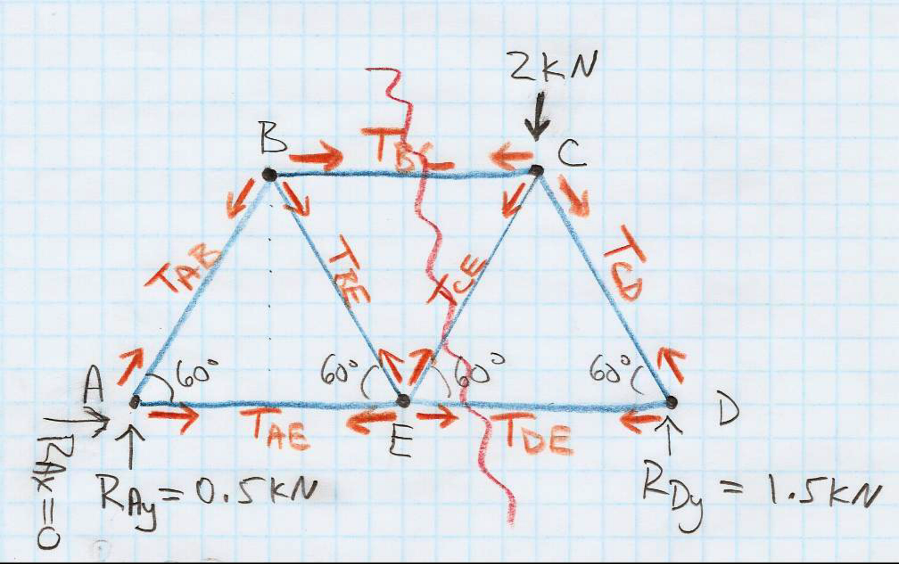
$T_{CD} = T_{CE} - 1.155\,kN = \boxed{-1.732\,kN = T_{CD}}$

Joint D: $-T_{DE} - T_{CD}\cos 60^\circ = 0 \Rightarrow \boxed{T_{DE} = +0.866\,kN}$

(check): $1.5\,kN + T_{CD}\sin 60^\circ = 0$ ✓

(check Joint E): $-T_{AE} + T_{DE} - T_{BE}\cos 60^\circ + T_{CE}\cos 60^\circ = 0$ ✓

$+T_{BE}\sin 60^\circ + T_{CE}\sin 60^\circ = 0$ ✓



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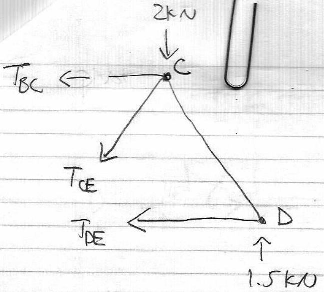
(We'll start next time with the Method of Sections for this truss.)



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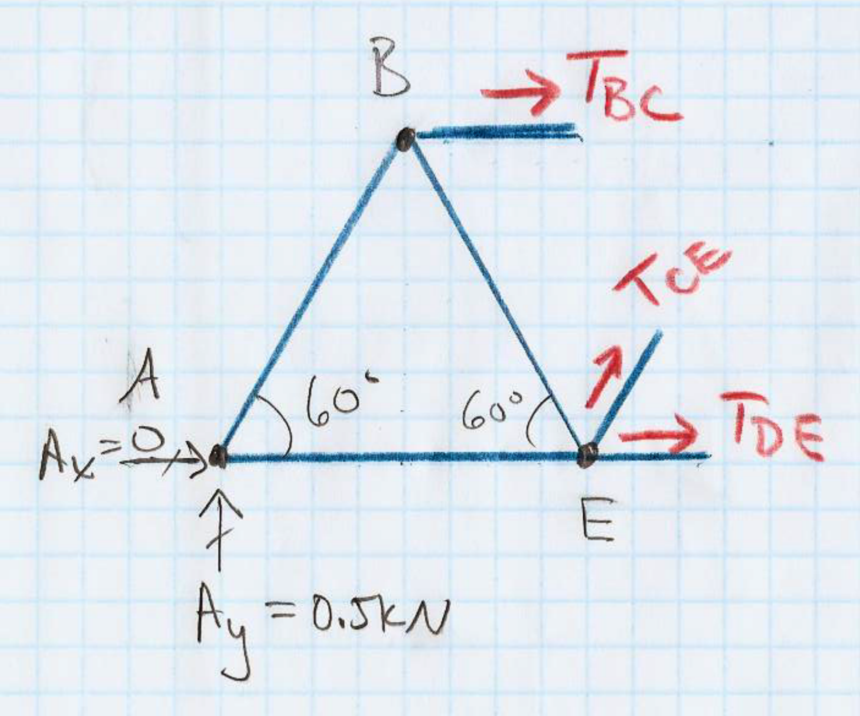

B
T_{BC}
T_{CE}
A
60°
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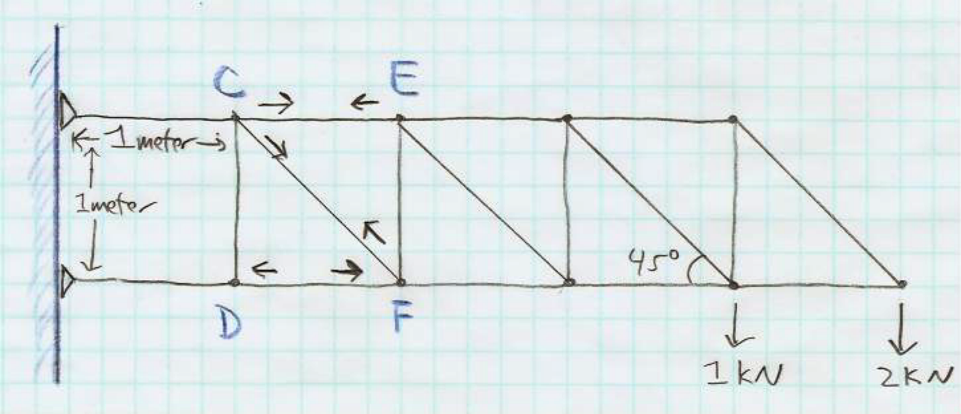
T_{DE}
$A_x = 0$
E
$A_y = 0.5kN$



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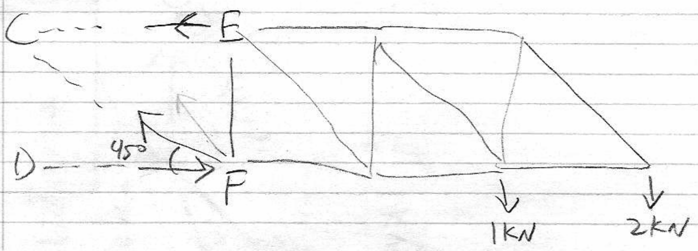

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$\Sigma M_{DF} = 0 = CE(1) - (1kN)(2) - (2kN)(3)$

$\Rightarrow \boxed{CE = 8kN}$

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$\boxed{DF = 11kN}$

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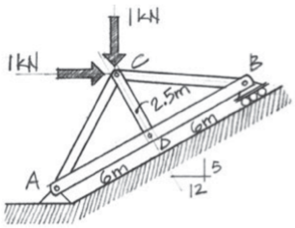

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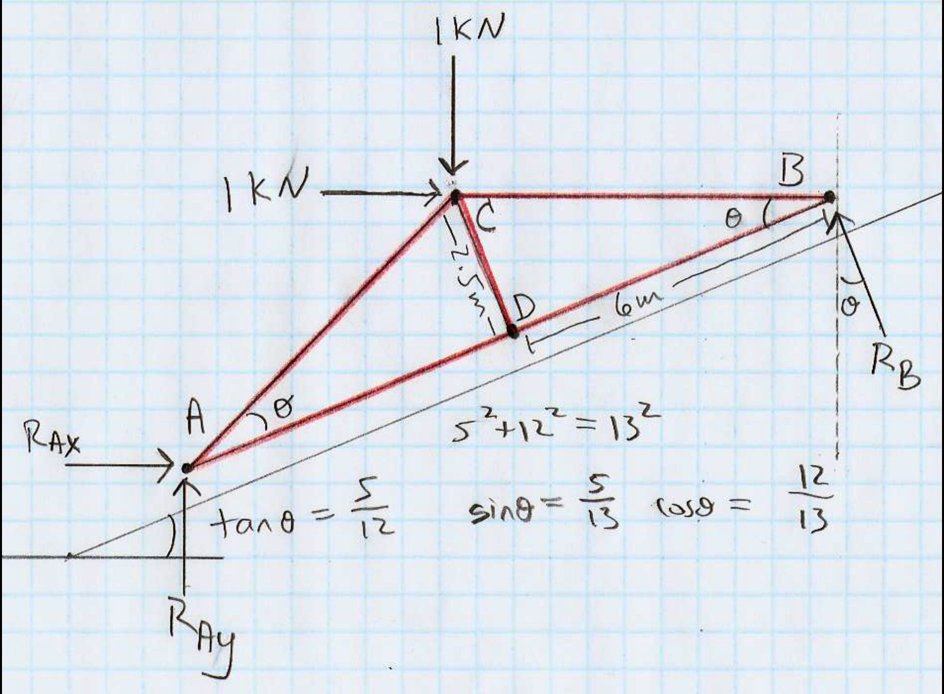
1 KN
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B
C
θ
2.5 m
D
6 m
θ
R_B
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θ
R_AX
$5^2 + 12^2 = 13^2$
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R_Ay

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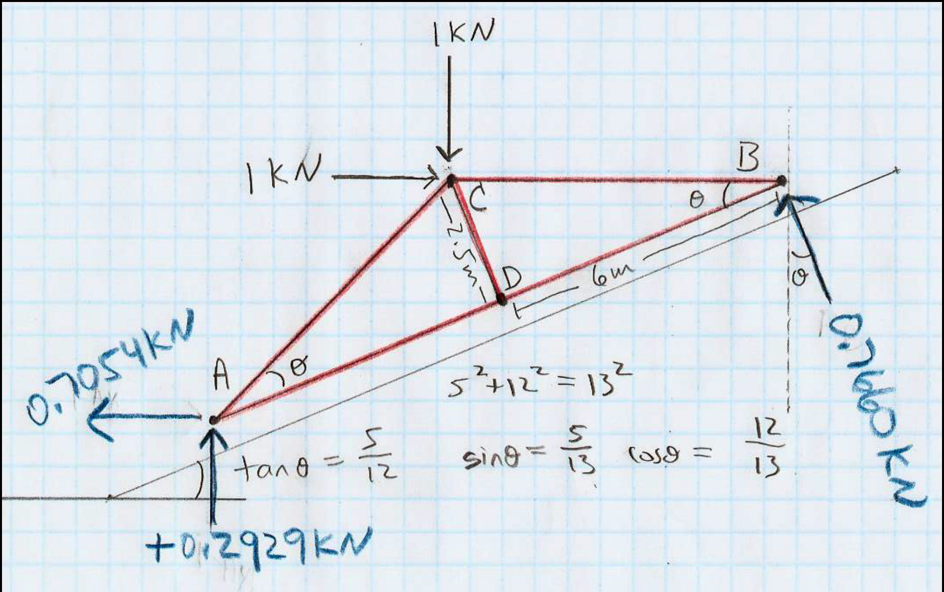
1KN
1KN
B
C
θ
2.5m
D
6m
θ
0.7054KN
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θ
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$5^2+12^2=13^2$
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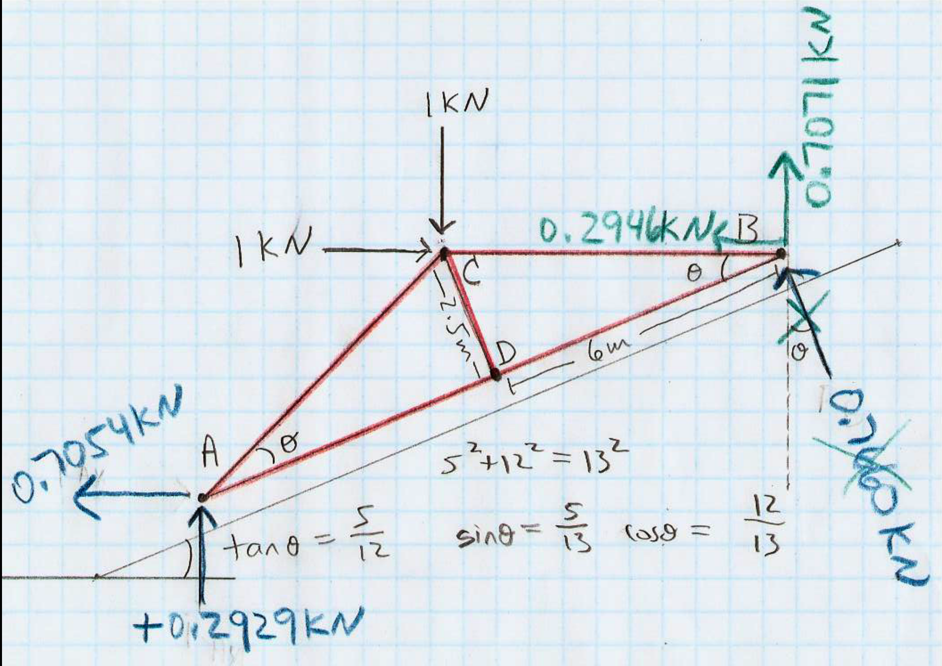
$\sin\theta = \frac{5}{13}$
$\cos\theta = \frac{12}{13}$
+0.2929KN


1KN
1KN
0.2946kN
B
0.7071 kN
C
θ
2.5m
D
6m
A
θ
0.7054kN
5²+12²=13²
tanθ = 5/12
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cosθ = 12/13
θ
+0.2929kN

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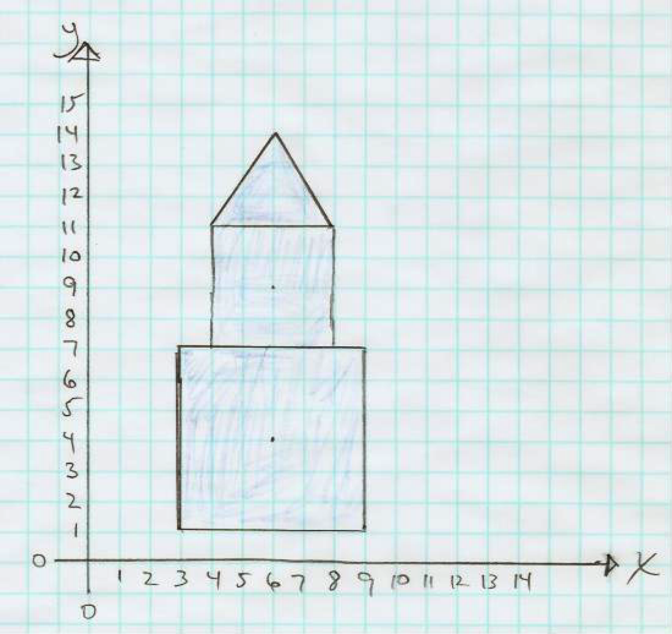

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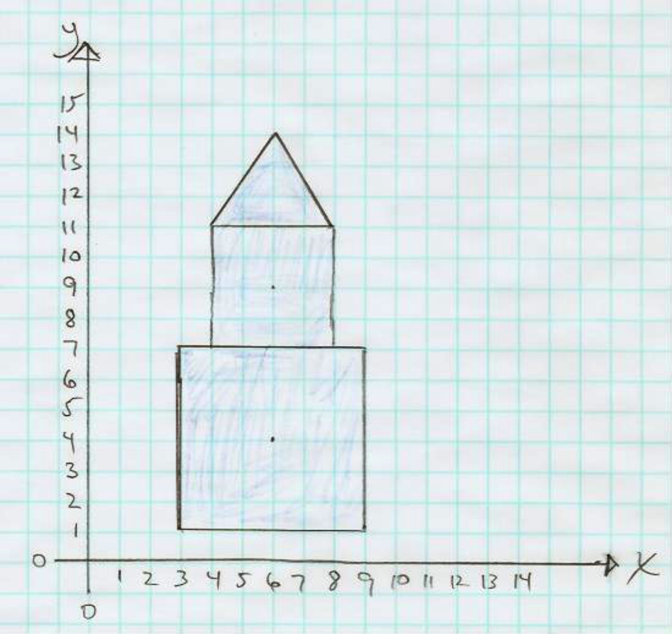

(A) 36, 16, 16
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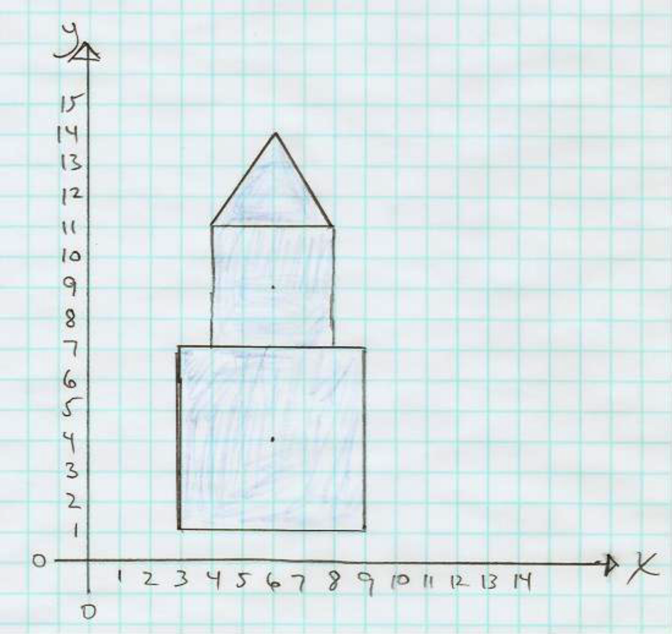
What are the Y_{centroid} values of the three individual polygons?



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(C) 4, 9, 12

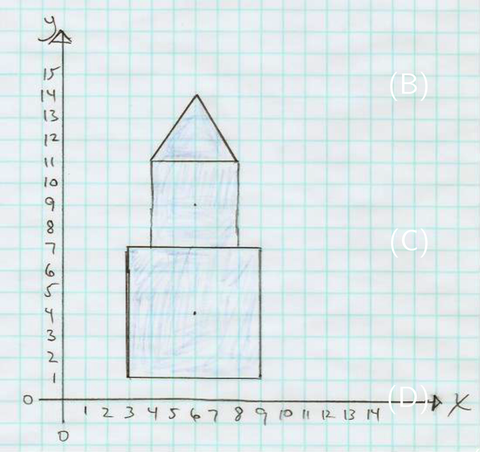
(D) 4, 9, 12.333

(E) 4, 9, 12.5

(F) 4, 9, 13

(G) 4, 9, 14

What is Y_{centroid} for the whole shaded area?



(A)

$$\frac{4+9+12}{3} = 8.33$$

(B)

$$\frac{(4)(36)+(9)(16)+(12)(6)}{36+16+6} = 6.21$$

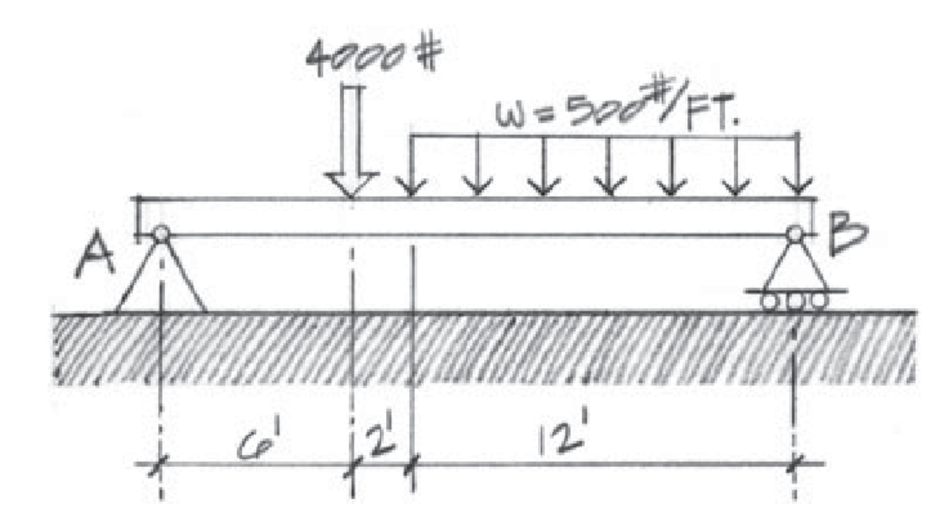
(C)

$$\frac{(4)(36)+(9)(16)+(12)(6)}{4+9+12} = 14.4$$

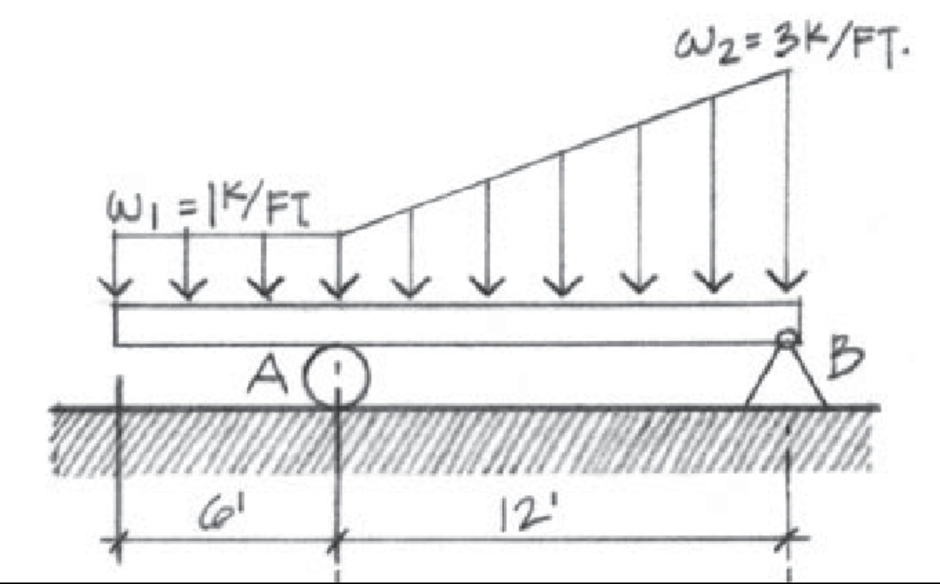
(D)

$$\frac{(4^2)(36)+(9^2)(16)+(12^2)(6)}{36+16+6} = 47.2$$

There's one problem similar to this (but using metric units) on HW(?): Determine the support reactions at A and B.



This one is harder than your homework, because the distributed load is non-uniform: Determine the support reactions at A and B.

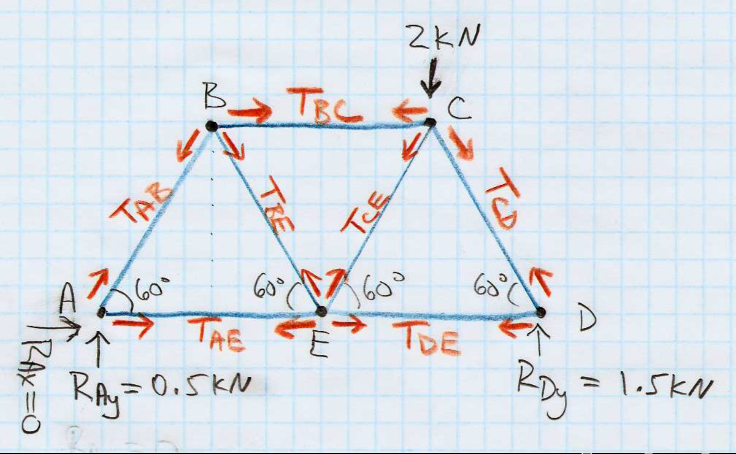


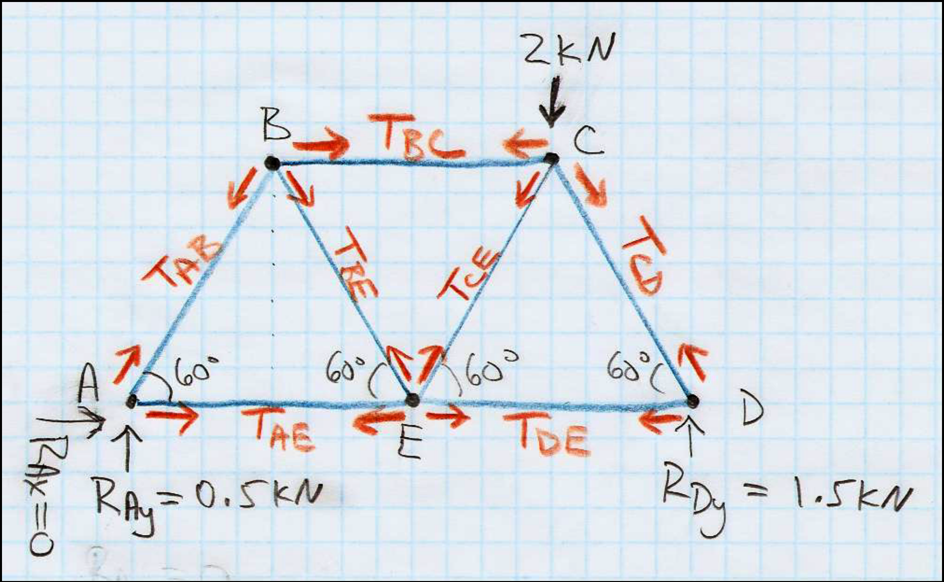
Physics 8 — Monday, November 11, 2019

- Last week, you skimmed Ch4 (load tracing) and read Ch5 (strength of materials) of Onouye/Kane. This week, you're reading Ch6 (cross-sectional properties) and Ch7 (simple beams).

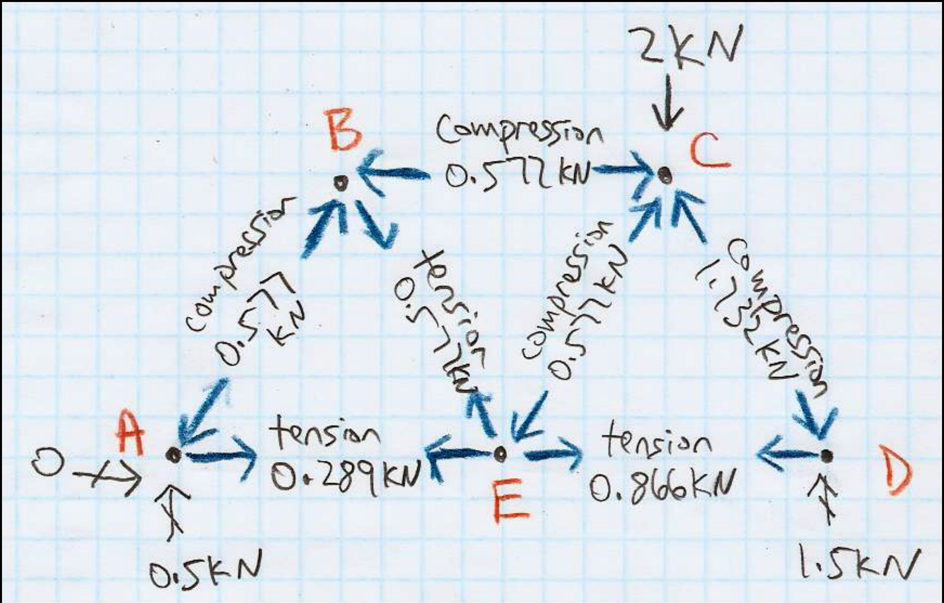
Physics 8 — Wednesday, November 13, 2019

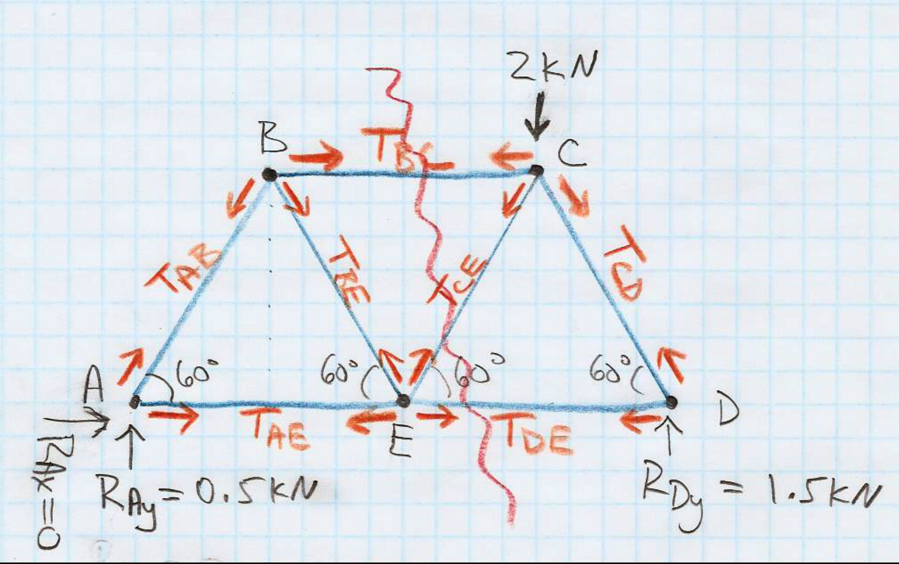
- Last week, you skimmed Ch4 (load tracing) and read Ch5 (strength of materials) of Onouye/Kane. This week, you read Ch6 (cross-sectional properties) and Ch7 (simple beams).
- HW10 due Friday. HW help: (Bill) Wed 4-6:30pm DRL 3C4, (Grace/Brooke) Thu 6-8pm DRL 2C4.



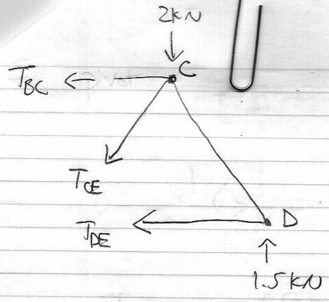


$T_{AB} = -0.577$ kN, $T_{AE} = +0.289$ kN, $T_{BE} = +0.577$ kN, $T_{BC} = -0.577$ kN, $T_{CE} = -0.577$ kN, $T_{CD} = -1.732$ kN, $T_{DE} = +0.866$ kN. My notation: tension > 0, compression < 0.


2KN
B
C
Compression
0.577kN
compression
0.577 kN
tension
0.577kN
compression
0.577kN
compression
1.732kN
A
E
D
tension
0.289kN
tension
0.866kN
0
0.5KN
1.5KN



Let's try drawing an EFBD for the **right** side of the cut ("section").



$$\sum F_x = 0 \Rightarrow -T_{BC} - T_{CE}\cos 60^\circ - T_{DE} = 0$$

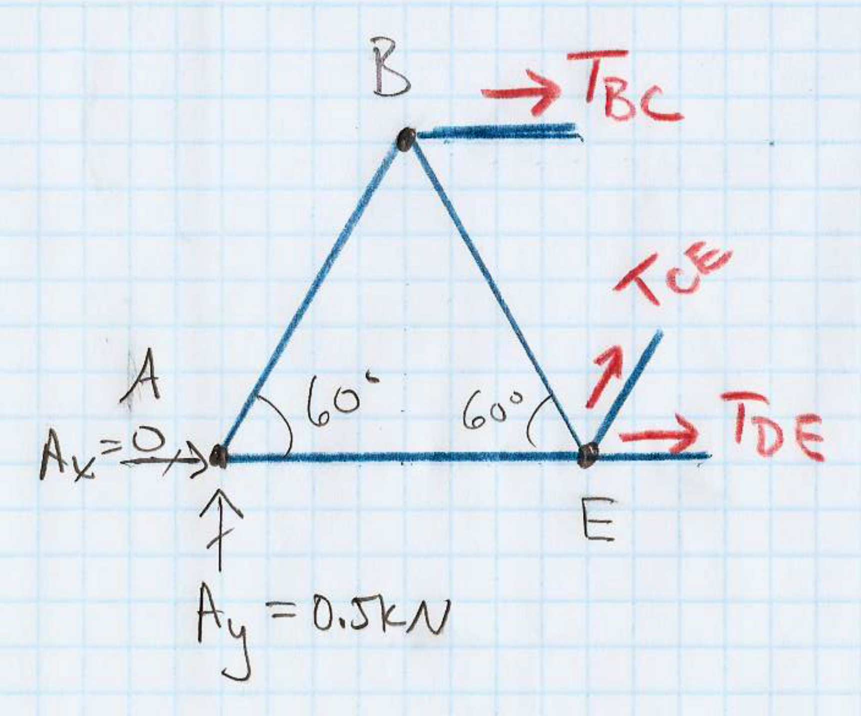
$$\sum F_y = 0 \Rightarrow -2kN + 1.5kN - T_{CE}\sin 60^\circ = 0$$

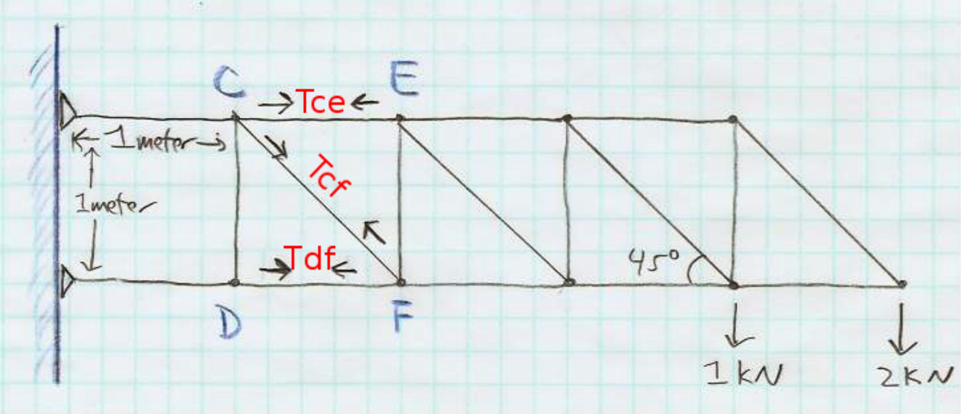
$$\sum M_C = 0 \Rightarrow +(1.5kN)\left(\frac{L}{2}\right) - T_{DE}\, L \sin 60^\circ = 0$$

$$T_{DE} = \frac{1.5kN}{2\sin 60^\circ} = +0.866kN$$

$$T_{CE} = \frac{-0.5kN}{\sin 60^\circ} = -0.577kN$$

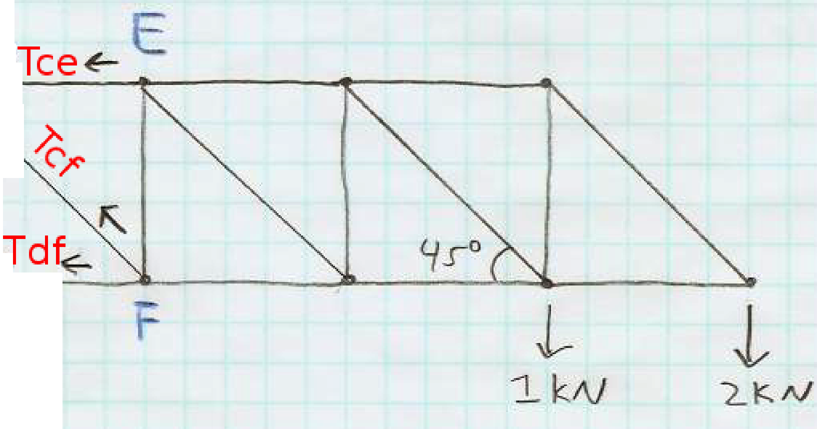
$$T_{BC} = -(T_{CE}\cos 60^\circ + T_{DE}) = -0.577$$


B
T_{BC}
T_{CE}
T_{DE}
A
60°
60°
$A_x = 0$
E
$A_y = 0.5kN$



Here's another truss problem that you can solve using the Method of Sections. Find forces in members **CE**, **CF**, and **DF**, with assumed force directions as shown.

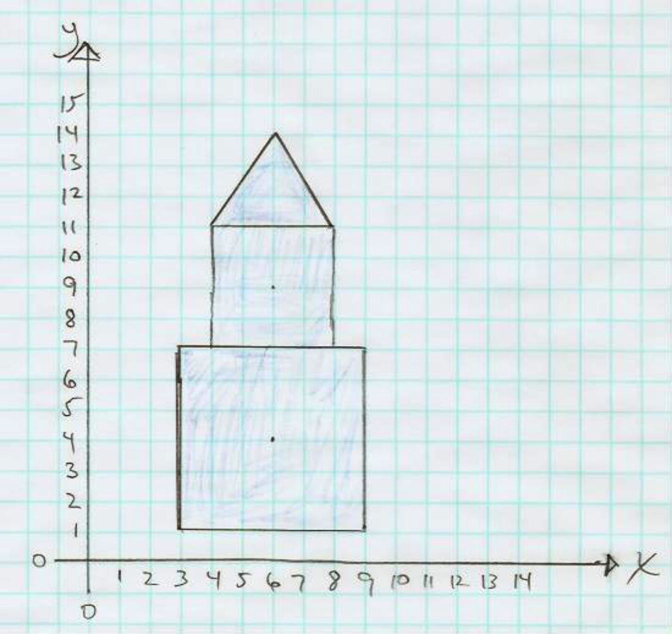
- What happens if an assumed force direction is backwards?
- Where should we "section" the truss?
- Then what do we do next?



If all goes well, we should get
$T_{CF} = +3\sqrt{2}$ kN, $T_{CE} = +8$ kN, $T_{DF} = -11$ kN.

- The idea of computing centroids of simple and composite shapes is very, very briefly introduced in O/K ch3 (in the context of "distributed loads"), and is discussed in much more detail in O/K ch6 (you read this week, for Monday).
- Let's go through one example using rectangles and triangles. It will help you in cases when you need to solve for the "reaction forces" on a beam that carries distributed loads. (Example coming up next.)

What is X_{centroid} for the shaded area?



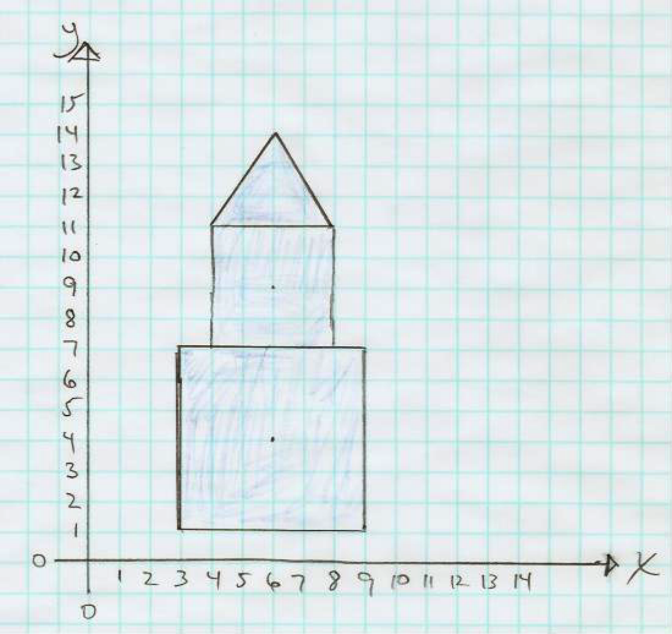
(A) 0

(B) 3

(C) 6

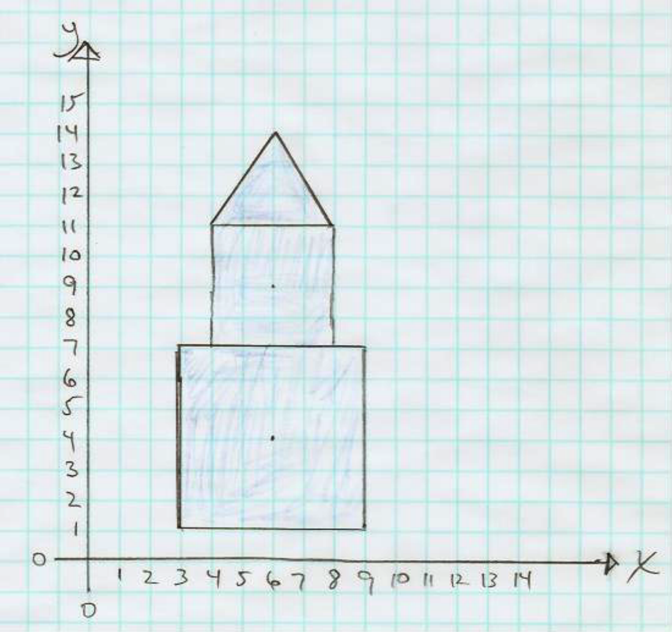
(D) 9

What are the areas of the three individual polygons?



(A) 36, 16, 16
(B) 36, 16, 12
(C) 36, 16, 8
(D) 36, 16, 6

What are the Y_{centroid} values of the three individual polygons?



(A) 4, 9, 11

(B) 4, 9, 11.667

(C) 4, 9, 12

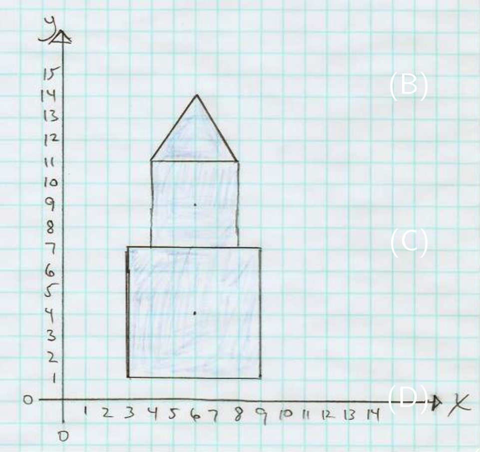
(D) 4, 9, 12.333

(E) 4, 9, 12.5

(F) 4, 9, 13

(G) 4, 9, 14

What is Y_{centroid} for the whole shaded area?



(A)

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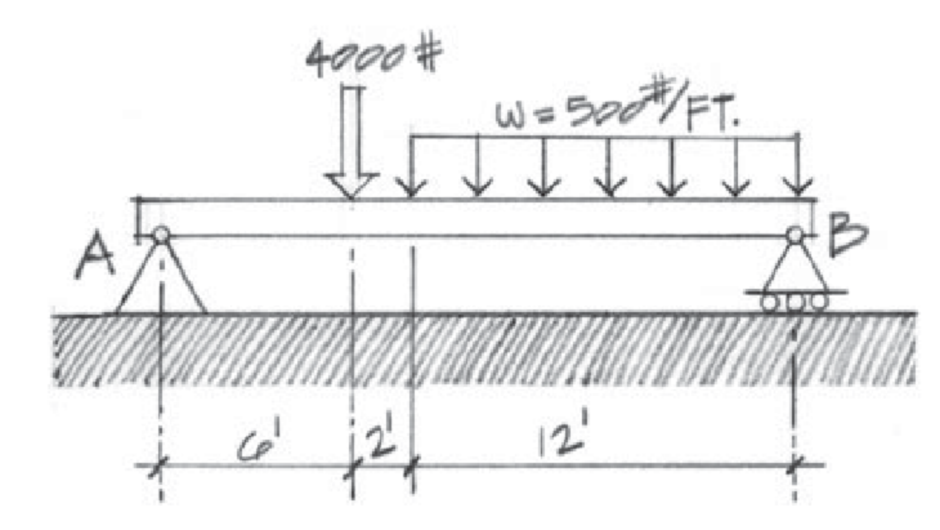
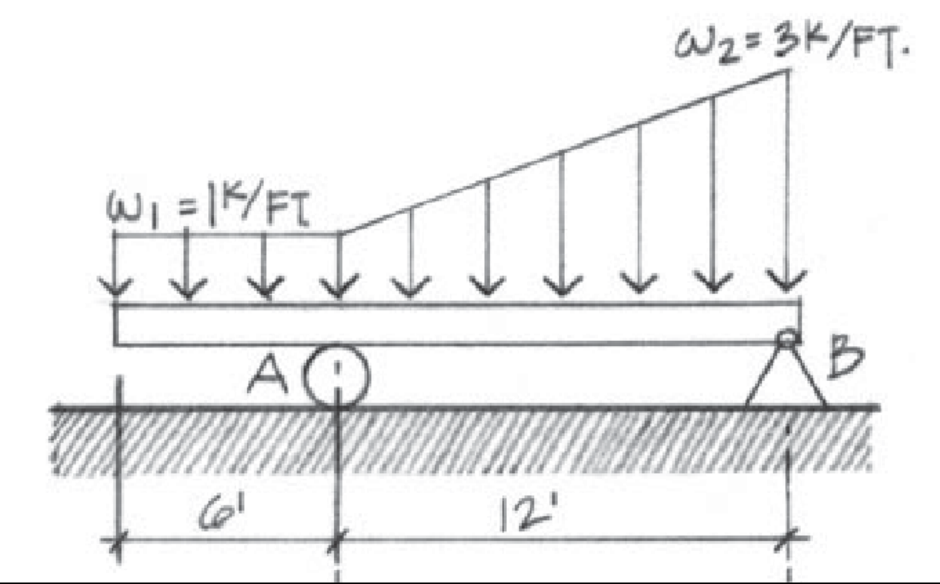
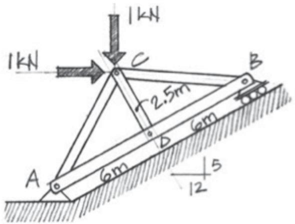
There's one problem similar to this (but using metric units) on HW(?): Determine the support reactions at A and B.



This one is harder than your homework, because the distributed load is non-uniform: Determine the support reactions at A and B.



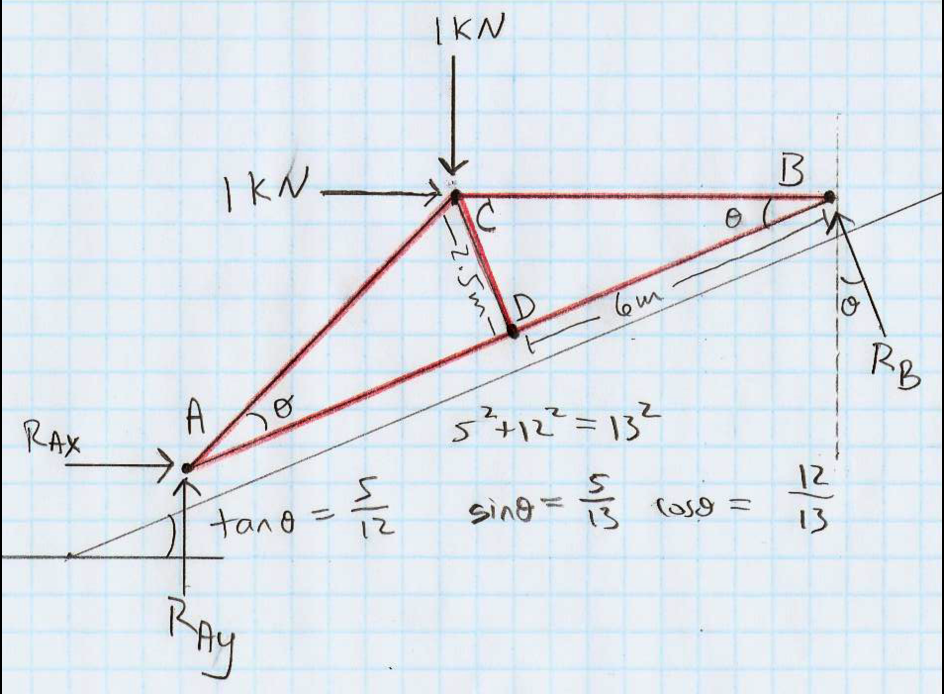
2.38 An inclined king-post truss supports a vertical and horizontal force at C. Determine the support reactions developed at *A* and *B*.



This is not really a “truss problem,” since we’re not asked to solve for the internal forces in the truss, but it is an example of a pretty tricky equilibrium problem.

Let’s try working through this together in class. (I think it’s deviously tricky!)

Notice, from the given dimensions: the angle of the incline is the same as the interior angle at joints A and B of the truss. Also notice pin/hinge support at A and roller support at B.


1 KN
1 KN
A
B
C
D
θ
θ
θ
2.5m
6m
R_{AX}
R_{Ay}
R_B
$5^2+12^2=13^2$
$\tan\theta = \frac{5}{12}$
$\sin\theta = \frac{5}{13}$
$\cos\theta = \frac{12}{13}$

① $0 = \sum_{\text{on truss}} F_x = R_{Ax} + 1\,kN - R_B \sin\theta$

$$\Rightarrow R_{Ax} = R_B \sin\theta - 1\,kN$$

② $0 = \sum_{\text{on truss}} F_y = R_{Ay} - 1\,kN + R_B \cos\theta$

$$\Rightarrow R_{Ay} = 1\,kN - R_B \cos\theta$$

③ $0 = \sum_{\circlearrowleft} M_A = R_B(12m) - 1\,kN(12m \sin\theta) - 1\,kN(6.5m \cos 2\theta)$

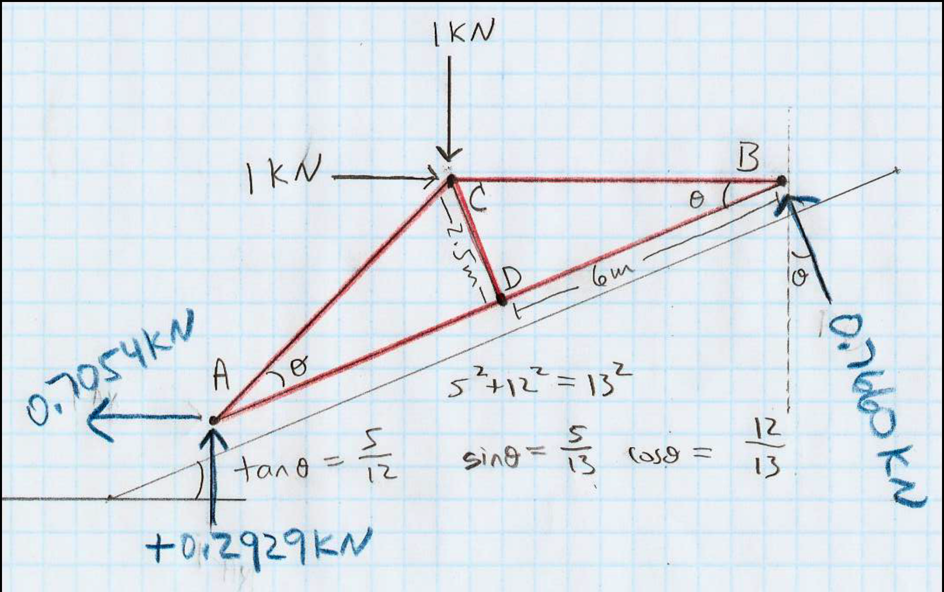
$$\Rightarrow R_B = 1\,kN(\sin\theta) + \frac{6.5}{12}\,kN(\cos 2\theta) = 0.7660\,kN$$

$$(R_B)_x = -R_B \sin\theta = -0.2946\,kN$$

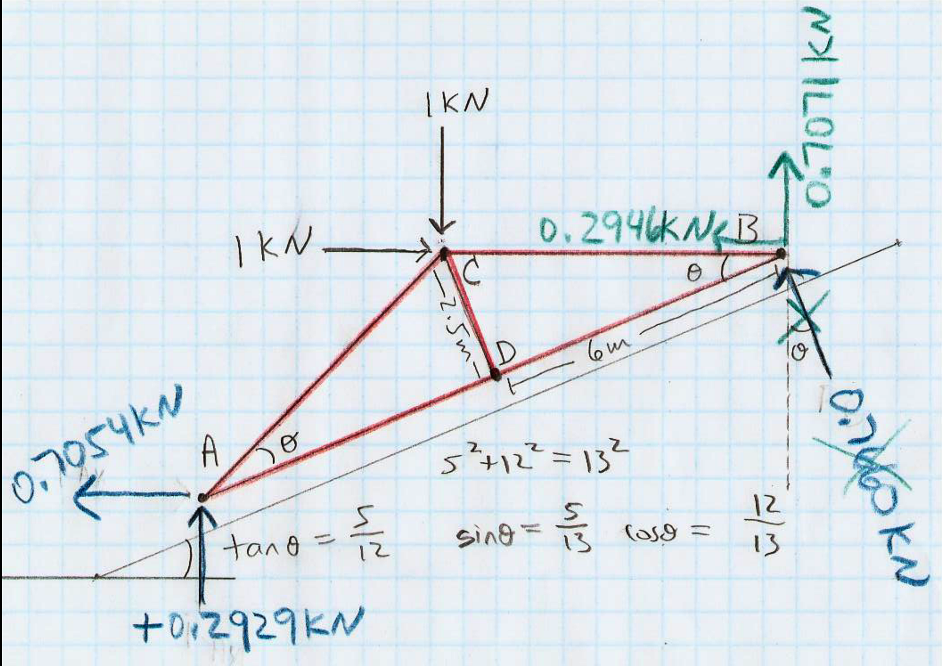
$$(R_B)_y = R_B \cos\theta = 0.7071\,kN$$

$$R_{Ax} = (0.2946 - 1)\,kN = -0.7054\,kN$$

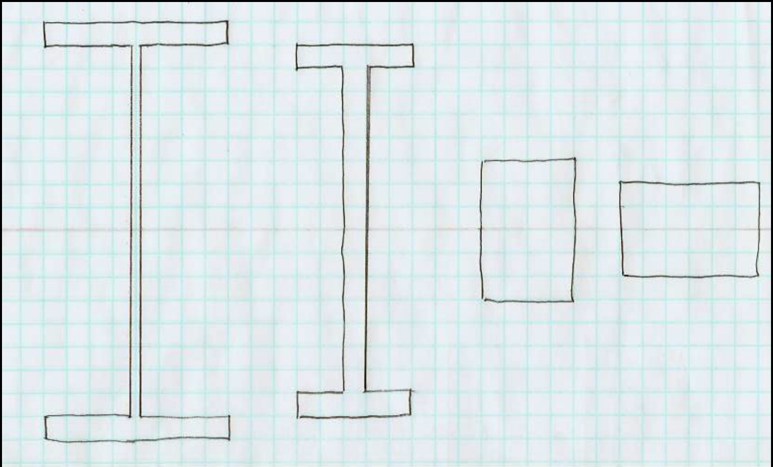
$$R_{Ay} = (1 - 0.7071)\,kN = 0.2929\,kN$$



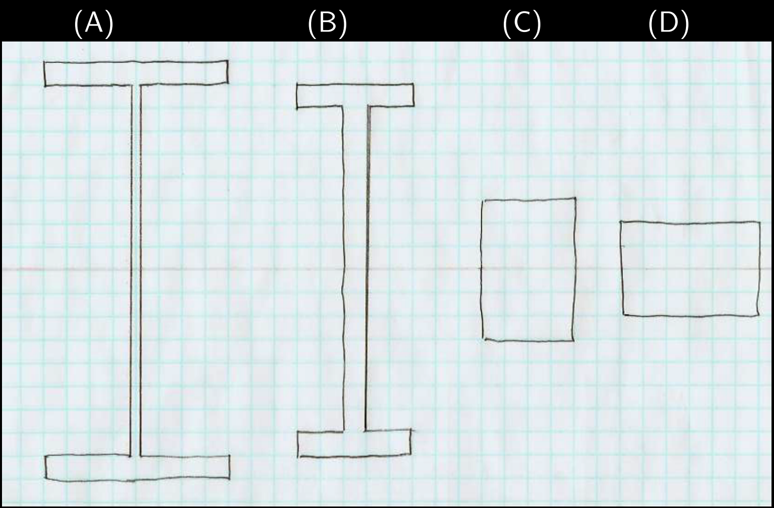
1KN
1KN
B
C
θ
2.5m
D
6m
θ
0.7054KN
A
θ
$5^2+12^2=13^2$
0.7660KN
$\tan\theta = \frac{5}{12}$
$\sin\theta = \frac{5}{13}$
$\cos\theta = \frac{12}{13}$
+0.2929KN


1KN
1KN
0.2946KN
B
0.7071KN
C
θ
2.5m
D
6m
θ
0.7071KN
0.7054KN
A
θ
5²+12²=13²
tan θ = 5/12
sin θ = 5/13
cos θ = 12/13
+0.2929KN

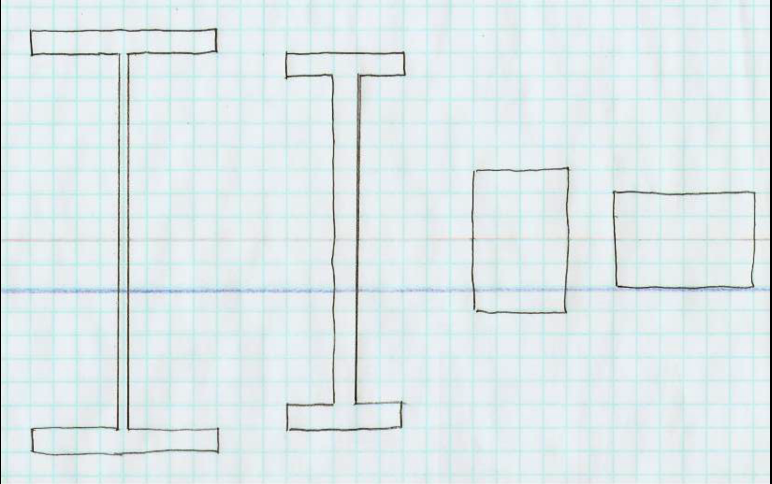
(A) (B) (C) (D)



Each shape has the same area: 24 squares. Which shape has the largest $I_x = \int y^2 \mathrm{d}A$ ("second moment of area about the x-axis"), with $y = 0$ given by the faint horizontal red line at the center?

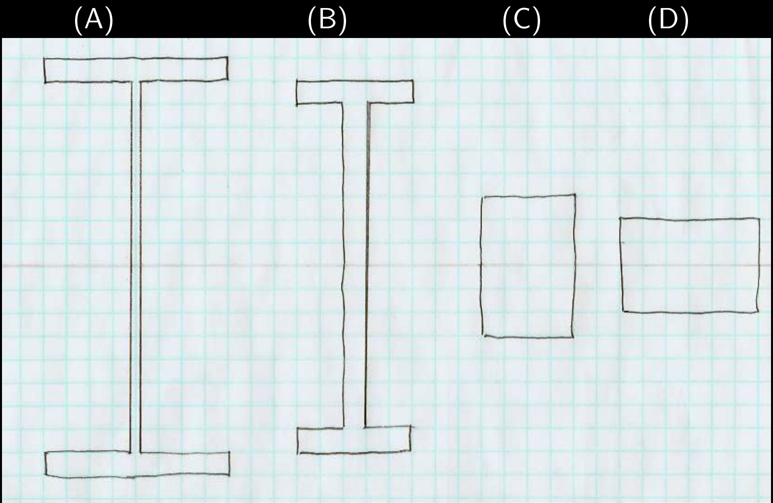


Each shape has the same area: 24 squares. Which shape has the **smallest** $I_x = \int y^2 \mathrm{d}A$ ("second moment of area about the x-axis"), with $y = 0$ given by the faint horizontal red line at the center?



If you moved the x-axis down by a couple of grid units, what would happen to $I_x = \int y^2 \mathrm{d}A$ for each shape? Would I_x change? Would I_x change by the same amount for each shape?

(Think: "parallel-axis theorem.")



Given that $I_x = \int y^2 \mathrm{d}A = \frac{1}{12} bh^3$ for a rectangle centered at $y = 0$, let's use the parallel-axis theorem to calculate I_x for shapes A, B, C, and D. For definiteness, let each graph-paper box be $1\ \mathrm{cm} \times 1\ \mathrm{cm}$. So the units will be cm^4.

Let's do the two rectangular shapes first, since they're quick.

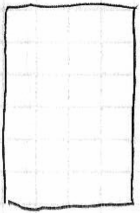
Then, the trick for the non-rectangular shapes is to use (from O/K §6.3) the "parallel-axis theorem:"

$$I_x = \sum I_{xc} + \sum A\, d_y^2$$

where each sum is over the simple shapes that compose the big shape.

- I_{xc} is the simple shape's own I_x value about its own centroid (which is $bh^3/12$ for a rectangle),
- A is the simple shape's area, and
- d_y is the vertical displacement of the simple shape's centroid from $y = 0$ (which should be the centroid of the big shape).

(C)



$b = 4\text{ cm}$

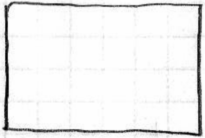
$h = 6\text{ cm}$

$A = 24\text{ cm}^2$

$y_c = 0$

$\frac{1}{12} bh^3 = \boxed{72\text{ cm}^4}$

(D)



$b = 6\text{ cm}$

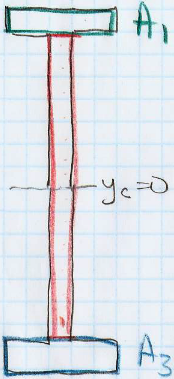
$h = 4\text{ cm}$

$A = 24\text{ cm}^2$

$y_c = 0$

$\frac{1}{12} bh^3 = \boxed{32\text{ cm}^4}$

(B)



$A_1 = 5\,cm^2$ $b_1 = 5\,cm$ $h_1 = 1\,cm$

$y_{c_1} = +7.5\,cm$

$A_2 = 14\,cm^2$ $b_2 = 1\,cm$

$h_2 = 14\,cm$

$y_{c_2} = 0$

$A_3 = 5\,cm^2$ $b_3 = 5\,cm$ $h_3 = 1\,cm$

$y_{c3} = -7.5\,cm$

$\frac{1}{12} b_1 h_1^3 = 0.417\,cm^4$

$A_1 y_{c_1}^2 = 281.25\,cm^4$

$\frac{1}{12} b_2 h_2^3 = 228.67\,cm^4$

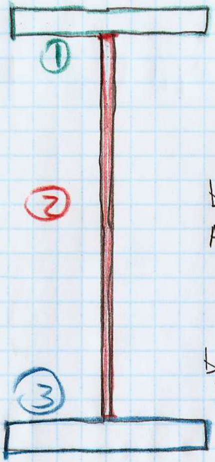
$A_2 y_{c_2}^2 = 0$

$\frac{1}{12} b_3 h_3^3 = 0.417\,cm^4$

$A_3 y_{c_3}^2 = 281.25\,cm^4$

$$I_B = \frac{1}{12} b_1 h_1^3 + \frac{1}{12} b_2 h_2^3 + \frac{1}{12} b_3 h_3^3 + A_1 y_{c_1}^2 + A_2 y_{c_2}^2 + A_3 y_{c_3}^2 = \boxed{792\,cm^4}$$

(A)



$b_1 = 8\,cm \quad h_1 = 1\,cm$

$A_1 = 8\,cm^2 \quad y_{c_1} = +8.5\,cm$

$\frac{1}{12} b_1 h_1^3 = 0.67\,cm^4$

$A_1 y_{c_1}^2 = 578\,cm^4$

$b_2 = 0.5\,cm \quad h_2 = 16\,cm$

$A_2 = 8\,cm^2 \quad y_{c_2} = 0$

$\frac{1}{12} b_2 h_2^3 = 170.67\,cm^4$

$A_2 y_{c_2}^2 = 0$

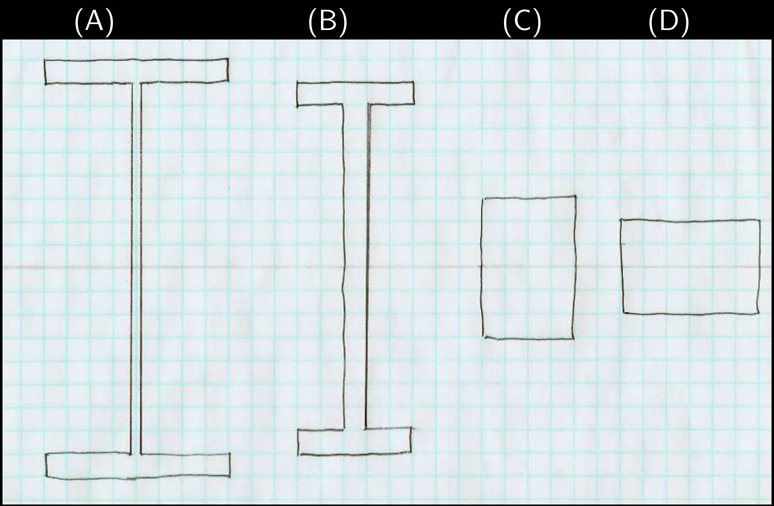
$b_3 = 8\,cm \quad h_3 = 1\,cm$

$A_3 = 8\,cm^2 \quad y_{c_3} = -8.5\,cm$

$\frac{1}{12} b_3 h_3^3 = 0.67\,cm^4$

$A_3 y_{c_3}^2 = 578\,cm^4$

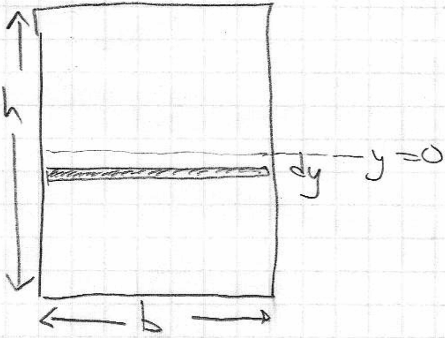
$$I_A = \frac{1}{12} b_1 h_1^3 + \frac{1}{12} b_2 h_2^3 + \frac{1}{12} b_3 h_3^3 + A_1 y_{c_1}^2 + A_2 y_{c_2}^2 + A_3 y_{c_3}^2 = \boxed{1328\,cm^4}$$



Each shape has same area $A = 24\ \mathrm{cm}^2$, but "second moment of area" is $I_A = 1328\ \mathrm{cm}^4$, $I_B = 792\ \mathrm{cm}^4$, $I_C = 72\ \mathrm{cm}^4$, $I_D = 32\ \mathrm{cm}^4$.

That's the motivation for the "I" shape of an I-beam: to get a large "second moment of area," $I = \int y^2\,\mathrm{d}A$. The deflection of a beam under load is inversely proportional to I.

rectangle

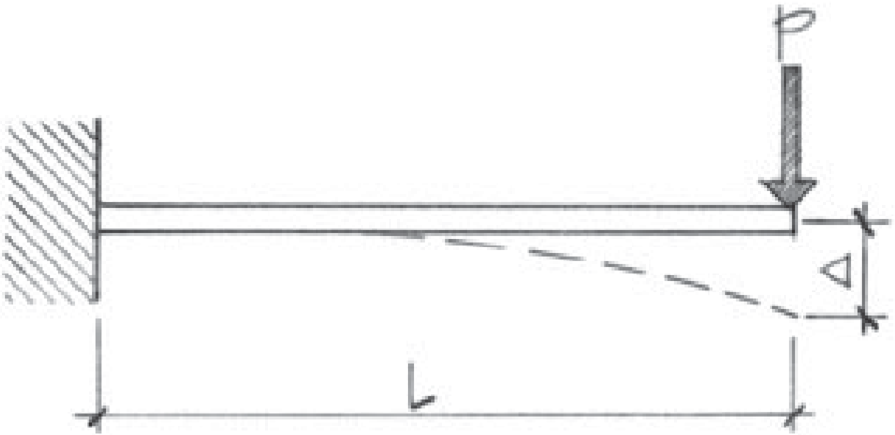


$$dA = b\,dy$$

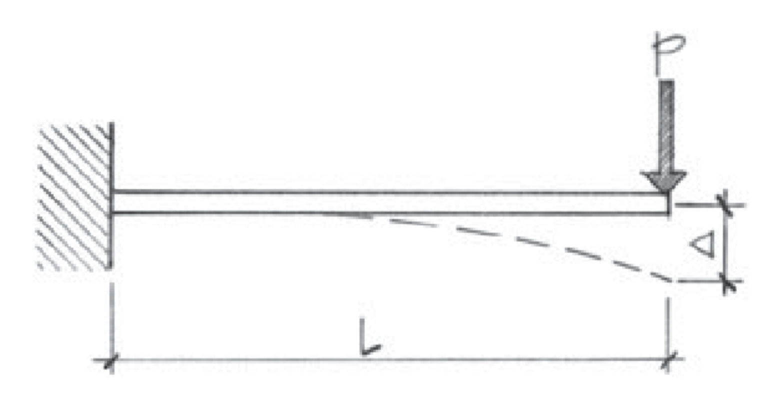
$$I = \int y^2\,dA = \int_{y=-\frac{h}{2}}^{y=h/2} y^2\, b\,dy = \left[\frac{by^3}{3}\right]_{y=-\frac{h}{2}}^{h/2}$$

$$= \frac{b(h/2)^3}{3} - \frac{b(-h/2)^3}{3} = \frac{bh^3}{12}$$

We can use the Method of Sections to study the internal forces and torques ("moments") within a beam. Consider this cantilever beam (whose own weight we neglect here) supporting a concentrated "load" force P at the far end. The left half is what holds up the right half. What force and torque ("moment") does the left half exert on the right half? Does the answer depend on where we "section" the beam?



We draw “load diagram” (basically a FBD for the beam), then the “shear (V) diagram” below that, then the “moment (M) diagram” below that. Sign conventions: $V > 0$ when beam LHS section is pulling up on beam RHS; $M > 0$ when beam is smiling.



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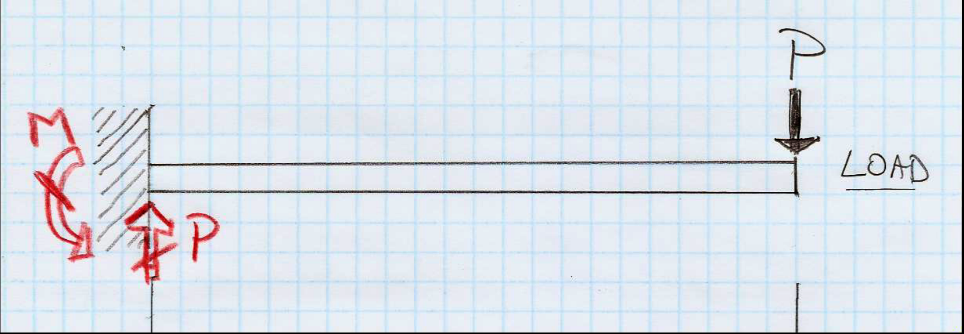

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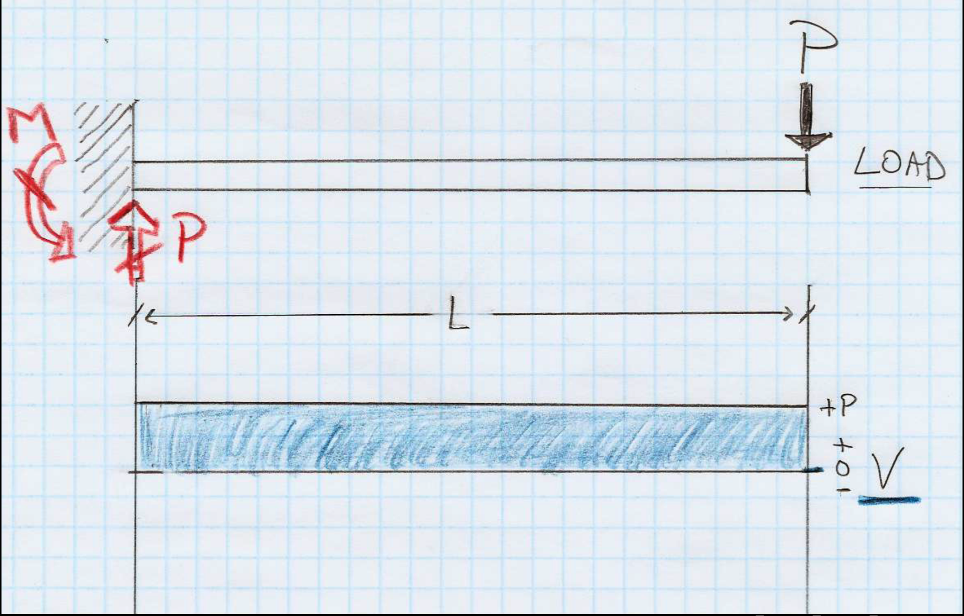

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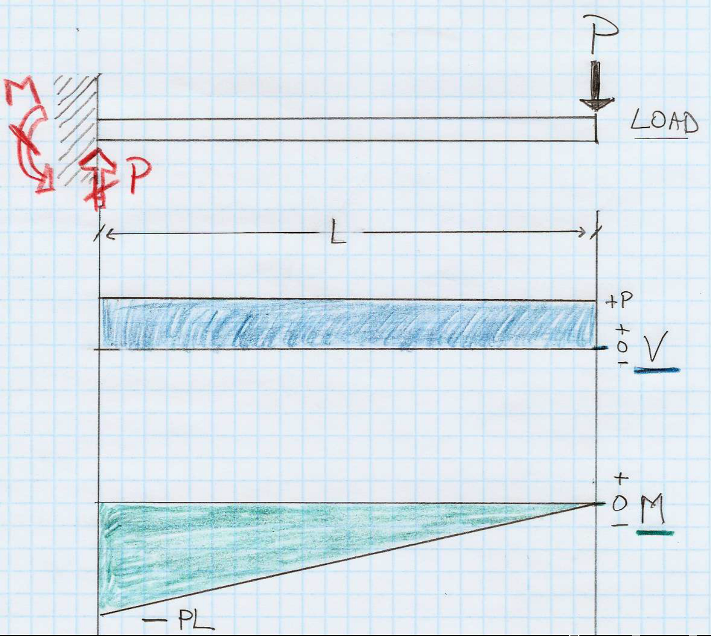
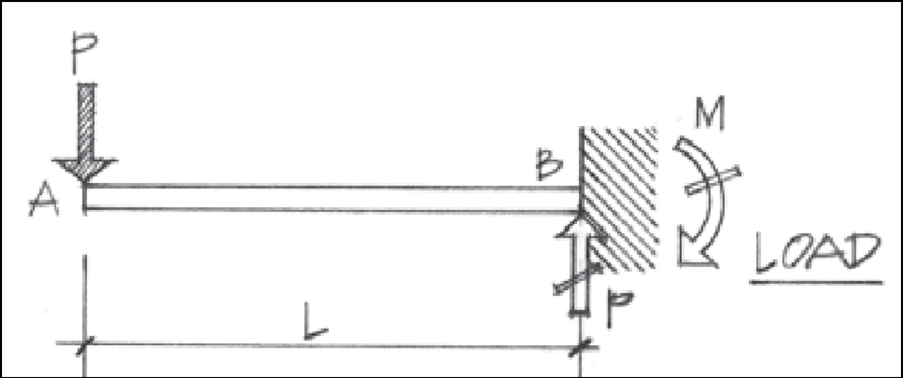
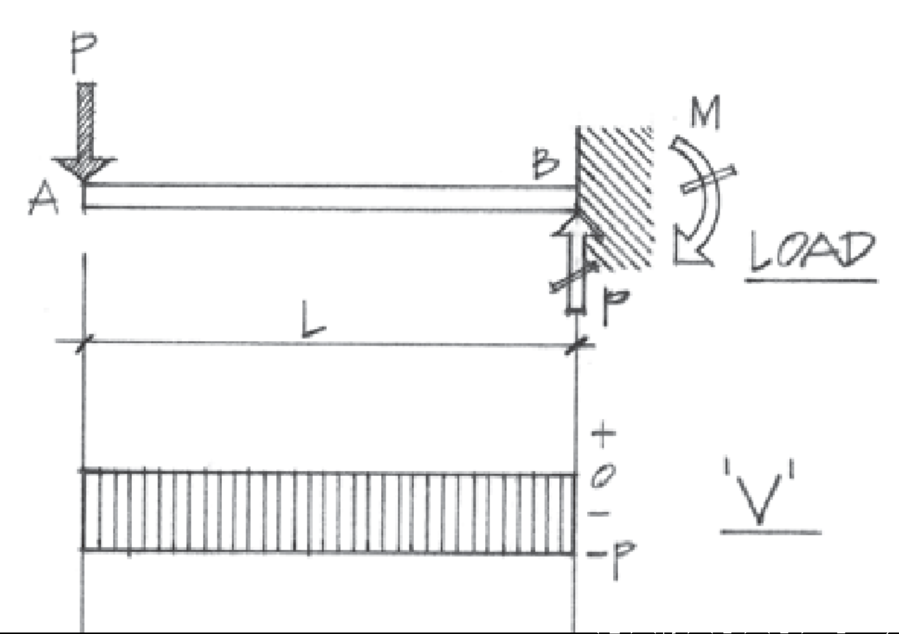


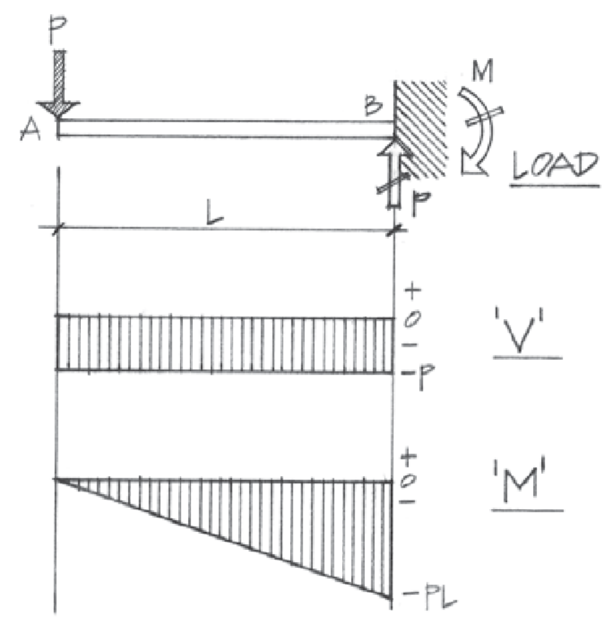
Let's try a mirror image of the same cantilever beam. Sign conventions: $V > 0$ when beam LHS section is pulling up on beam RHS; $M > 0$ when beam is smiling.



Sign conventions: $V > 0$ when beam LHS section is pulling up on beam RHS; $M > 0$ when beam is smiling.



Sign conventions: $V > 0$ when beam LHS section is pulling up on beam RHS; $M > 0$ when beam is smiling.



Physics 8 — Wednesday, November 13, 2019

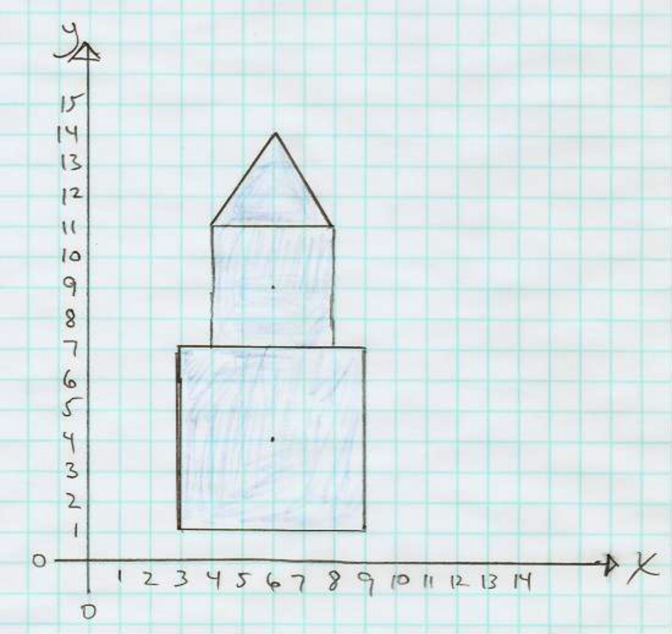
- Last week, you skimmed Ch4 (load tracing) and read Ch5 (strength of materials) of Onouye/Kane. This week, you read Ch6 (cross-sectional properties) and Ch7 (simple beams).
- HW10 due Friday. HW help: (Bill) Wed 4-6:30pm DRL 3C4, (Grace/Brooke) Thu 6-8pm DRL 2C4.

Physics 8 — Friday, November 15, 2019

- Turn in HW10. Pick up HW11 handout. HW11 is "due" next Friday, but you can turn it in on Monday, Nov 25, just in case it takes us an extra day to get through the material on beams.
- This week, you read Ch6 (cross-sectional properties) and Ch7 (simple beams). Next week, you'll read Ch8 (more details on beams).

- The idea of computing centroids of simple and composite shapes is very, very briefly introduced in O/K ch3 (in the context of "distributed loads"), and is discussed in much more detail in O/K ch6 (you read this week, for Monday).
- Let's go through one example using rectangles and triangles. It will help you in cases when you need to solve for the "reaction forces" on a beam that carries distributed loads. (Example coming up next.)

What is X_{centroid} for the shaded area?



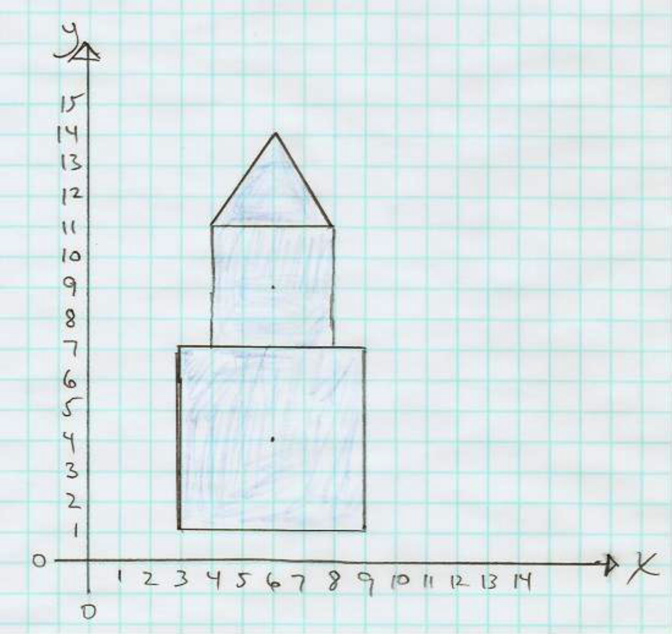
(A) 0

(B) 3

(C) 6

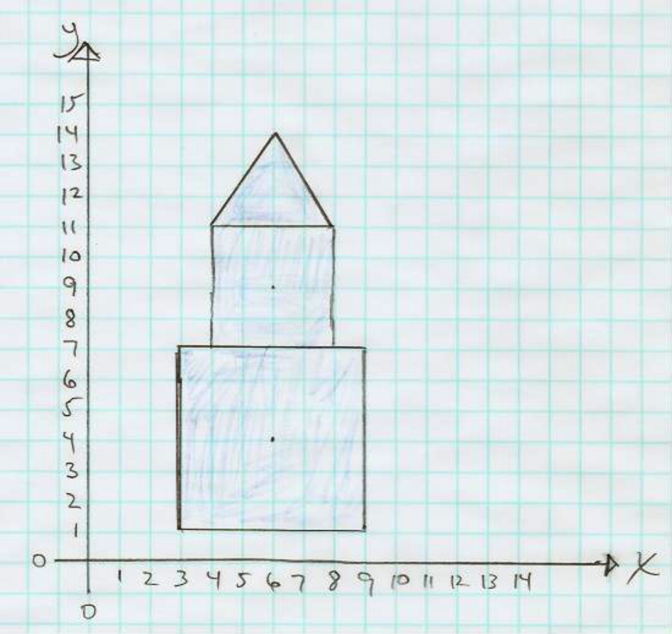
(D) 9

What are the areas of the three individual polygons?



(A) 36, 16, 16
(B) 36, 16, 12
(C) 36, 16, 8
(D) 36, 16, 6

What are the Y_{centroid} values of the three individual polygons?



(A) 4, 9, 11

(B) 4, 9, 11.667

(C) 4, 9, 12

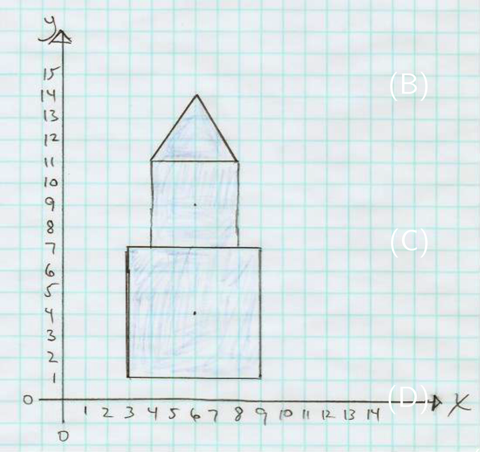
(D) 4, 9, 12.333

(E) 4, 9, 12.5

(F) 4, 9, 13

(G) 4, 9, 14

What is Y_{centroid} for the whole shaded area?



(A)

$$\frac{4+9+12}{3} = 8.33$$

(B)

$$\frac{(4)(36)+(9)(16)+(12)(6)}{36+16+6} = 6.21$$

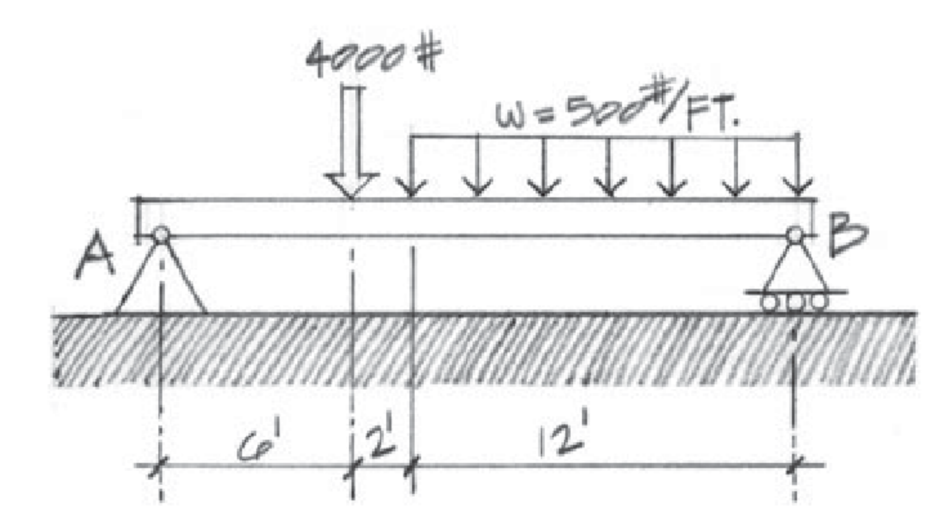
(C)

$$\frac{(4)(36)+(9)(16)+(12)(6)}{4+9+12} = 14.4$$

(D)

$$\frac{(4^2)(36)+(9^2)(16)+(12^2)(6)}{36+16+6} = 47.2$$

There was one problem similar to this (but using metric units) on HW10: Determine the support reactions at A and B.



```
ClearAll["Global`*"];
load1Force = 4000.0 pound;
load1X = 6.0 foot; Measure positions w.r.t. support A;
load2Force = (500.0 pound / foot) * (12 foot)
```

```
6000. pound
```

```
Find centroid of distributed load, for equivalent concentrated load;
load2X = (6.0 + 2.0 + 12.0 / 2) foot
```

```
14. foot
```

```
Evaluate moments about pivot A;
Solve[0 == By (6.0 + 2.0 + 12.0) foot - load1Force * load1X -
   load2Force * load2X, By]
```

```
{{By → 5400. pound}}
```

```
By = By /. First[%]
```

```
5400. pound
```

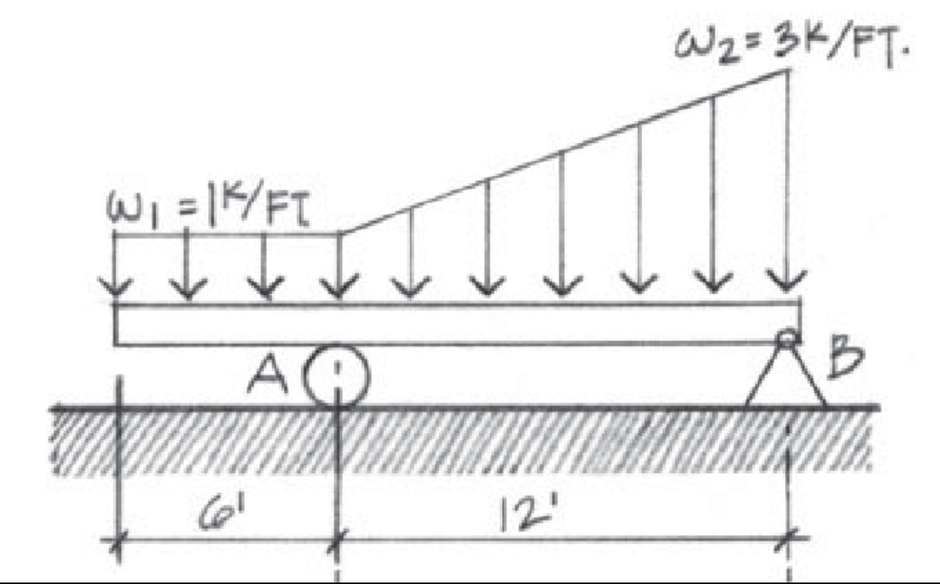
```
Solve[0 == Ay + By - load1Force - load2Force, Ay]
```

```
{{Ay → 4600. pound}}
```

```
Solve[0 == Ax]
```

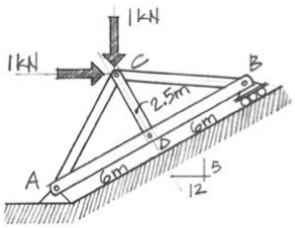
```
{{Ax → 0}}
```

This one is harder than your homework, because the distributed load is non-uniform: Determine the support reactions at A and B.



```
ClearAll["Global`*"];
foot = Quantity[1.0, "foot"];
pound = Quantity[1.0, "pound"];
Rectangle for uniform 1k/foot load spanning entire beam.;
load1X = 0.5 (6 foot + 12 foot)

9. ft

load1Force = (1000 pound / foot) * (18.0 foot)

18000. lb

Trianglular load that sits above uniform load.;
load2X = 18.0 foot - (12.0 foot) / 3

14. ft

load2Force = 0.5 * (12.0 foot) * (2000 pound / foot)

12000. lb

Moments about B;
L = 18.0 foot;
Solve[
 0 == load1Force * (L - load1X) + load2Force * (L - load2X) -
   Ay * (12 foot), Ay]

{{Ay → 17500. lb}}

Ay = Ay /. First[%]

17500. lb

Solve[0 == Ay + By - load1Force - load2Force, By]

{{By → 12500. lb}}
```

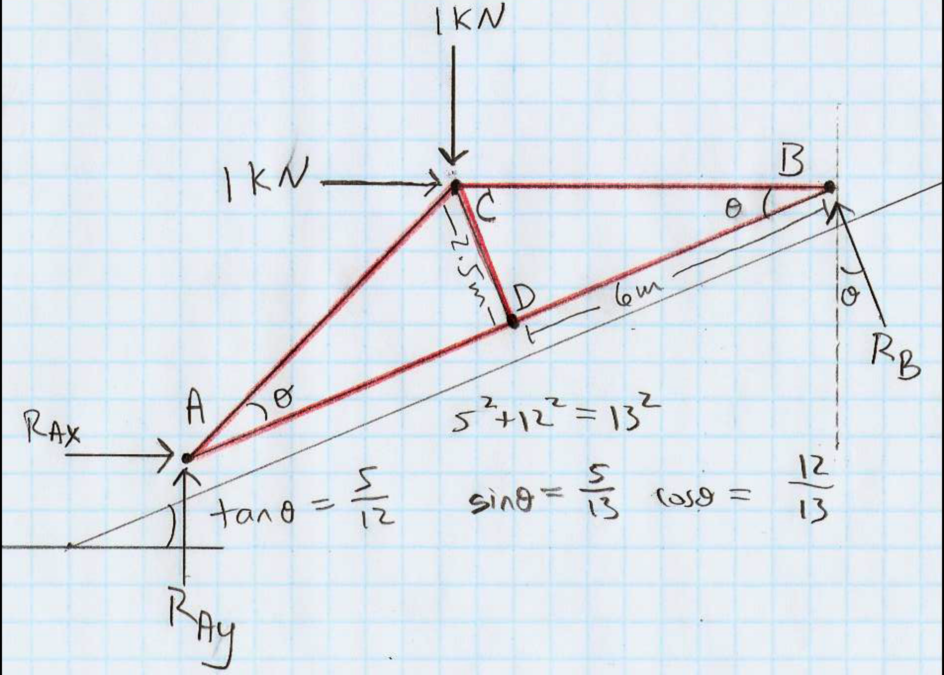
2.38 An inclined king-post truss supports a vertical and horizontal force at C. Determine the support reactions developed at A and B.



This is not really a "truss problem," since we're not asked to solve for the internal forces in the truss, but it is an example of a pretty tricky equilibrium problem.

Let's try working through this together in class. (I think it's deviously tricky!)

Notice, from the given dimensions: the angle of the incline is the same as the interior angle at joints A and B of the truss. Also notice pin/hinge support at A and roller support at B.



$$5^2 + 12^2 = 13^2$$

$$\tan\theta = \frac{5}{12} \qquad \sin\theta = \frac{5}{13} \qquad \cos\theta = \frac{12}{13}$$

① $0 = \sum_{\text{on truss}} F_x = R_{Ax} + 1\,kN - R_B \sin\theta$

$$\Rightarrow R_{Ax} = R_B \sin\theta - 1\,kN$$

② $0 = \sum_{\text{on truss}} F_y = R_{Ay} - 1\,kN + R_B \cos\theta$

$$\Rightarrow R_{Ay} = 1\,kN - R_B \cos\theta$$

③ $0 = \sum_{\circlearrowleft} M_A = R_B(12m) - 1\,kN(12m \sin\theta) - 1\,kN(6.5m \cos 2\theta)$

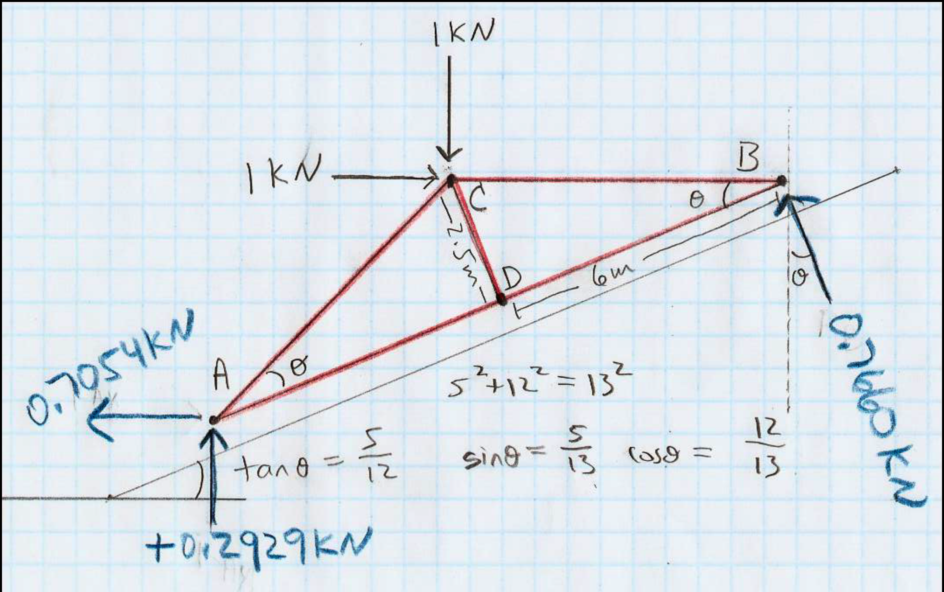
$$\Rightarrow R_B = 1\,kN(\sin\theta) + \frac{6.5}{12}\,kN(\cos 2\theta) = 0.7660\,kN$$

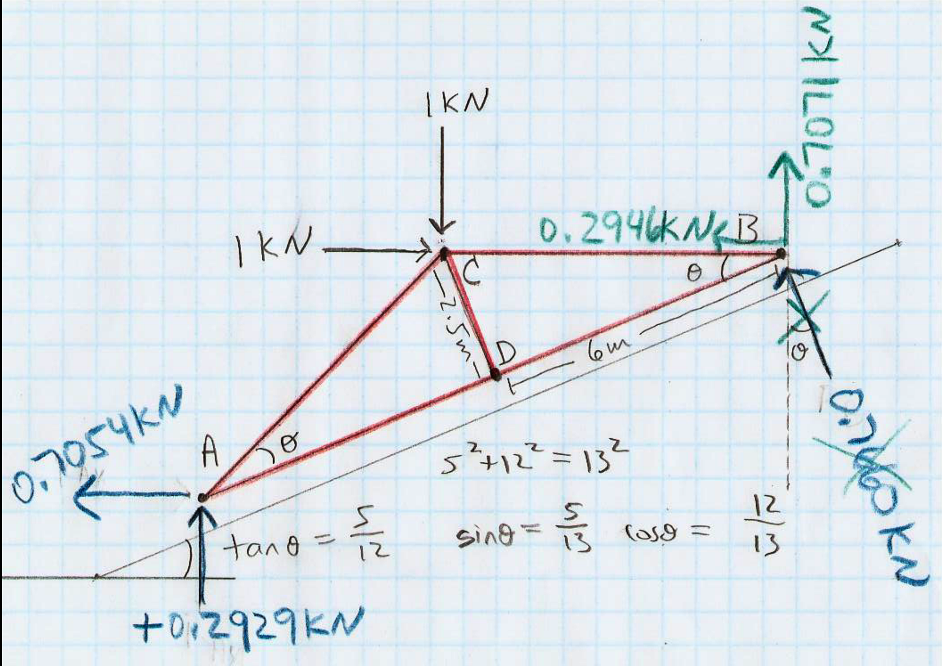
$$(R_B)_x = -R_B \sin\theta = -0.2946\,kN$$

$$(R_B)_y = R_B \cos\theta = 0.7071\,kN$$

$$R_{Ax} = (0.2946 - 1)\,kN = -0.7054\,kN$$

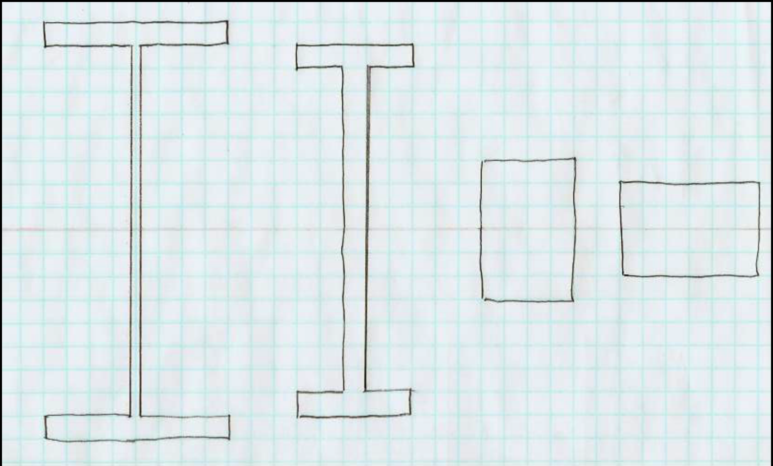
$$R_{Ay} = (1 - 0.7071)\,kN = 0.2929\,kN$$


1KN
1KN
B
C
θ
2.5m
D
6m
θ
A
θ
0.7054KN
0.7660KN
$5^2+12^2=13^2$
$\tan\theta = \frac{5}{12}$
$\sin\theta = \frac{5}{13}$
$\cos\theta = \frac{12}{13}$
+0.2929KN



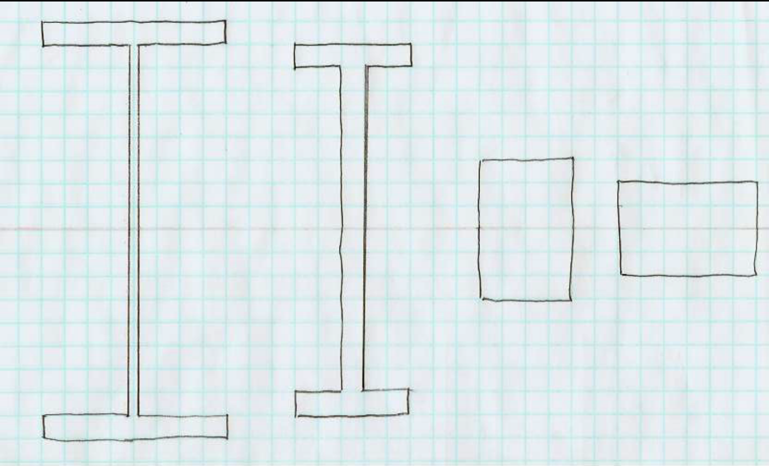
1KN
1KN
0.2946KN
B
0.7071KN
C
θ
2.5m
D
6m
θ
0.7071KN
A
θ
0.7054KN
$5^2+12^2=13^2$
+0.2929KN
$\tan\theta = \frac{5}{12}$
$\sin\theta = \frac{5}{13}$
$\cos\theta = \frac{12}{13}$

(A) (B) (C) (D)

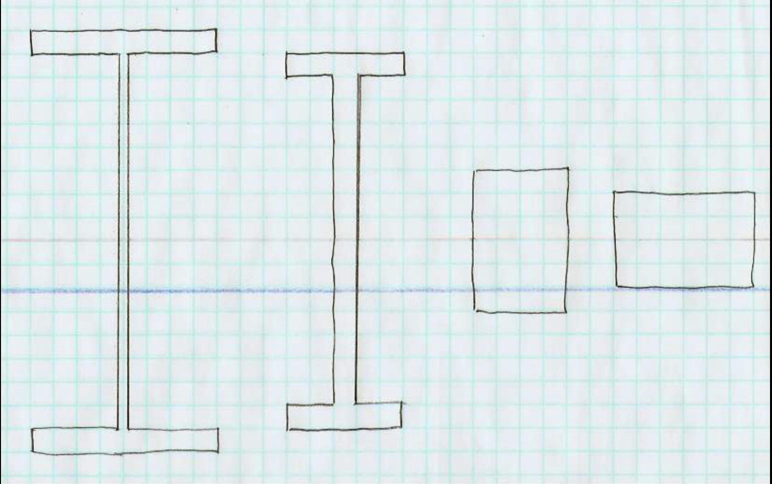


Each shape has the same area: 24 squares. Which shape has the largest $I_x = \int y^2 \mathrm{d}A$ ("second moment of area about the x-axis"), with $y = 0$ given by the faint horizontal red line at the center?

(A) (B) (C) (D)



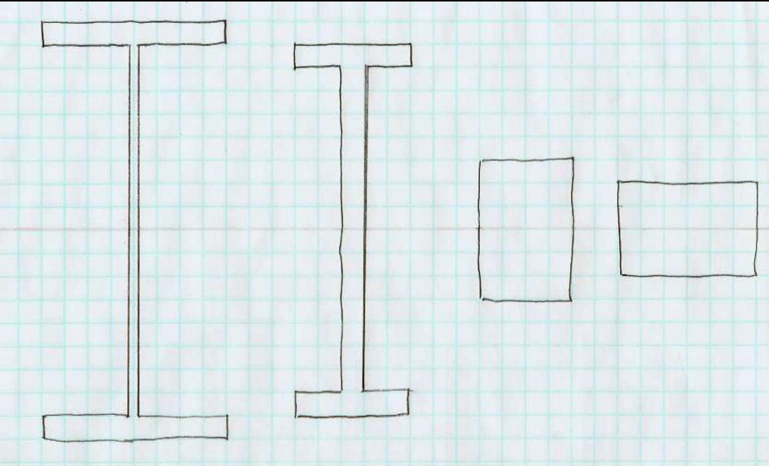
Each shape has the same area: 24 squares. Which shape has the **smallest** $I_x = \int y^2 \mathrm{d}A$ ("second moment of area about the x-axis"), with $y = 0$ given by the faint horizontal red line at the center?



If you moved the x-axis down by a couple of grid units, what would happen to $I_x = \int y^2 \mathrm{d}A$ for each shape? Would I_x change? Would I_x change by the same amount for each shape?

(Think: "parallel-axis theorem.")

(A) (B) (C) (D)



Given that $I_x = \int y^2 \mathrm{d}A = \frac{1}{12} bh^3$ for a rectangle centered at $y = 0$, let's use the parallel-axis theorem to calculate I_x for shapes A, B, C, and D. For definiteness, let each graph-paper box be $1\ \mathrm{cm} \times 1\ \mathrm{cm}$. So the units will be cm^4.

Let's do the two rectangular shapes first, since they're quick.

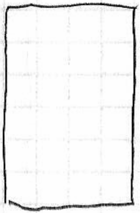
Then, the trick for the non-rectangular shapes is to use (from O/K §6.3) the "parallel-axis theorem:"

$$I_x = \sum I_{xc} + \sum A\, d_y^2$$

where each sum is over the simple shapes that compose the big shape.

- I_{xc} is the simple shape's own I_x value about its own centroid (which is $bh^3/12$ for a rectangle),
- A is the simple shape's area, and
- d_y is the vertical displacement of the simple shape's centroid from $y = 0$ (which should be the centroid of the big shape).

(C)



$b = 4\,cm$

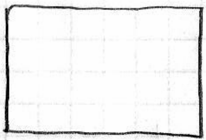
$h = 6\,cm$

$A = 24\,cm^2$

$y_c = 0$

$\frac{1}{12} b h^3 = \boxed{72\,cm^4}$

(D)



$b = 6\,cm$

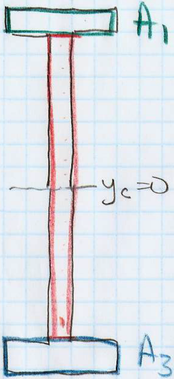
$h = 4\,cm$

$A = 24\,cm^2$

$y_c = 0$

$\frac{1}{12} b h^3 = \boxed{32\,cm^4}$

(B)



$A_1 = 5\,cm^2$ $b_1 = 5\,cm$ $h_1 = 1\,cm$

$y_{c_1} = +7.5\,cm$

$A_2 = 14\,cm^2$ $b_2 = 1\,cm$

$h_2 = 14\,cm$

$y_{c_2} = 0$

$A_3 = 5\,cm^2$ $b_3 = 5\,cm$ $h_3 = 1\,cm$

$y_{c_3} = -7.5\,cm$

$\frac{1}{12} b_1 h_1^3 = 0.417\,cm^4$

$A_1 y_{c_1}^2 = 281.25\,cm^4$

$\frac{1}{12} b_2 h_2^3 = 228.67\,cm^4$

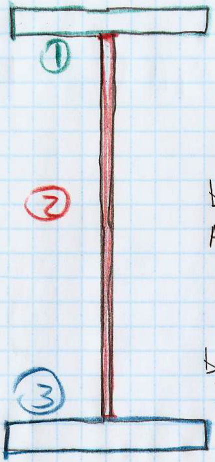
$A_2 y_{c_2}^2 = 0$

$\frac{1}{12} b_3 h_3^3 = 0.417\,cm^4$

$A_3 y_{c_3}^2 = 281.25\,cm^4$

$$I_B = \frac{1}{12} b_1 h_1^3 + \frac{1}{12} b_2 h_2^3 + \frac{1}{12} b_3 h_3^3 + A_1 y_{c_1}^2 + A_2 y_{c_2}^2 + A_3 y_{c_3}^2 = \boxed{792\,cm^4}$$

(A)



$b_1 = 8\,cm \quad h_1 = 1\,cm$

$A_1 = 8\,cm^2 \quad y_{c_1} = +8.5\,cm$

$\frac{1}{12} b_1 h_1^3 = 0.67\,cm^4$

$A_1\, y_{c_1}^2 = 578\,cm^4$

$b_2 = 0.5\,cm \quad h_2 = 16\,cm$

$A_2 = 8\,cm^2 \quad y_{c_2} = 0$

$\frac{1}{12} b_2 h_2^3 = 170.67\,cm^4$

$A_2\, y_{c_2}^2 = 0$

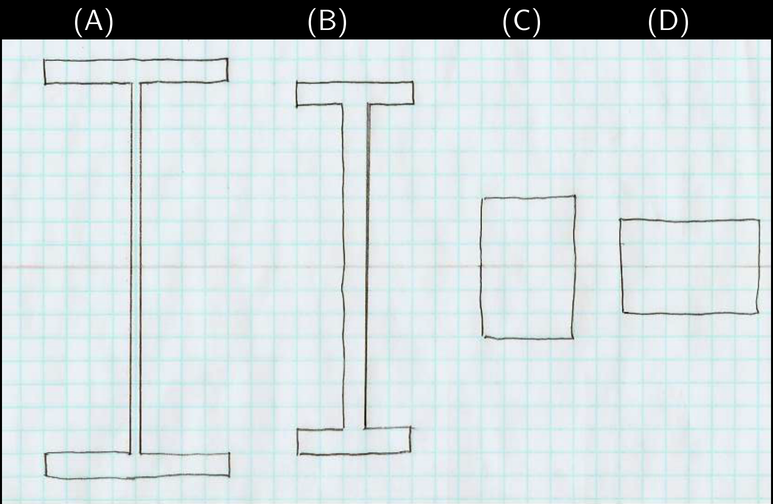
$b_3 = 8\,cm \quad h_3 = 1\,cm$

$A_3 = 8\,cm^2 \quad y_{c_3} = -8.5\,cm$

$\frac{1}{12} b_3 h_3^3 = 0.67\,cm^4$

$A_3\, y_{c_3}^2 = 578\,cm^4$

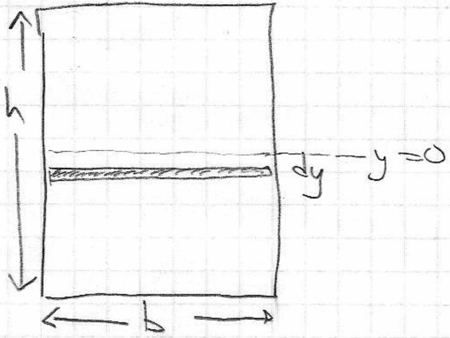
$$I_A = \frac{1}{12} b_1 h_1^3 + \frac{1}{12} b_2 h_2^3 + \frac{1}{12} b_3 h_3^3 + A_1 y_{c_1}^2 + A_2 y_{c_2}^2 + A_3 y_{c_3}^2 = \boxed{1328\,cm^4}$$



Each shape has same area $A = 24\ \mathrm{cm}^2$, but "second moment of area" is $I_A = 1328\ \mathrm{cm}^4$, $I_B = 792\ \mathrm{cm}^4$, $I_C = 72\ \mathrm{cm}^4$, $I_D = 32\ \mathrm{cm}^4$.

That's the motivation for the "I" shape of an I-beam: to get a large "second moment of area," $I = \int y^2\, \mathrm{d}A$. The deflection of a beam under load is inversely proportional to I.

rectangle

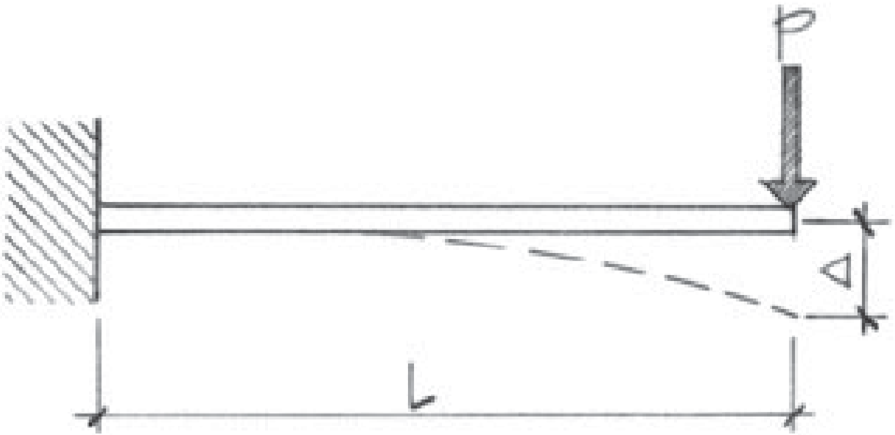


$dA = b\,dy$

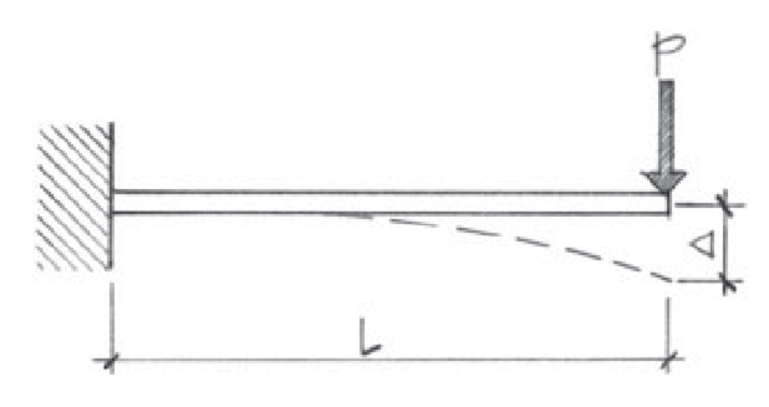
$$I = \int y^2\,dA = \int_{y=-\frac{h}{2}}^{y=h/2} y^2\, b\, dy = \left[\frac{by^3}{3}\right]_{y=-\frac{h}{2}}^{h/2}$$

$$= \frac{b(h/2)^3}{3} - \frac{b(-h/2)^3}{3} = \frac{bh^3}{12}$$

We can use the Method of Sections to study the internal forces and torques ("moments") within a beam. Consider this cantilever beam (whose own weight we neglect here) supporting a concentrated "load" force P at the far end. The left half is what holds up the right half. What force and torque ("moment") does the left half exert on the right half? Does the answer depend on where we "section" the beam?



We draw “load diagram” (basically a FBD for the beam), then the “shear (V) diagram” below that, then the “moment (M) diagram” below that. Sign conventions: $V > 0$ when beam LHS section is pulling up on beam RHS; $M > 0$ when beam is smiling.



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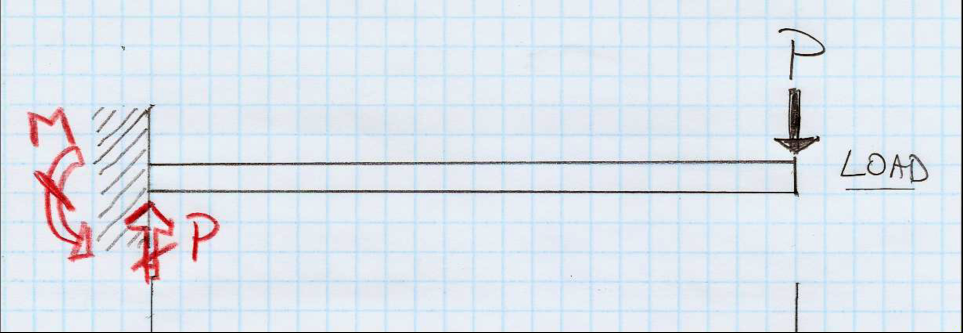
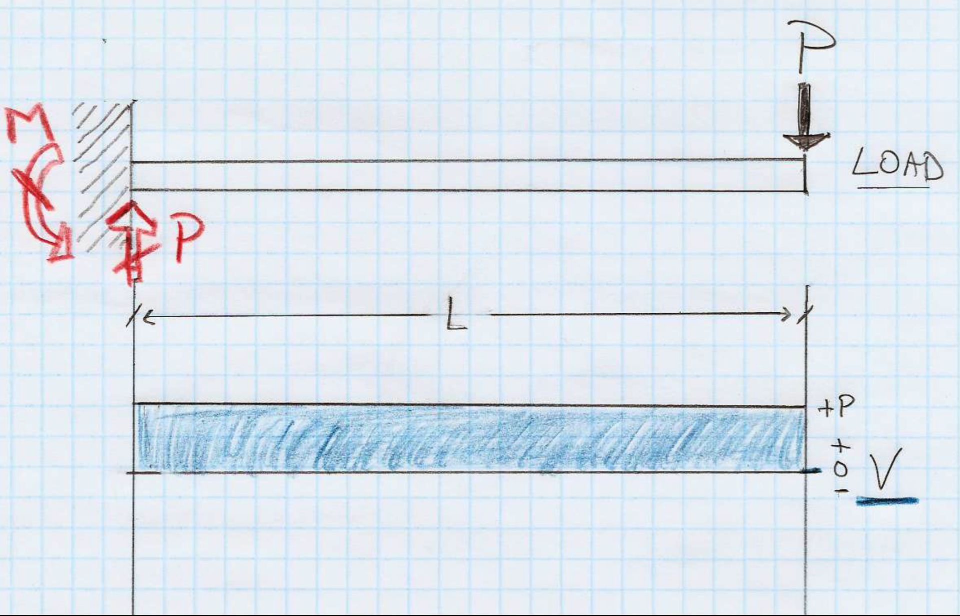
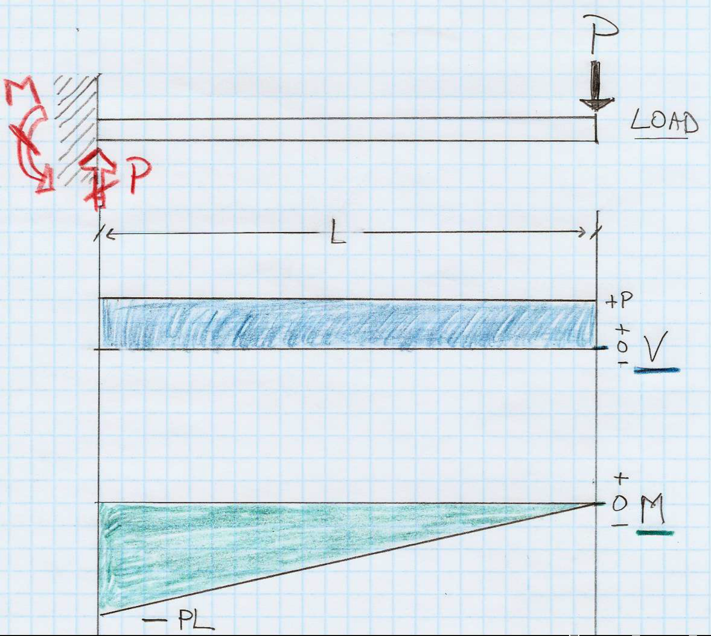


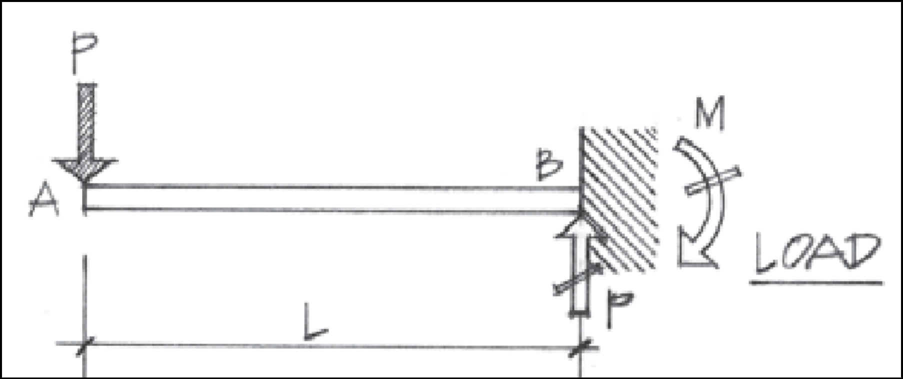
Sign conventions: $V > 0$ when beam LHS section is pulling up on beam RHS; $M > 0$ when beam is smiling.



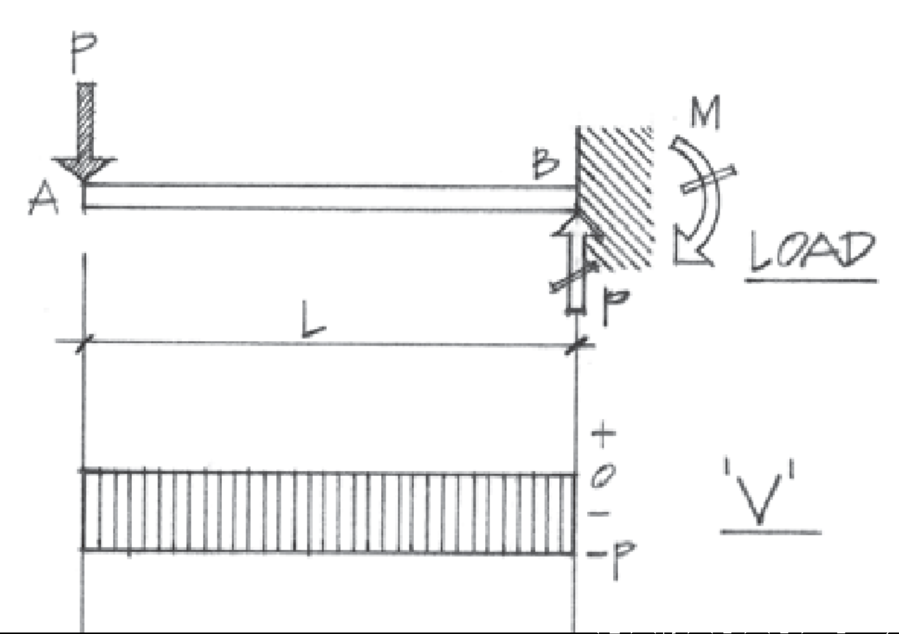
Sign conventions: $V > 0$ when beam LHS section is pulling up on beam RHS; $M > 0$ when beam is smiling.



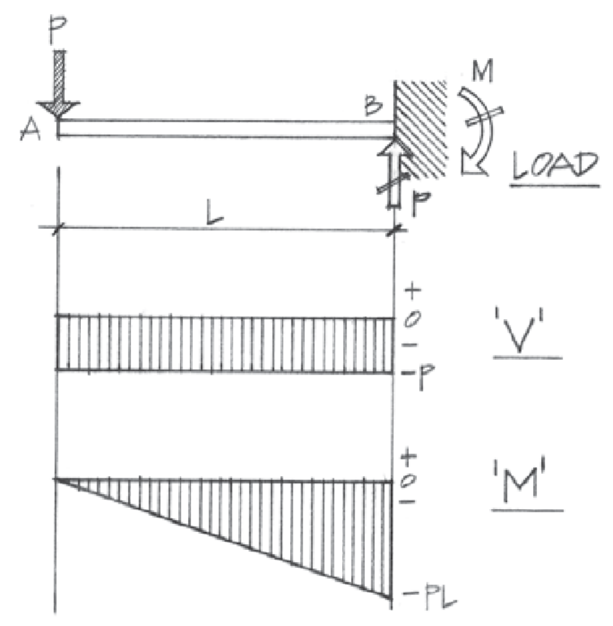
Let's try a mirror image of the same cantilever beam. Sign conventions: $V > 0$ when beam LHS section is pulling up on beam RHS; $M > 0$ when beam is smiling.



Sign conventions: $V > 0$ when beam LHS section is pulling up on beam RHS; $M > 0$ when beam is smiling.



Sign conventions: $V > 0$ when beam LHS section is pulling up on beam RHS; $M > 0$ when beam is smiling.

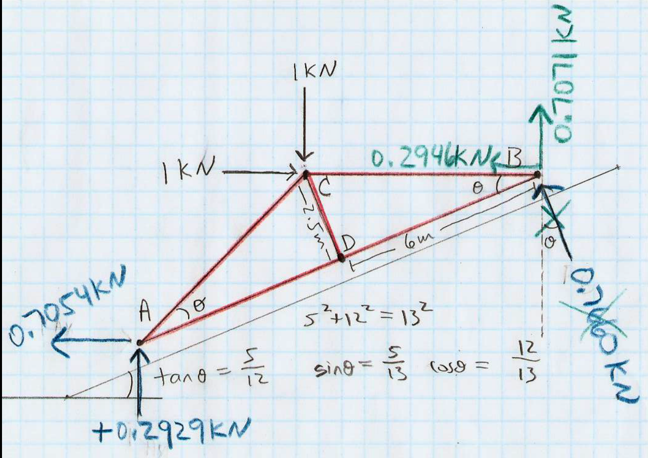


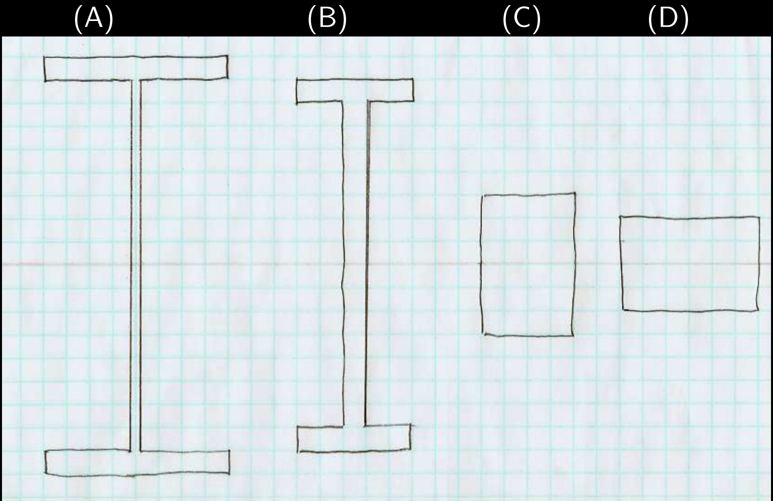
Physics 8 — Friday, November 15, 2019

- Turn in HW10. Pick up HW11 handout. HW11 is "due" next Friday, but you can turn it in on Monday, Nov 25, just in case it takes us an extra day to get through the material on beams.
- This week, you read Ch6 (cross-sectional properties) and Ch7 (simple beams). Next week, you'll read Ch8 (more details on beams).

Physics 8 — Monday, November 18, 2019

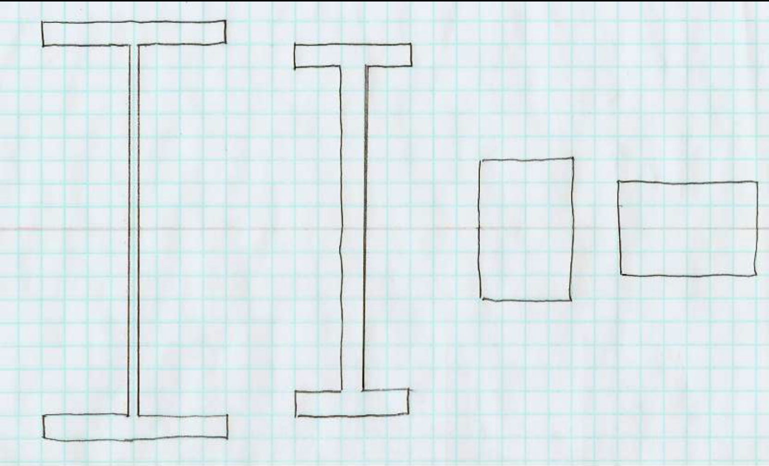
- HW11 is "due" on Friday, but you can turn it in on Monday, Nov 25, just in case it takes us an extra day to get through the material on beams.
- Last week, you read Ch6 (cross-sectional properties) and Ch7 (simple beams). This week, read Ch8 (more about beams).



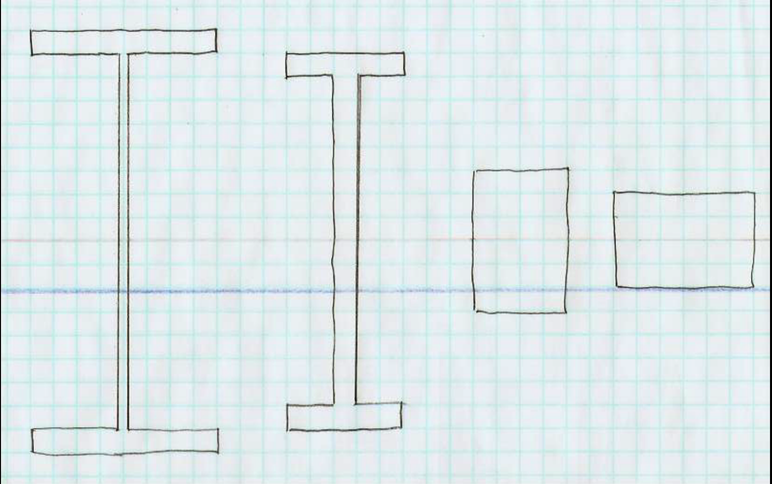


Each shape has the same area: 24 squares. Which shape has the largest $I_x = \int y^2 \mathrm{d}A$ ("second moment of area about the x-axis"), with $y = 0$ given by the faint horizontal red line at the center?

(A) (B) (C) (D)



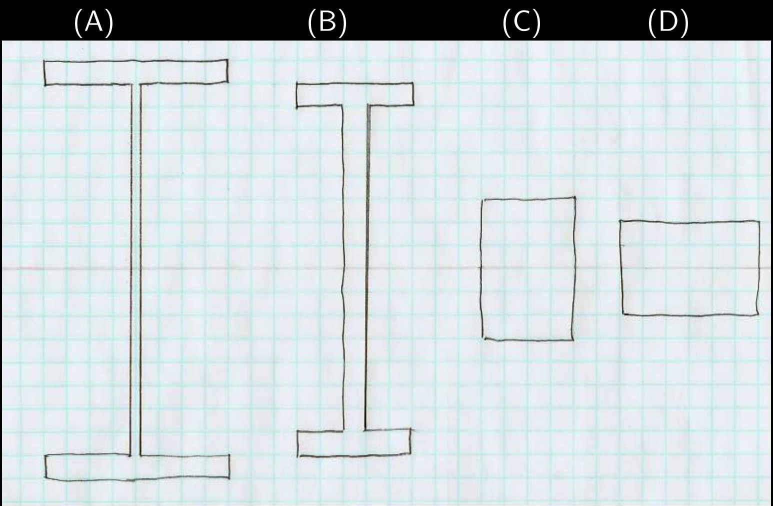
Each shape has the same area: 24 squares. Which shape has the **smallest** $I_x = \int y^2 \mathrm{d}A$ ("second moment of area about the x-axis"), with $y = 0$ given by the faint horizontal red line at the center?



If you moved the x-axis down by a couple of grid units, what would happen to $I_x = \int y^2 \mathrm{d}A$ for each shape? Would I_x change? Would I_x change by the same amount for each shape?

(A) yes (B) no

(Think: "parallel-axis theorem.")



Given that $I_x = \int y^2 \mathrm{d}A = \frac{1}{12} bh^3$ for a rectangle centered at $y = 0$, let's use the parallel-axis theorem to calculate I_x for shapes A, B, C, and D. For definiteness, let each graph-paper box be $1\ \mathrm{cm} \times 1\ \mathrm{cm}$. So the units will be cm^4.

Let's do the two rectangular shapes first, since they're quick.

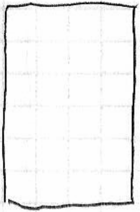
Then, the trick for the non-rectangular shapes is to use (from O/K §6.3) the "parallel-axis theorem:"

$$I_x = \sum I_{xc} + \sum A\, d_y^2$$

where each sum is over the simple shapes that compose the big shape.

- I_{xc} is the simple shape's own I_x value about its own centroid (which is $bh^3/12$ for a rectangle),
- A is the simple shape's area, and
- d_y is the vertical displacement of the simple shape's centroid from $y = 0$ (which should be the centroid of the big shape).

(C)



$b = 4\,cm$ h =

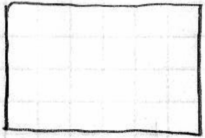
$h = 6\,cm$

$A = 24\,cm^2$

$y_c = 0$

$\frac{1}{12} b h^3 = \boxed{72\,cm^4}$

(D)



$b = 6\,cm$

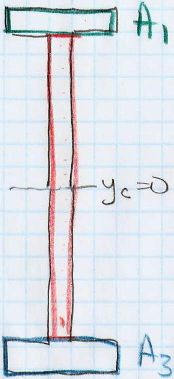
$h = 4\,cm$

$A = 24\,cm^2$

$y_c = 0$

$\frac{1}{12} b h^3 = \boxed{32\,cm^4}$

(B)



$A_1 = 5\,cm^2 \quad b_1 = 5\,cm \quad h_1 = 1\,cm$

$y_{c_1} = +7.5\,cm$

$A_2 = 14\,cm^2 \quad b_2 = 1\,cm$

$h_2 = 14\,cm$

$y_{c_2} = 0$

$A_3 = 5\,cm^2 \quad b_3 = 5\,cm \quad h_3 = 1\,cm$

$y_{c3} = -7.5\,cm$

$\frac{1}{12} b_1 h_1^3 = 0.417\,cm^4$

$A_1 y_{c_1}^2 = 281.25\,cm^4$

$\frac{1}{12} b_2 h_2^3 = 228.67\,cm^4$

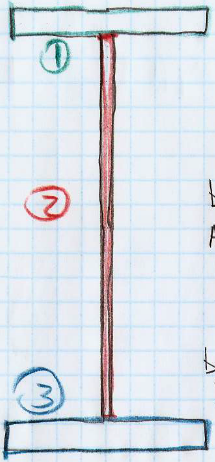
$A_2 y_{c_2}^2 = 0$

$\frac{1}{12} b_3 h_3^3 = 0.417\,cm^4$

$A_3 y_{c_3}^2 = 281.25\,cm^4$

$$I_B = \frac{1}{12} b_1 h_1^3 + \frac{1}{12} b_2 h_2^3 + \frac{1}{12} b_3 h_3^3 + A_1 y_{c_1}^2 + A_2 y_{c_2}^2 + A_3 y_{c_3}^2 = \boxed{792\,cm^4}$$

(A)



$b_1 = 8\text{cm} \quad h_1 = 1\text{cm}$

$A_1 = 8\text{cm}^2 \quad y_{c_1} = +8.5\text{cm}$

$\frac{1}{12} b_1 h_1^3 = 0.67\text{cm}^4$

$A_1 y_{c_1}^2 = 578\text{cm}^4$

$b_2 = 0.5\text{cm} \quad h_2 = 16\text{cm}$

$A_2 = 8\text{cm}^2 \quad y_{c_2} = 0$

$\frac{1}{12} b_2 h_2^3 = 170.67\text{cm}^4$

$A_2 y_{c_2}^2 = 0$

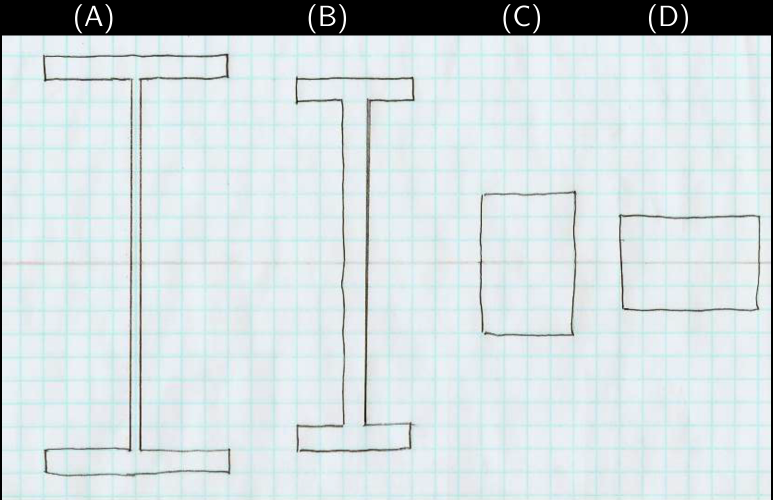
$b_3 = 8\text{cm} \quad h_3 = 1\text{cm}$

$A_3 = 8\text{cm}^2 \quad y_{c_3} = -8.5\text{cm}$

$\frac{1}{12} b_3 h_3^3 = 0.67\text{cm}^4$

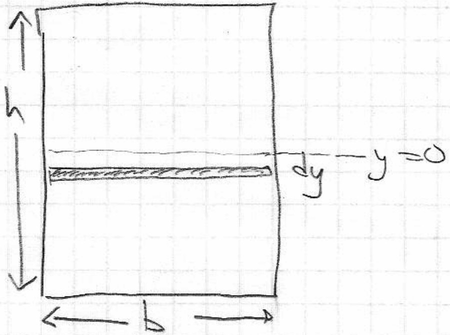
$A_3 y_{c_3}^2 = 578\text{cm}^4$

$$I_A = \frac{1}{12} b_1 h_1^3 + \frac{1}{12} b_2 h_2^3 + \frac{1}{12} b_3 h_3^3 + A_1 y_{c_1}^2 + A_2 y_{c_2}^2 + A_3 y_{c_3}^2 = \boxed{1328\text{cm}^4}$$



Each shape has same area $A = 24\ \mathrm{cm}^2$, but "second moment of area" is $I_A = 1328\ \mathrm{cm}^4$, $I_B = 792\ \mathrm{cm}^4$, $I_C = 72\ \mathrm{cm}^4$, $I_D = 32\ \mathrm{cm}^4$.
That's the motivation for the "I" shape of an I-beam: to get a large "second moment of area," $I = \int y^2\, \mathrm{d}A$. The deflection of a beam under load is inversely proportional to I.

rectangle

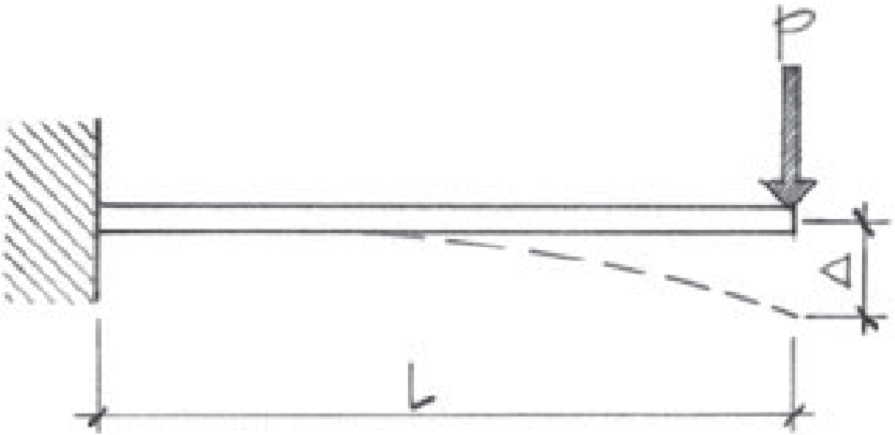


$dA = b\,dy$

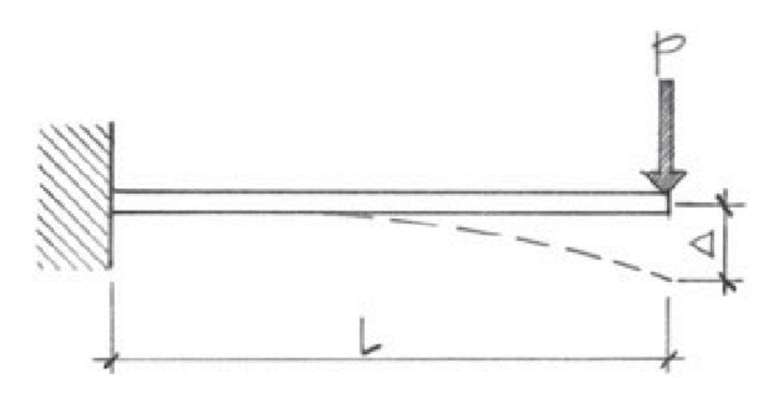
$$I = \int y^2\, dA = \int_{y=-\frac{h}{2}}^{y=h/2} y^2\, b\, dy = \left[\frac{by^3}{3}\right]_{y=-\frac{h}{2}}^{h/2}$$

$$= \frac{b(h/2)^3}{3} - \frac{b(-h/2)^3}{3} = \frac{bh^3}{12}$$

We can use the Method of Sections to study the internal forces and torques ("moments") within a beam. Consider this cantilever beam (whose own weight we neglect here) supporting a concentrated "load" force P at the far end. The left half is what holds up the right half. What force and torque ("moment") does the left half exert on the right half? Does the answer depend on where we "section" the beam?

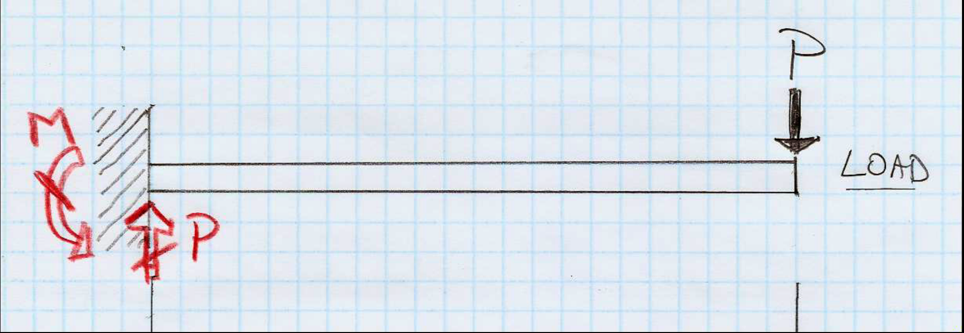


We draw "load diagram" (basically a FBD for the beam), then the "shear (V) diagram" below that, then the "moment (M) diagram" below that. Sign conventions: $V > 0$ when beam LHS section is pulling up on beam RHS; $M > 0$ when beam is smiling.

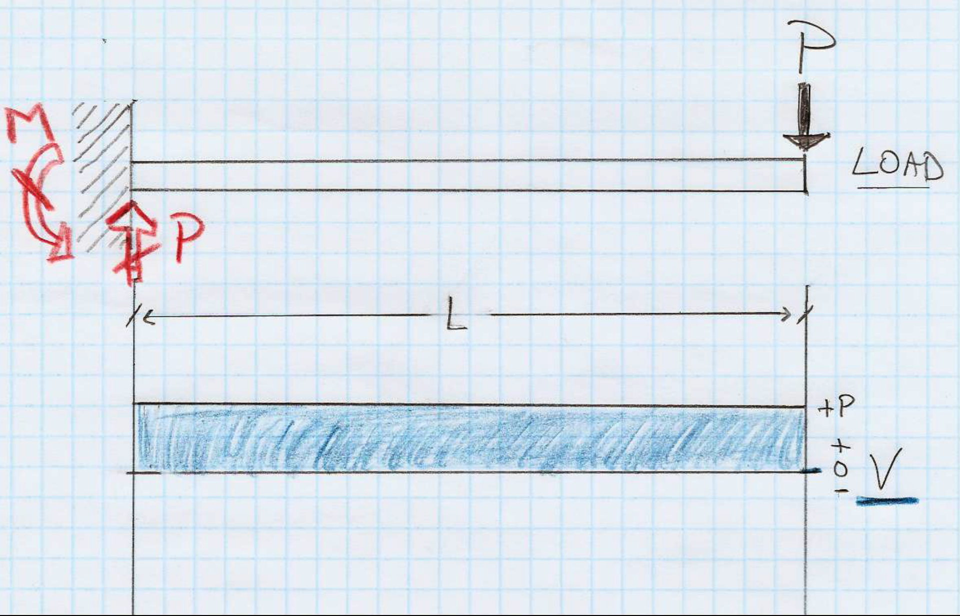


Another way to state the $V(x)$ sign convention: $V(x)$ is the running sum of all (upward minus downward) forces exerted on the beam, from the left side up to and including x.

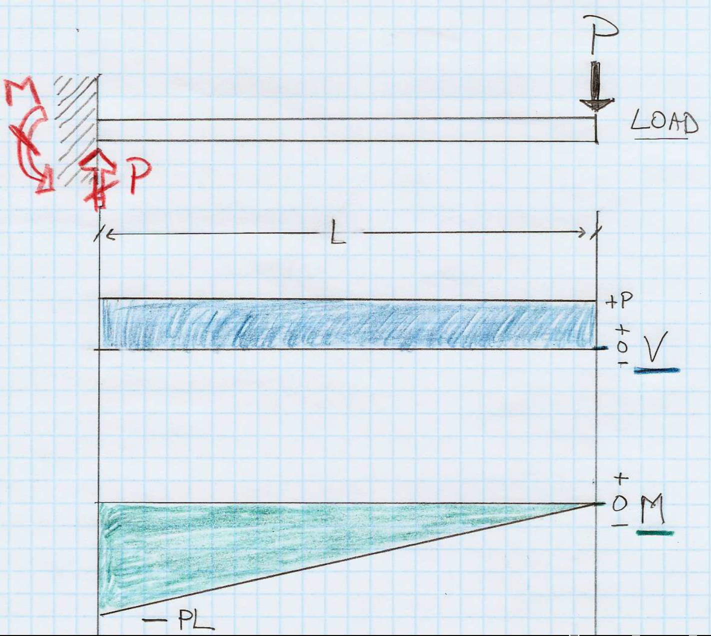
We draw "load diagram" (basically a FBD for the beam), then the "shear (V) diagram" below that, then the "moment (M) diagram" below that. Sign conventions: $V > 0$ when beam LHS section is pulling up on beam RHS; $M > 0$ when beam is smiling.



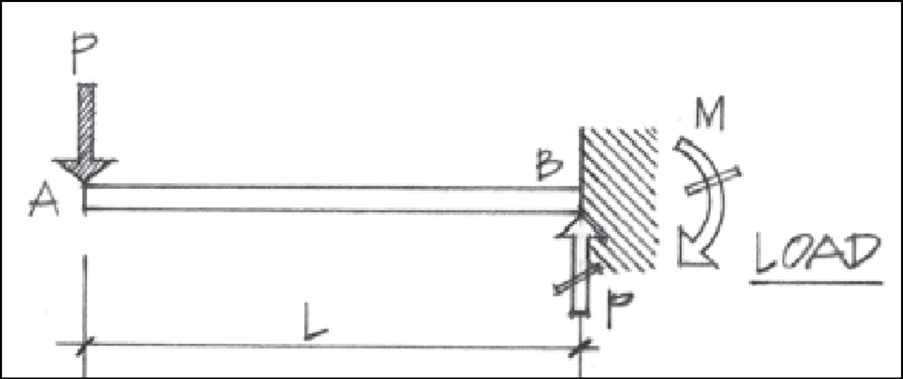
Sign conventions: $V > 0$ when beam LHS section is pulling up on beam RHS; $M > 0$ when beam is smiling.



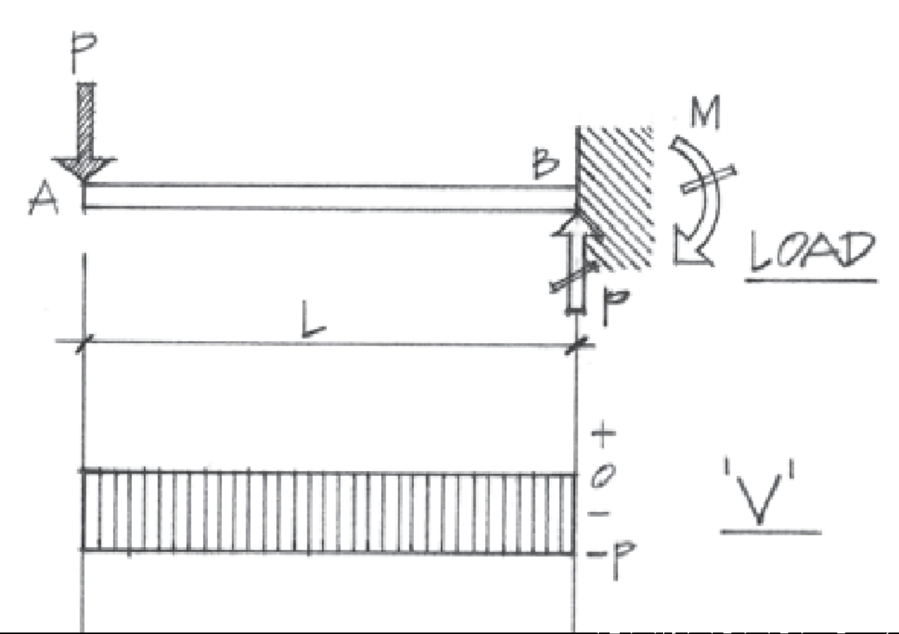
Sign conventions: $V > 0$ when beam LHS section is pulling up on beam RHS; $M > 0$ when beam is smiling.



Let's try a mirror image of the same cantilever beam. Sign conventions: $V > 0$ when beam LHS section is pulling up on beam RHS; $M > 0$ when beam is smiling.



Sign conventions: $V > 0$ when beam LHS section is pulling up on beam RHS; $M > 0$ when beam is smiling.



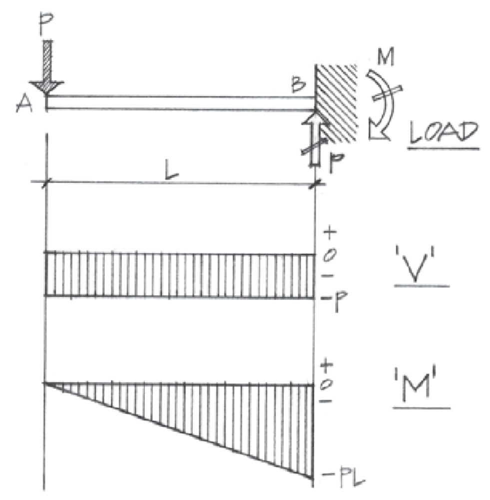
Sign conventions: $V(x) > 0$ when beam left of x is pulling up on beam right of x. $M(x) > 0$ when beam is smiling.

Transverse shear $V(x)$ is the running sum of forces on beam, from $0 \ldots x$, where upward = positive.

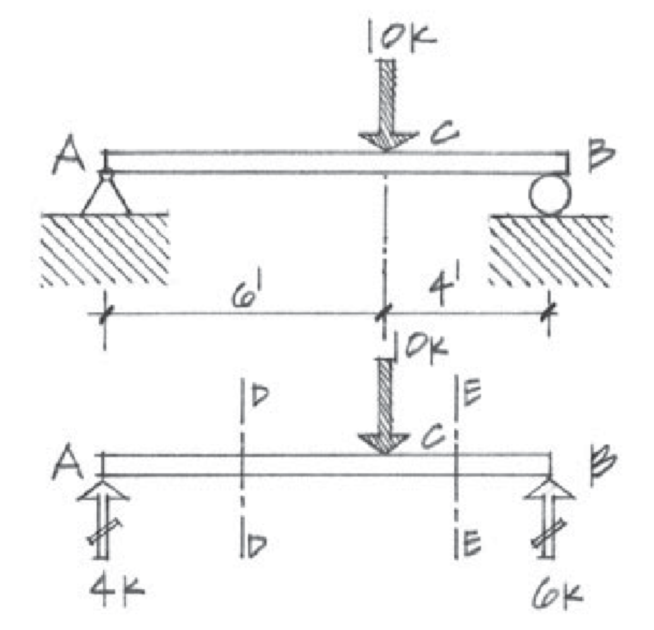
Bending moment $M(x)$ is the torque exerted by each side of the beam, cut at x, on the other side; but beware of sign convention.

$$V(x) = \frac{\mathrm{d}}{\mathrm{d}x} M(x)$$

The V diagram graphs the slope of the M diagram.



Draw V and M for this "simply supported" beam: $V(x) > 0$ when beam $0 \dots x$ pulls up on beam $x \dots L$. $M > 0$ when beam smiles.

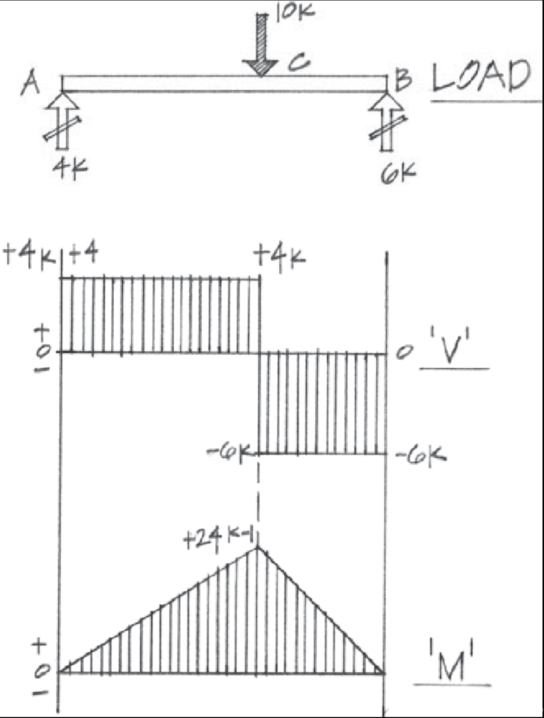


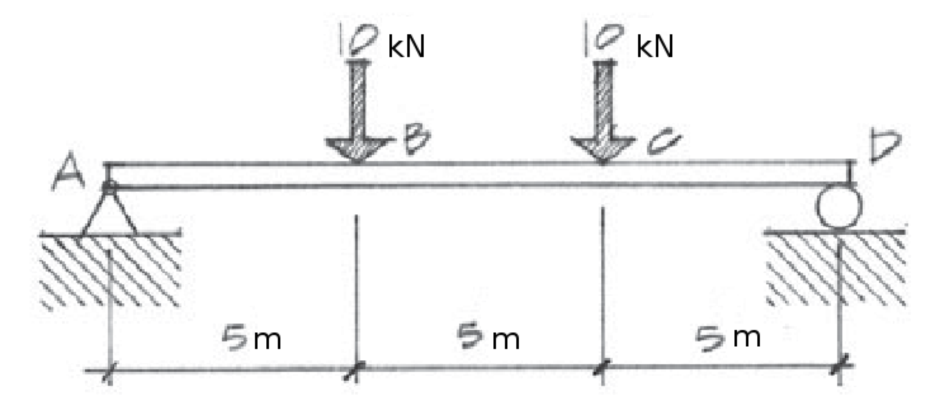
$$V(x) = \frac{\mathrm{d}}{\mathrm{d}x} M(x)$$

The shear (V) diagram equals the slope of the moment (M) diagram.

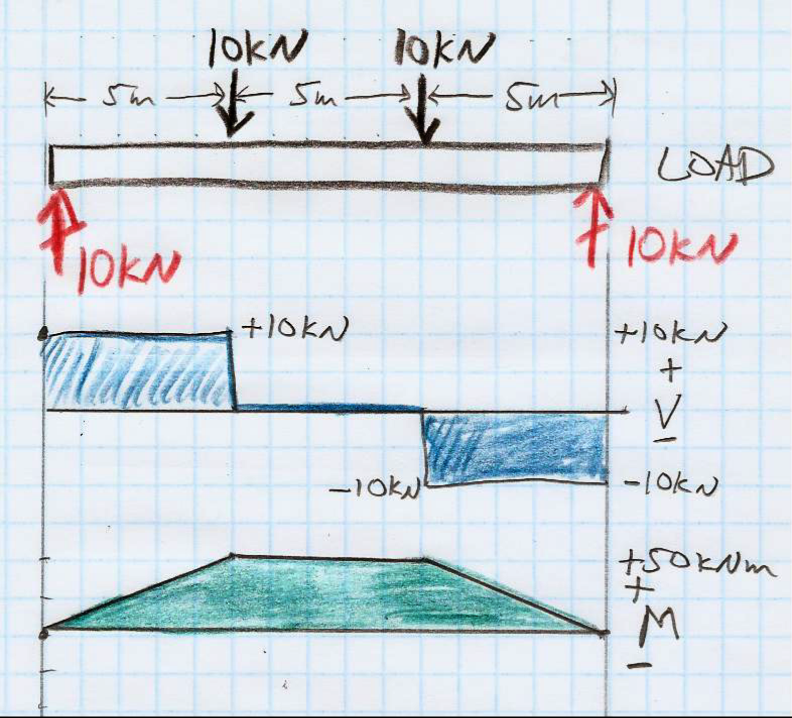
$$M(x) = \int V(x)\,\mathrm{d}x$$

But be careful about the M values at the ends — depends how the beam is supported. A free, hinged, or roller-supported end has $M = 0$: support exerts no torque on that end. Fixed end of cantilever has $M \neq 0$.





Let's try drawing load, V, and M diagrams for this simply-supported beam. Pretend the units are meters and kilonewtons rather than the original drawing's feet and kilopounds ("kips").


10kN
10kN
5m
5m
5m
LOAD
10kN
10kN
+10kN
+10kN
+
V
−
−10kN
−10kN
+50kNm
+
M
−

Shear (V) and moment (M) diagrams:

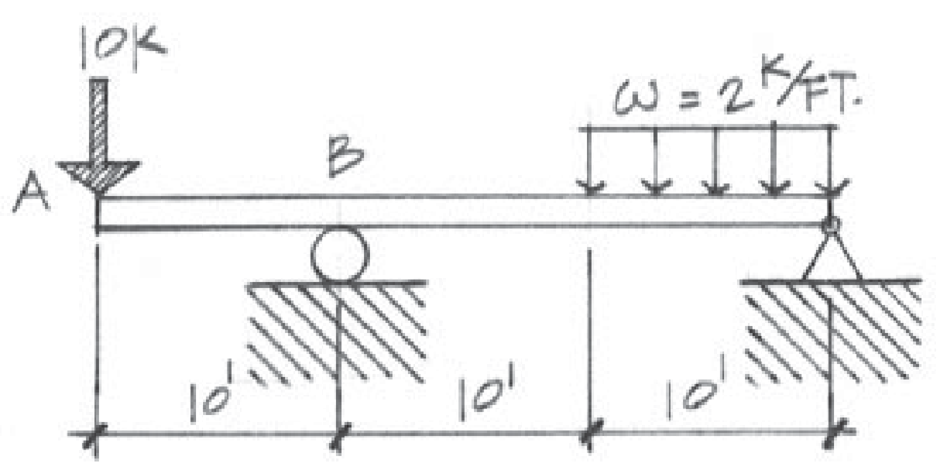
- First draw a "load diagram," which is an EFBD that shows all of the vertical forces acting on the beam.
- The "shear diagram" $V(x)$ graphs the running sum of all vertical forces (both supports and loads) acting on the beam, from the left side up to x, where upward = positive, downward = negative.
- To draw the "moment diagram" $M(x)$, note that V is the slope of M:

$$V(x) = \frac{\mathrm{d}}{\mathrm{d}x} M(x)$$

- The change in M from x_1 to x_2 is given by

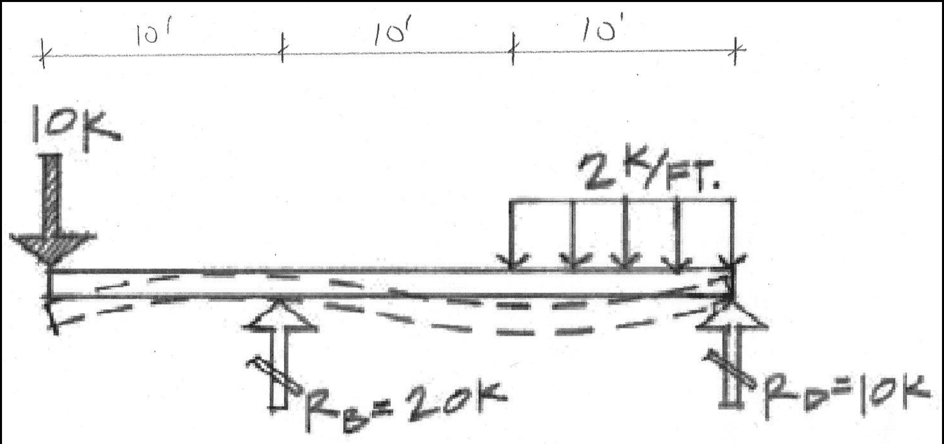
$$M_2 - M_1 = (x_2 - x_1)\, V_{1\to 2}^{\mathrm{average}}$$

- If an end of a beam is unsupported ("free"), is hinge/pin supported, or is roller supported, then $M = 0$ at that end. You can only have $M \neq 0$ at an end if the support at that end is capable of exerting a torque on the beam — for example, the fixed end of a cantilever has $M \neq 0$.

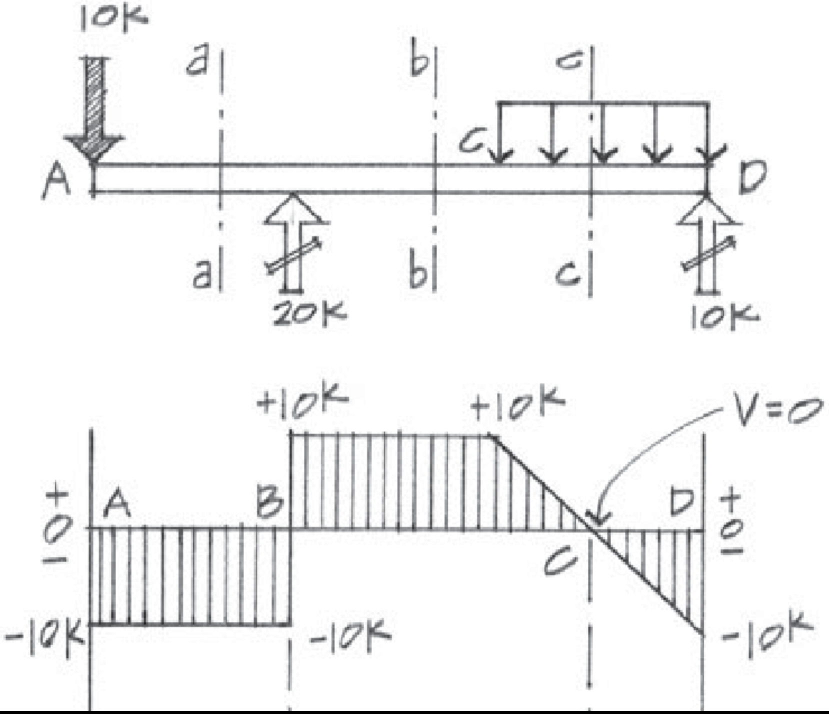


Draw shear (V) and moment (M) diagrams for this beam! Tricky! First one needs to solve for the support ("reaction") forces.

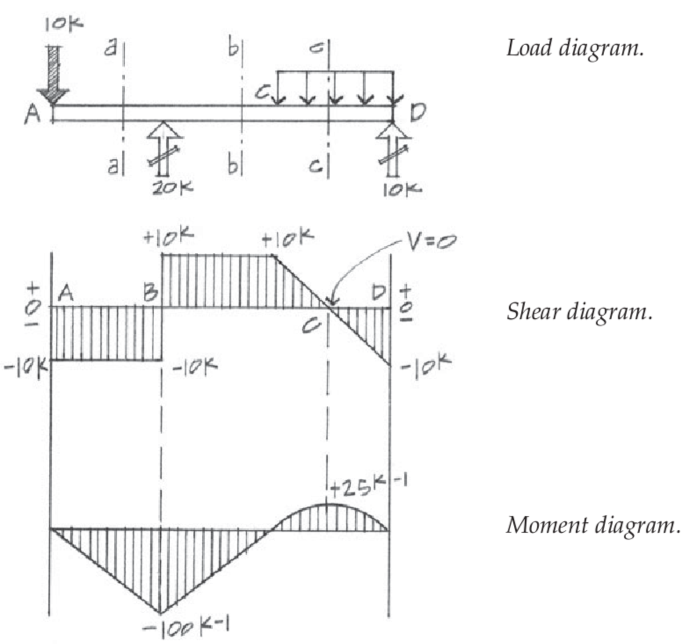
Note: in solving for the support forces, you replace distributed load w with equivalent point load. But when you draw the load diagram to find V and M, you need to keep w in its original form.



Remember that $V(x)$ is the running sum, from LHS to x, of vertical forces acting on the beam, with upward=positive.

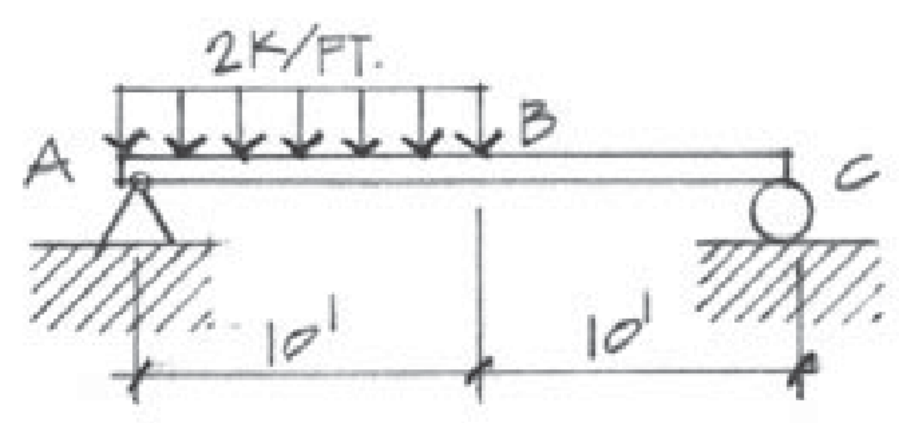


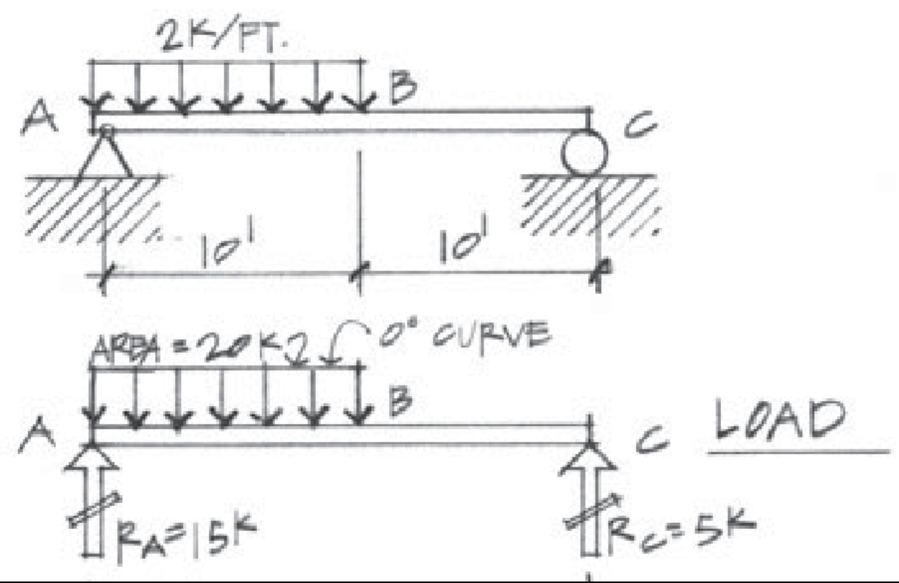
10K
a
b
c
c
A
D
a
b
c
20K
10K
+10K
+10K
V=0
+
0
−
A
B
D
C
+
0
−
−10K
−10K
−10K

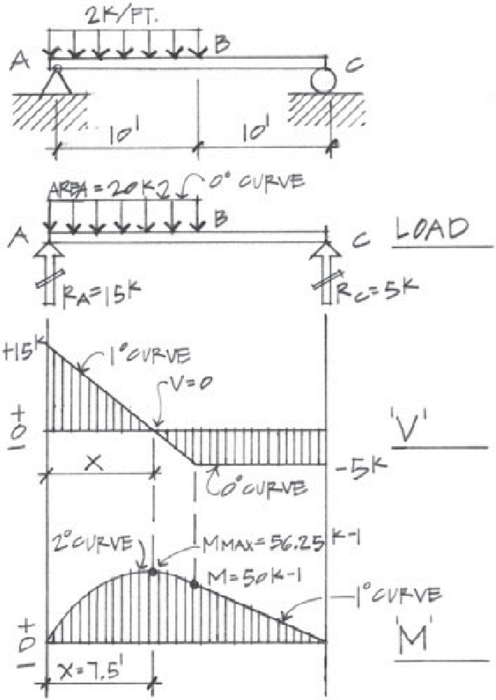


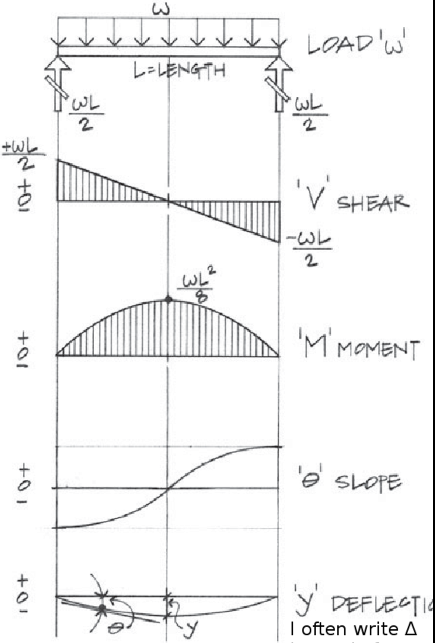
Neat trick: $M_2 - M_1 = (V_{1\to2}^{\text{average}})(x_2 - x_1)$

Draw load, V, and M diagrams for this simply supported beam with a partial uniform load.




2K/FT.
A
B
C
10'
10'
AREA = 20K
0° CURVE
A
B
C
LOAD
RA=15K
RC=5K

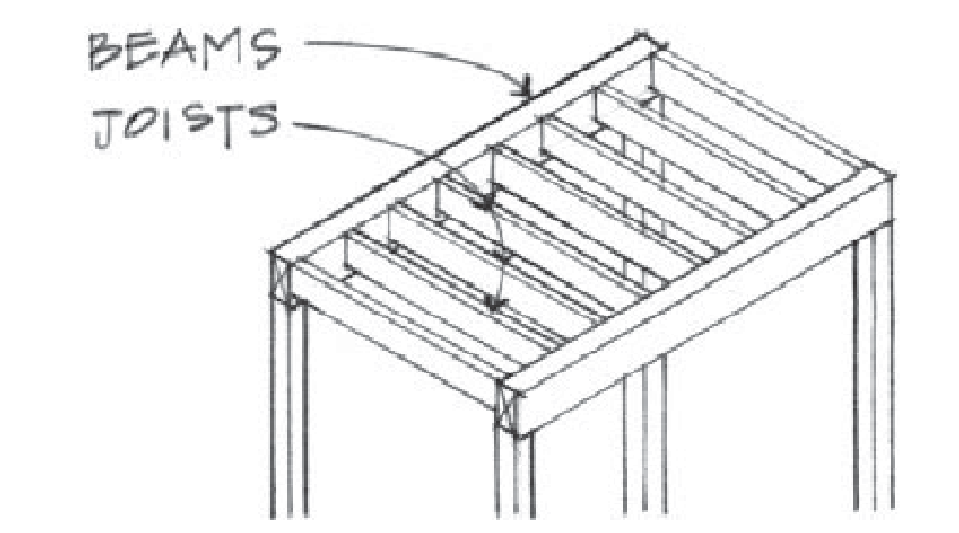

2K/FT.
A
B
C
10'
10'
AREA = 20K
0° CURVE
LOAD
$R_A = 15^K$
$R_C = 5^K$
$+15^K$
1° CURVE
V = 0
+
0
−
'V'
X
-5^K
0° CURVE
2° CURVE
MMAX = 56.25 K-I
M = 50K-I
1° CURVE
'M'
X = 7.5'



I often write Δ instead of y

Figure 7.11 Relationship of load, shear, moment, slope, and deflection diagrams.

Why do we care about these beam diagrams, anyway? Usually the floor of a structure must carry a specified weight per unit area. The beams (beams, girders, joists, etc.) must be strong enough to support this load without failing and must be stiff enough to support this load without excessive deflection.



Beam criteria:

- Normal stress in the extreme fibers of the beam (farthest from neutral surface) must be smaller than the allowable bending stress, F_b, which depends on the material (wood, steel, etc.).
- This happens where $M(x)$ has largest magnitude.
- Shear stress (in both y ("transverse") and x ("longitudinal") must be smaller than the allowable shear stress, F_v, which is also a property of the material (wood, steel, etc.).
- This happens where $V(x)$ has largest magnitude, and (surprisingly) is largest near the neutral surface.
- The above two are "strength" criteria. The third one is a "stiffness" criterion:
- The maximum deflection under load must satisfy the building code: typically $\Delta y_{\max} < L/360$.
- For a uniform load, this happens farthest away from the supports. If deflection is too large, plaster ceilings develop cracks, floors feel uncomfortably bouncy or sloped.
- The book also notes buckling as a beam failure mode.

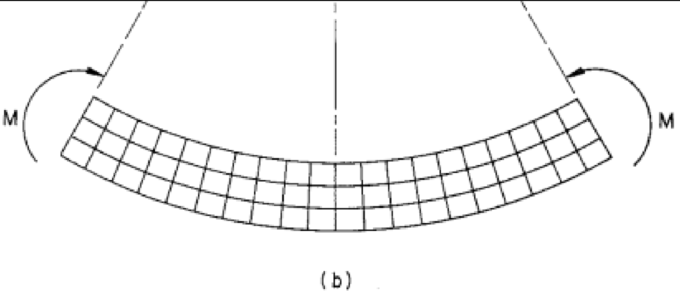
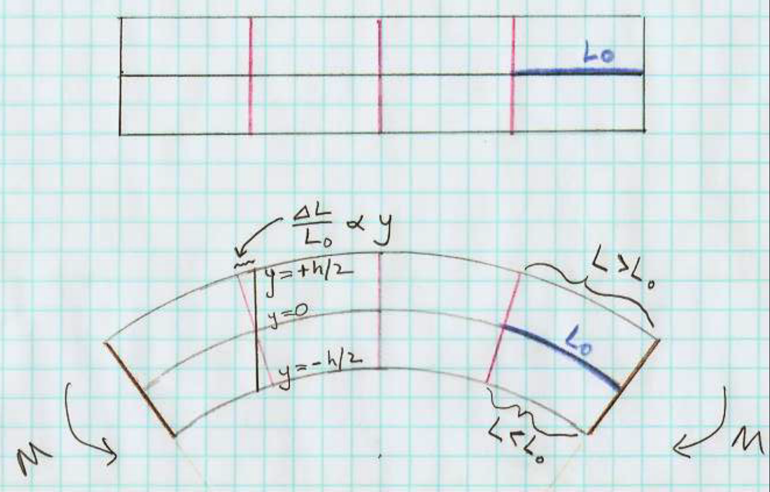


Fig. 6.5.
Navier's assumption. Originally plane and parallel sections (a) remain plane after bending (b), but converge onto a common center of curvature. This assumption can be illustrated with a rubber beam.

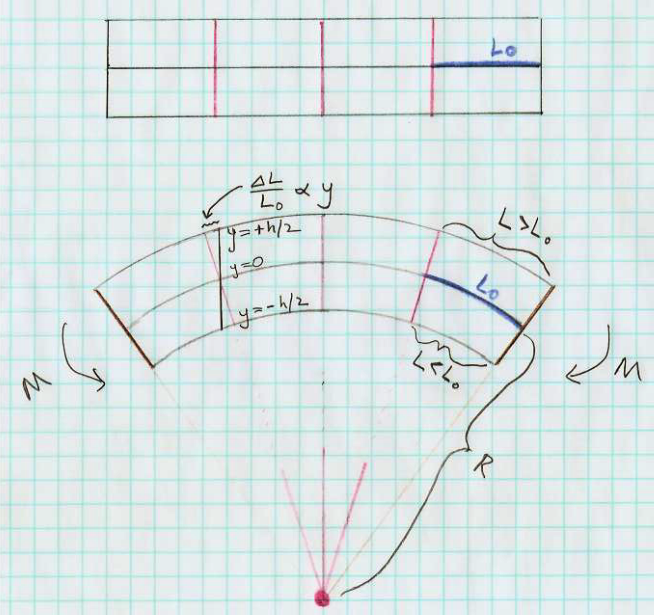
(In this illustration, bottom is in tension, top is in compression, as in a "simply supported" beam.)

A big topic from this week's reading was to see how an initially horizontal beam responds to the bending moment $M(x)$ by deforming into a curved shape.



(In this illustration, top is in tension, as in a cantilever.)

Key idea: bending moment $M \propto \frac{1}{R}$, where R is the radius of curvature of the beam. For constant M, vertical lines converge toward common center of curvature.



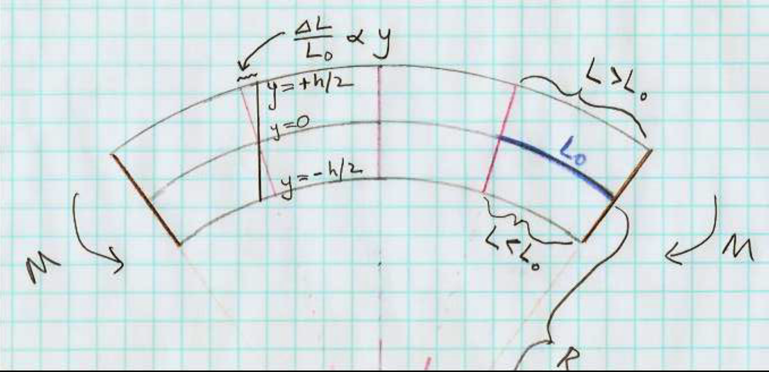
$$\text{strain} = \frac{\Delta L}{L_0} = \frac{y}{R}$$

where $y = 0$ is the neutral surface.

So in this case $y > 0$ is in tension and $y < 0$ is in compression.

If you think of wood fibers running along the beam's axis, then the fibers above the neutral surface ($y > 0$) are stretched in proportion to y, and the fibers below the neutral surface ($y < 0$) are compressed in proportion to $|y|$.

$$\text{strain} = \frac{\Delta L}{L} = \frac{y}{R}$$



Now remember that $\frac{\Delta L}{L}$ is called (axial) *strain*, and force per unit area is called *stress*. For an elastic material, strain (e) $\propto$ stress (f).

$$\frac{\Delta L}{L} = \frac{1}{E} \times \frac{\text{Force}}{\text{Area}} = \frac{1}{E} \times f \qquad e = \frac{1}{E} \times f$$

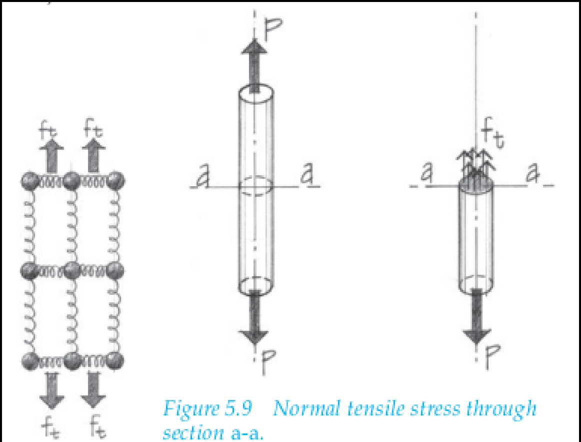
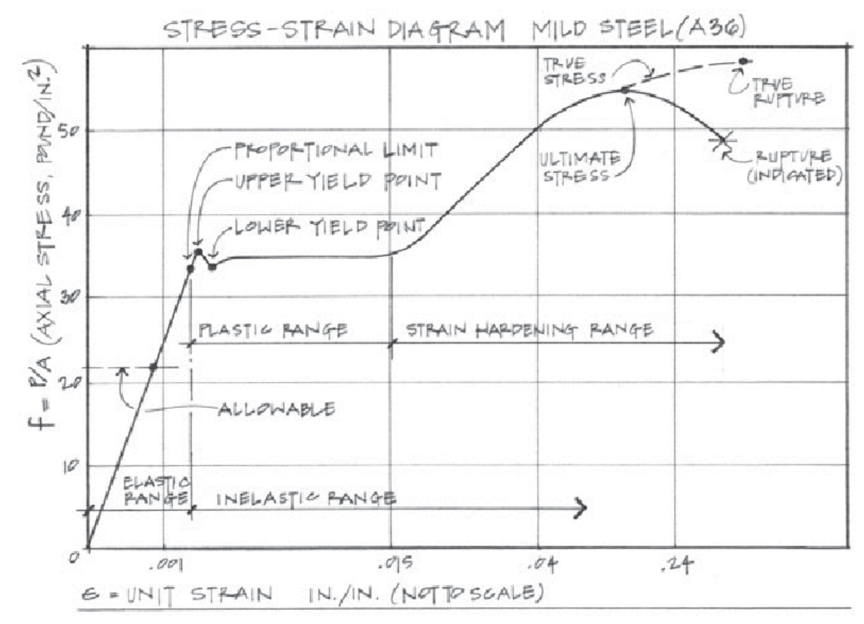


Figure 5.9 Normal tensile stress through section a-a.

In the elastic region, strain ($e = \Delta L/L$) is proportional to stress ($f = F/A$). $\boxed{f = Ee}$. The slope E is Young's modulus.

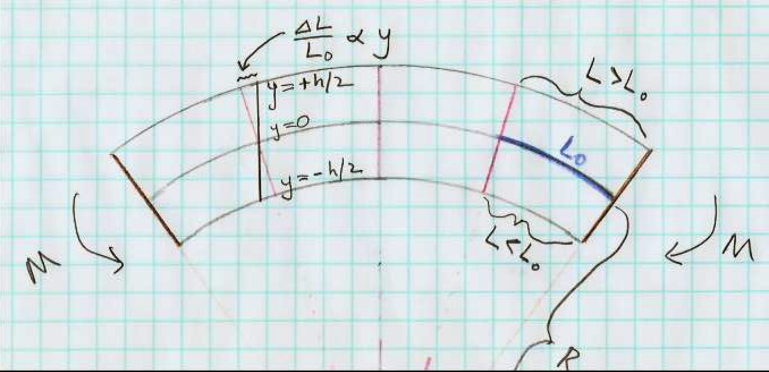


Plugging in $f = Ee$ to the bending-beam diagram:

$$\frac{y}{R} = \frac{\Delta L}{L} = e = \frac{f}{E}$$

we find the force-per-unit area (stress) exerted by the fibers is

$$f = \frac{Ey}{R}$$

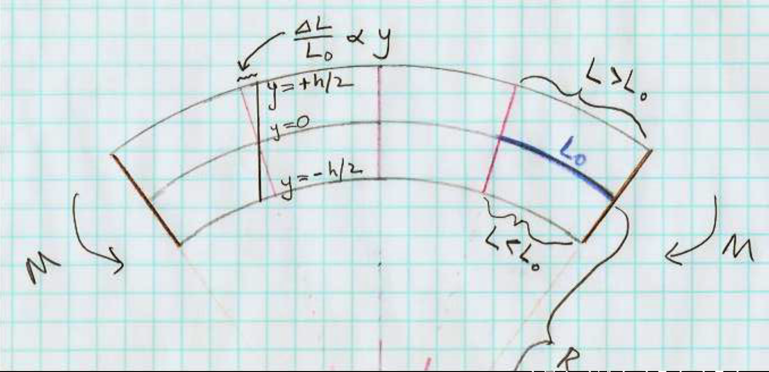


The force-per-unit area (stress) exerted by the fibers is

$$f = \frac{Ey}{R}$$

while the torque (bending moment $\mathrm{d}M$, pivot about N.A.) exerted by each tiny fiber of area $\mathrm{d}A$ is proportional to its lever arm y

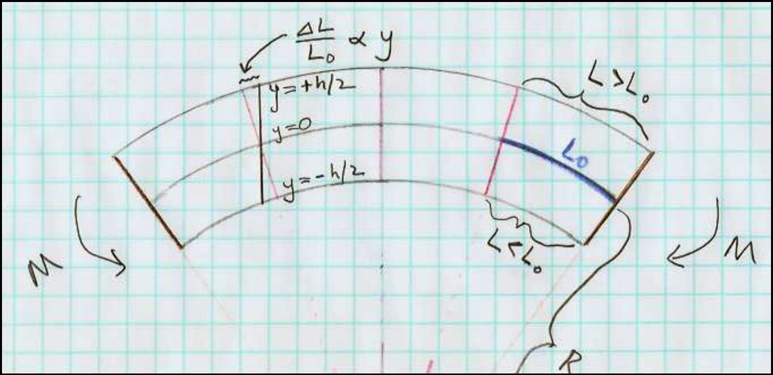
$$\mathrm{d}M = y\ \mathrm{d}F = y\ f\mathrm{d}A = y\left(\frac{Ey}{R}\right)\mathrm{d}A = \frac{E}{R}\ y^2\ \mathrm{d}A$$



So the bending moment M exerted by a curved beam is

$$M = \frac{E}{R} \int y^2 \, \mathrm{d}A = \frac{EI}{R}$$

where R is the curved beam's radius of curvature and $I = \int y^2 \, \mathrm{d}A$ is the "second moment of area" a.k.a. "area moment of inertia."



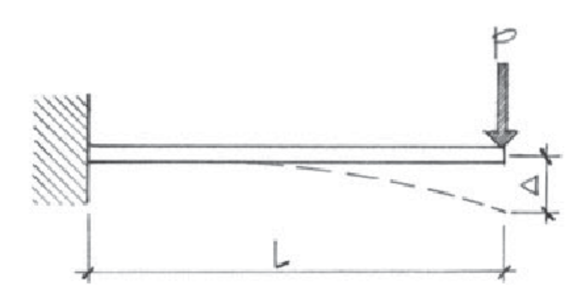
$$M = \frac{E}{R}\int y^2 \mathrm{d}A = \frac{EI}{R}$$

Meanwhile, the vertical *deflection* Δ of a point along the beam is related to its curvature by (in limit where $\Delta \ll R$)

$$\frac{1}{R} \approx \frac{\mathrm{d}^2\Delta}{\mathrm{d}x^2}$$

so you can integrate the $M(x)$ curve twice to get deflection

$$\frac{\mathrm{d}^2\Delta}{\mathrm{d}x^2} = \frac{M}{EI} \quad \Rightarrow \quad \Delta(x) = \frac{1}{EI}\int \mathrm{d}x \int M(x)\,\mathrm{d}x$$



Calculus digression (not important — but you may be curious):

You may have seen in calculus that the "curvature" (which means $1/R$, where R is the radius of curvature) of a function $y = f(x)$ is

$$\frac{1}{R} = \frac{y''}{(1 + (y')^2)^{3/2}}$$

We are working in the limit $y' \ll 1$, so

$$\frac{1}{R} \approx y''$$

That's how we arrived at

$$\frac{1}{R} \approx \frac{\mathrm{d}^2\Delta}{\mathrm{d}x^2}$$

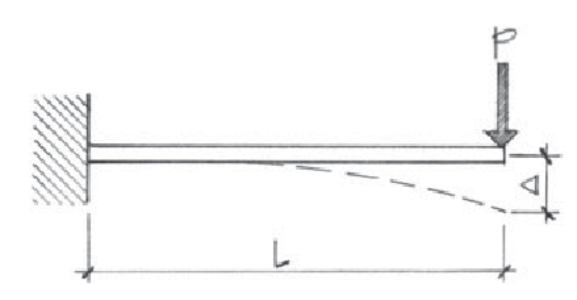
$$M = \frac{E}{R} \int y^2 \mathrm{d}A = \frac{EI}{R}$$

Meanwhile, the vertical *deflection* Δ of a point along the beam is related to its curvature by (in limit where $\Delta \ll R$)

$$\frac{1}{R} \approx \frac{\mathrm{d}^2\Delta}{\mathrm{d}x^2}$$

so you can **integrate the $M(x)$ curve twice to get deflection**

$$\frac{\mathrm{d}^2\Delta}{\mathrm{d}x^2} = \frac{M}{EI} \quad \Rightarrow \quad \boxed{\Delta(x) = \frac{1}{EI} \int \mathrm{d}x \int M(x)\, \mathrm{d}x}$$



This Onouye/Kane figure writes "y" here for deflection, but I wrote "Δ" for deflection on the preceding pages (and they usually do, too), because we were already using y for "distance above the neutral surface."

So you integrate $M(x)/EI$ twice w.r.t. x to get the deflection $\Delta(x)$.

The bending moment $M(x) = EI\ \mathrm{d}^2\Delta/\mathrm{d}x^2$, where E is Young's modulus and I is second moment of area.

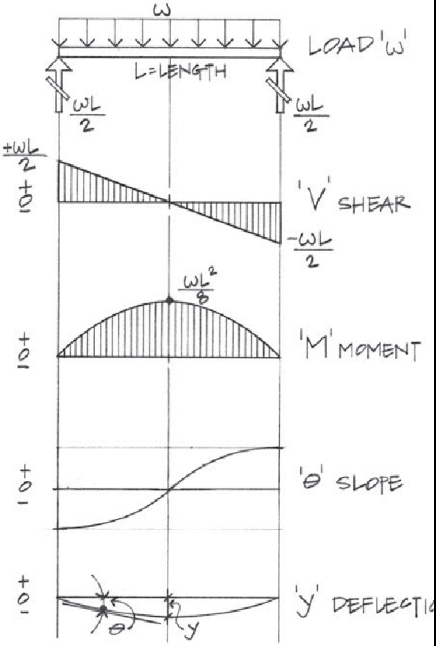
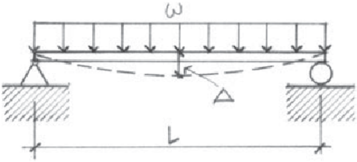
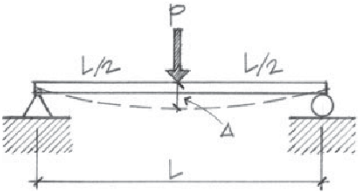
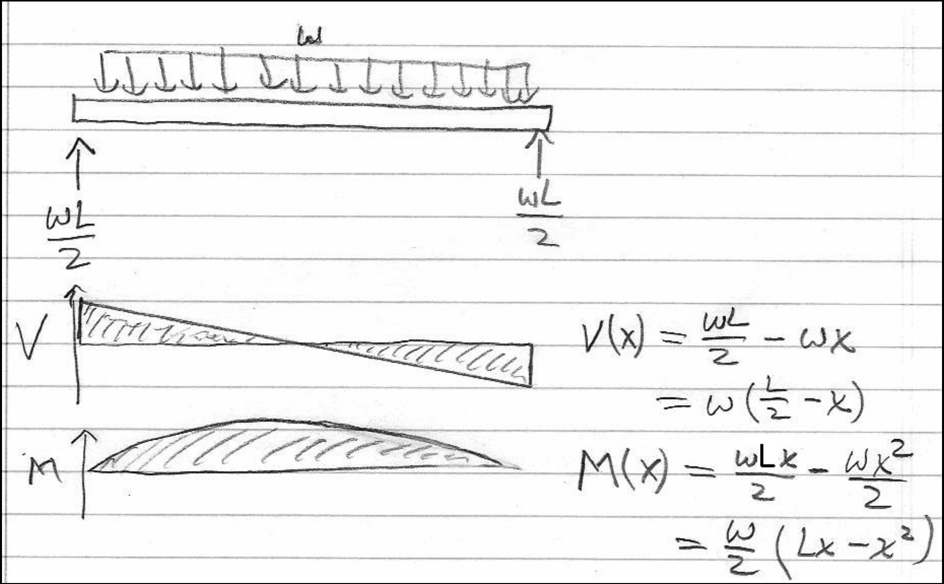


Figure 7.11 Relationship of load, shear, moment, slope, and deflection diagrams.

The most common deflection results can be found in tables.

Beam Load and Support	**Actual Deflection***
 (a) Uniform load, simple span	$\Delta_{max} = \dfrac{5\omega L^4}{384EI}$ (at the centerline)
 (b) Concentrated load at midspan	$\Delta_{max} = \dfrac{PL^3}{48EI}$ (at the centerline)

FYI, here's where that crazy $(5wL^4)/(384EI)$ comes from!



(continued on next page)

Here's where that crazy $(5wL^4)/(384EI)$ comes from!

$$\Delta(x) = -\frac{1}{EI}\int dx \int M(x)\,dx$$

$$= -\frac{w}{2EI}\int dx \int (Lx - x^2)\,dx = -\frac{w}{2EI}\int dx\left(\frac{Lx^2}{2} - \frac{x^3}{3} + C_1\right)$$

$$= -\frac{w}{2EI}\left(\frac{Lx^3}{6} - \frac{x^4}{12} + C_1 x + C_2\right)$$

$$\Delta(0) = 0 \Rightarrow \boxed{C_2 = 0}$$

$$\Delta(L) = 0 \Rightarrow \frac{L^4}{6} - \frac{L^4}{12} + C_1 L = 0 \Rightarrow \boxed{C_1 = -\frac{L^3}{12}}$$

$$\Delta(x) = -\frac{w}{2EI}\left(\frac{Lx^3}{6} - \frac{x^4}{12} - \frac{L^3 x}{12}\right)$$

$$\Delta_{max} = \Delta\left(\frac{L}{2}\right) = -\frac{w}{2EI}\left(\frac{L(L/2)^3}{6} - \frac{(L/2)^4}{12} - \frac{L^3(L/2)}{12}\right)$$

$$= -\frac{wL^4}{2EI}\left(\frac{1}{48} - \frac{1}{192} - \frac{1}{24}\right) = -\frac{wL^4}{2EI}\left(-\frac{5}{192}\right) = \boxed{\frac{5wL^4}{384EI}}$$

The 2 integration constants can be tricky. Simply supported: $\Delta(0) = \Delta(L) = 0$. (For cantilever, $\Delta(0) = \Delta'(0) = 0$ instead.)

Maximum deflection is one of several beam-design criteria. You can see now how it relates to the load and $M(x)$ diagrams: $\Delta_{\max}$ comes from integrating $M(x)/(EI)$ twice w.r.t. x to get $\Delta(x)$.

For point load P at the end of a cantilever (for example), you get

$$\Delta_{\max} = \frac{PL^3}{3EI}$$

For uniform load w on simply-supported beam, you get

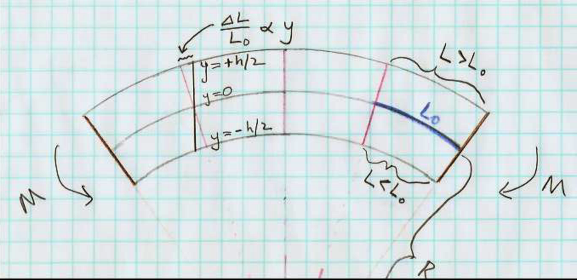
$$\Delta_{\max} = \frac{5wL^4}{384EI}$$

You just look these results up, or use a computer to calculate them.

Deflection is proportional to load, and inversely proportional to Young's modulus and to the second moment of area.

- More load $\rightarrow$ more deflection
- Stiffer material $\rightarrow$ less deflection
- Cross-section with larger $I = \int y^2 \, \mathrm{d}A$ $\rightarrow$ less deflection

Another beam-design criterion is maximum bending stress: the fibers farthest from the neutral surface experience the largest tension or compression, hence largest bending stress.

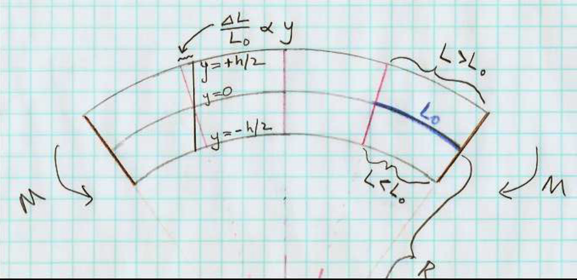


When we section the beam at x, bending moment $M(x)$ is

$$M = \frac{EI}{R}$$

which we can solve for the radius of curvature $R = EI/M$. Then the stress a distance y above the neutral surface is

$$f = Ee = E\,\frac{y}{R} = \frac{E\,y}{(EI/M)} = \frac{M\,y}{I}$$



The bending stress a distance y above the neutral surface is

$$f = \frac{M\,y}{I}$$

The largest bending stress happens in the fibers farthest above or below the neutral surface. Call this largest distance $y_{\mathrm{max}} \equiv c$.

$$f_{\mathrm{max}} = \frac{M\,c}{I} = \frac{M}{(I/c)} = \frac{M}{S}$$

The ratio $S = I/c$ is called "section modulus."

Bending stress in fibers farthest from neutral surface:

$$f_{\max} = \frac{M}{(I/c)} = \frac{M}{S}$$

So you sketch your load, V, and M diagrams, and you find $M_{\max}$, i.e. the largest magnitude of $M(x)$.

Then, the material you are using for beams (wood, steel, etc.) has a maximum allowable bending stress, F_b.

So then you look in your table of beam cross-sections and choose

$$S \geq S_{\text{required}} = \frac{M_{\max}}{F_b}$$

Beam criteria:

- Normal stress in the extreme fibers of the beam (farthest from neutral surface) must be smaller than the allowable bending stress, F_b, which depends on the material (wood, steel, etc.).
- This happens where $M(x)$ has largest magnitude.
- Shear stress (in both y ("transverse") and x ("longitudinal") must be smaller than the allowable shear stress, F_v, which is also a property of the material (wood, steel, etc.).
- This happens where $V(x)$ has largest magnitude, and (surprisingly) is largest near the neutral surface.
- The above two are "strength" criteria. The third one is a "stiffness" criterion:
- The maximum deflection under load must satisfy the building code: typically $\Delta y_{\mathrm{max}} < L/360$.
- For a uniform load, this happens farthest away from the supports. If deflection is too large, plaster ceilings develop cracks, floors feel uncomfortably bouncy or sloped.
- The book also notes longitudinal buckling as a failure mode.

Maximum deflection is one of several beam-design criteria. $\Delta_{\max}$ comes from integrating $M(x)/(EI)$ twice w.r.t. x to get $\Delta(x)$.

For uniform load w on simply-supported beam, you get

$$\Delta_{\max} = \frac{5wL^4}{384EI}$$

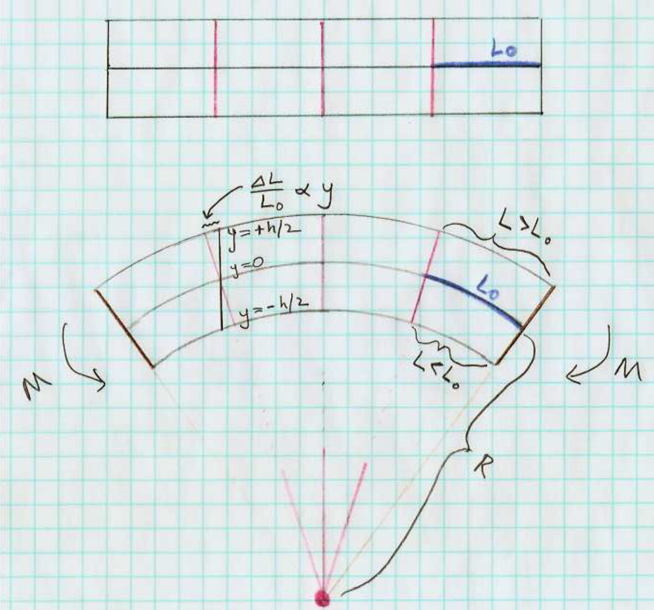
You just look these results up, or use a computer to calculate them. But I had great fun calculating the $5/384$ myself!

Deflection is proportional to load, and inversely proportional to Young's modulus and to the second moment of area.

- More load $\rightarrow$ more deflection
- Stiffer material $\rightarrow$ less deflection
- Cross-section with larger $I = \int y^2 \, \mathrm{d}A$ $\rightarrow$ less deflection

Notice that putting a column in the middle of a long, uniformly loaded beam reduces $\Delta_{\max}$ by a factor of $2^4 = 16$. Alternatively, if you want to span a large, open space without intermediate columns or bearing walls, you need beams with large I.

Bending beam into circular arc of radius R gives $\boxed{e = \frac{\Delta L}{L_0} = \frac{y}{R}}$, strain e vs. distance y above the neutral surface.



Hooke's Law $\boxed{f = Ee}$

gives stress $\boxed{f = \frac{E\,y}{R}}$

Torque exerted by fibers of beam is

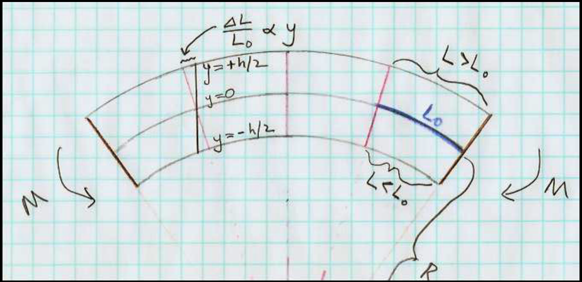
$$M = \int y\,(f\,\mathrm{d}A) =$$

$$y\,\frac{E\,y}{R}\,\mathrm{d}A = \frac{E}{R}\,y^2\,\mathrm{d}A$$

$$\boxed{M = \frac{EI}{R}}$$

Eliminate $R \Rightarrow$

$$\boxed{f = \frac{M\,y}{I} = \frac{M}{I/y}}$$



The bending stress a distance y above the neutral surface is

$$f = \frac{M\,y}{I}$$

The largest bending stress happens in the fibers farthest above or below the neutral surface. Call this largest distance $y_{\max} \equiv c$.

$$f_{\max} = \frac{M_{\max}\,c}{I} = \frac{M_{\max}}{(I/c)} = \frac{M_{\max}}{S}$$

The ratio $S = I/c$ is called "section modulus." The load diagram gives you $M_{\max}$. Each material (wood, steel, etc.) has allowed bending stress $f_{\max}$. Then $S_{\min}$ tells you how big a beam you need.

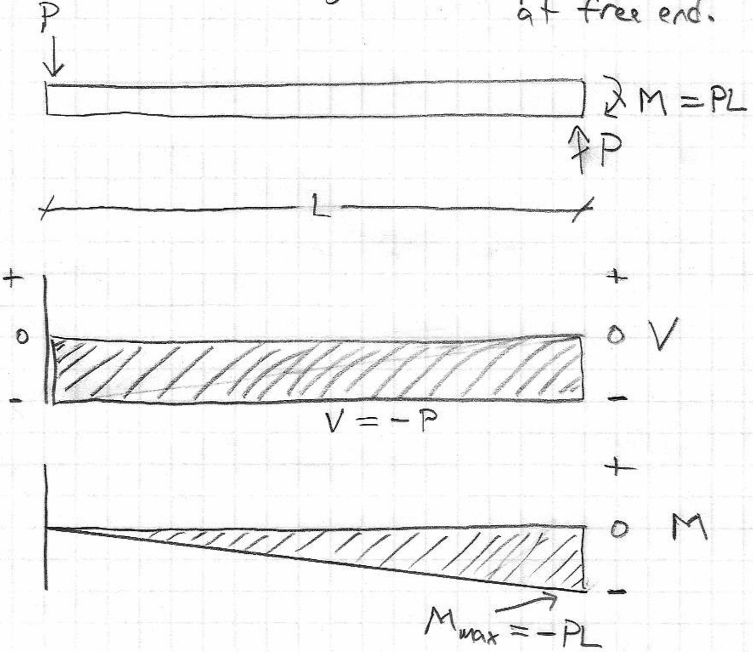
Example (using metric units!): A cantilever beam has a span of 3.0 m with a single concentrated load of 100 kg at its unsupported end. If the beam is made of timber having allowable bending stress $F_b = 1.1 \times 10^7\ \mathrm{N/m^2}$ (was 1600 psi in US units), what minimum section modulus is required?

What is the smallest "2×" dimensional lumber (width = 1.5 inch = 0.038 m) whose cross-section satisfies this strength criterion?

Would this beam also satisfy a $\Delta_{\mathrm{max}} < L/240$ (maximum deflection) stiffness criterion? If not, what standard "2×" cross-section is needed instead?

$\Delta_{\mathrm{max}} = PL^3/(3EI)$ for a cantilever with concentrated load at end. Use Young's modulus $E = 1.1 \times 10^{10}\ \mathrm{N/m^2}$ for southern pine.

Cantilever of length L with point load P at free end.



$$S_{\min} = \frac{|M_{\max}|}{f_{\text{allowed}}} = \frac{PL}{F_b} = \frac{(980\,\mathrm{N})(3\,\mathrm{m})}{1.1 \times 10^7\,\mathrm{N/m^2}} = 26.7 \times 10^{-5}\,\mathrm{m}^3$$

$$\Delta_{\text{allowed}} = \frac{L}{240} = \frac{3.0\,\mathrm{m}}{240} = 0.0125\ \mathrm{m}$$

$$\Delta_{\max} = \frac{PL^3}{3EI} \quad \Rightarrow \quad I_{\min} = \frac{PL^3}{3E\Delta_{\text{allowed}}} = 64.2 \times 10^{-6}\,\mathrm{m}^4$$

I worked out b, h, I, and $S = I/c$ values in metric units for standard "2×" dimensional lumber.

	b	b	h	h	$I = bh^3/12$	$S = bh^2/6$
2×4	1.5 in	.038 m	3.5 in	.089 m	$2.23 \times 10^{-6}\ \mathrm{m}^4$	$5.02 \times 10^{-5}\ \mathrm{m}^3$
2×6	1.5 in	.038 m	5.5 in	.140 m	$8.66 \times 10^{-6}\ \mathrm{m}^4$	$12.4 \times 10^{-5}\ \mathrm{m}^3$
2×8	1.5 in	.038 m	7.5 in	.191 m	$21.9 \times 10^{-6}\ \mathrm{m}^4$	$23.0 \times 10^{-5}\ \mathrm{m}^3$
2×10	1.5 in	.038 m	9.5 in	.241 m	$44.6 \times 10^{-6}\ \mathrm{m}^4$	$37.0 \times 10^{-5}\ \mathrm{m}^3$
2×12	1.5 in	.038 m	11.5 in	.292 m	$79.1 \times 10^{-6}\ \mathrm{m}^4$	$54.2 \times 10^{-5}\ \mathrm{m}^3$

The numbers are nicer if you use centimeters instead of meters, but then you have the added hassle of remembering to convert back to meters in calculations.

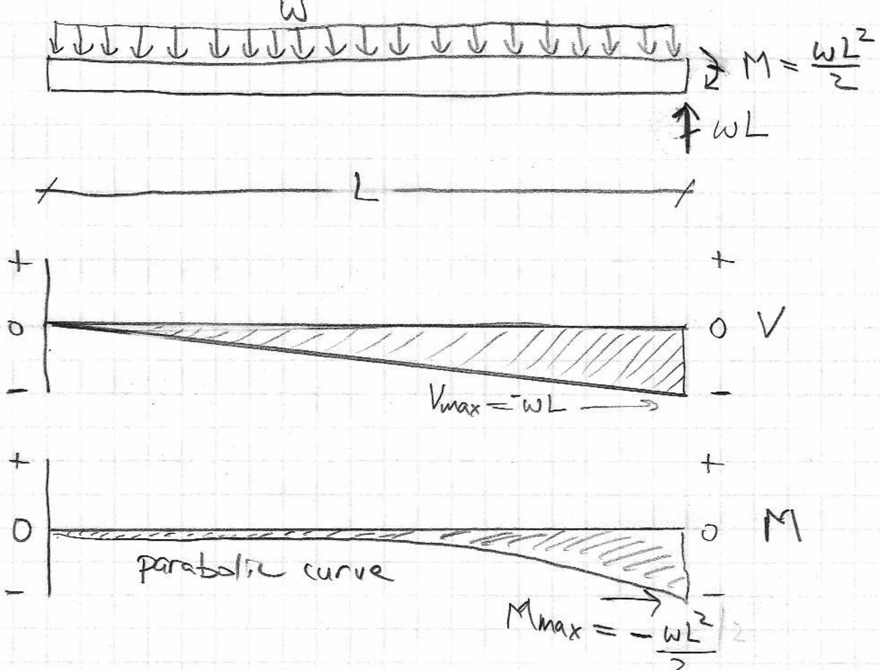
	b	b	h	h	$I = bh^3/12$	$S = bh^2/6$
2×4	1.5 in	3.8 cm	3.5 in	8.9 cm	$223\ \mathrm{cm}^4$	$50.2\ \mathrm{cm}^3$
2×6	1.5 in	3.8 cm	5.5 in	14.0 cm	$866\ \mathrm{cm}^4$	$124\ \mathrm{cm}^3$
2×8	1.5 in	3.8 cm	7.5 in	19.1 cm	$2195\ \mathrm{cm}^4$	$230\ \mathrm{cm}^3$
2×10	1.5 in	3.8 cm	9.5 in	24.1 cm	$4461\ \mathrm{cm}^4$	$370\ \mathrm{cm}^3$
2×12	1.5 in	3.8 cm	11.5 in	29.2 cm	$7913\ \mathrm{cm}^4$	$542\ \mathrm{cm}^3$

Minor variation on same problem: A cantilever beam has a span of 3.0 m with a uniform distributed load of 33.3 $\mathrm{kg/m}$ along its entire length. If we use timber with allowable bending stress $F_b = 1.1 \times 10^7\ \mathrm{N/m^2}$, what minimum section modulus is required?

What is the smallest "2×" dimensional lumber (width $=$ 1.5 inch $=$ 0.038 m) whose cross-section satisfies this strength criterion?

Would this beam also satisfy a $\Delta_{\max} < L/240$ (maximum deflection) stiffness criterion? If not, what standard "2×" cross-section is needed instead?

$\Delta_{\max} = wL^4/(8EI)$ for a cantilever with uniform load. Use Young's modulus $E = 1.1 \times 10^{10}\ \mathrm{N/m^2}$ for southern pine.


w
M = wL²/2
wL
L
+
0
−
V
Vmax = −wL
M
parabolic curve
Mmax = −wL²/2

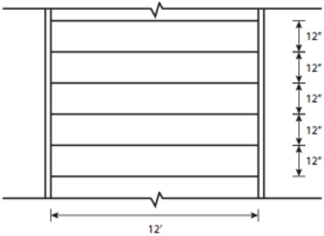
$$S_{\mathrm{min}} = \frac{|M_{\mathrm{max}}|}{f_{\mathrm{allowed}}} = \frac{wL^2/2}{F_b} = \frac{(326\,\mathrm{N/m})(3\,\mathrm{m})^2/2}{1.1 \times 10^7\,\mathrm{N/m^2}} = 13.3 \times 10^{-5}\,\mathrm{m}^3$$

$$\Delta_{\mathrm{allowed}} = \frac{L}{240} = \frac{3.0\,\mathrm{m}}{240} = 0.0125\ \mathrm{m}$$

$$\Delta_{\mathrm{max}} = \frac{wL^4}{8EI} \quad \Rightarrow \quad I_{\mathrm{min}} = \frac{wL^4}{8E\Delta_{\mathrm{allowed}}} = 24.0 \times 10^{-6}\,\mathrm{m}^4$$

2) Size a wood joist for a row house floor which spans 12 feet. Joists are spaced at 16 inches on center.

f = 1,300 psi
f = 85 psi
E = 1.7 x 10 psi
LL = 60 psf
DL = 30 psf



Plan View

Hint: remember that a "2 x 4" wood joist is only nominal; its true dimensions are "1.5 x 3.5" inches. (4 = 1.5, 6 = 5.5, 8 = 7.25, 10 = 9.25 inches)

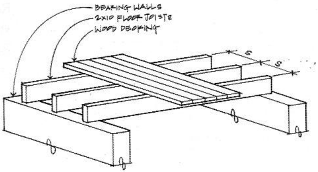
(Here's a homework problem from ARCH 435.)

Actually, Home Depot's 2×10 really is 9.5 inches deep, not 9.25 inches, and 2×12 really is 11.5 inches deep.

A timber floor system uses joists made of "2×10" dimensional lumber. Each joist spans a length of 4.27 m (simply supported). The floor carries a load of 2400 $\mathrm{N/m^2}$. At what spacing should the joists be placed, in order not to exceed allowable bending stress $F_b = 10000\ \mathrm{kN/m^2}$ $(1.0 \times 10^7\ \mathrm{N/m^2})$?

(We should get an answer around 24 inches = 0.61 meters.)

8.6 A timber floor system utilizing 2 × 10 S4S joists spans a length of 14′ (simply supported). The floor carries a load of 50 psf (DL + LL). At what spacing should the joists be placed? Assume Douglas Fir–Larch No. 2 (F_b = 1,450 psi).



Solution:

Based on the allowable stress criteria:

$$f = \frac{Mc}{I} = \frac{M}{S}$$

$$M_{max} = S \times f_b = (21.4 \text{ in.}^3)(1.45 \text{ k/in.}^3) = 31 \text{ k-in.}$$

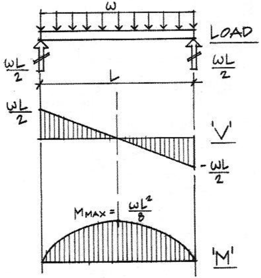
$$M = \frac{31 \text{ k-in.}}{12 \text{ in./ft.}} = 2.58 \text{ k-ft.}$$

Based on the bending moment diagram:

$$M_{max} = \frac{\omega L^2}{8}$$

Therefore,

$$\omega = \frac{8M}{L^2}$$



Substituting for M obtained previously,

$$\omega = \frac{8(2.58 \text{ k-ft.})}{(14')^2} = 0.105 \text{ k/ft.} = 105 \text{ \#/ft.}$$

But

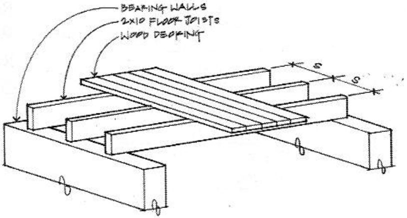
$$\omega = \text{\#/ft.}^2 \times \text{tributary width (joist spacing } s)$$

$$s = \frac{\omega}{50 \text{ psf}} = \frac{105 \text{ \#/ft.}}{50 \text{ \#/ft.}^2} = 2.1'$$

$$s = 25'' \text{ spacing}$$

Use 24″ o.c. spacing.

8.6 A timber floor system utilizing 2 × 10 S4S joists spans a length of 14′ (simply supported). The floor carries a load of 50 psf (DL + LL). At what spacing should the joists be placed? Assume Douglas Fir–Larch No. 2 (F_b = 1,450 psi).



Solution:

Based on the allowable stress criteria:

$$f = \frac{Mc}{I} = \frac{M}{S}$$

$$M_{max} = S \times f_b = (21.4 \text{ in.}^3)(1.45 \text{ k/in.}^3) = 31 \text{ k-in.}$$

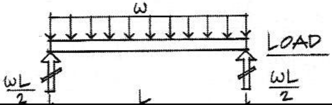
$$M = \frac{31 \text{ k-in.}}{12 \text{ in./ft.}} = 2.58 \text{ k-ft.}$$

Based on the bending moment diagram:

$$M_{max} = \frac{\omega L^2}{8}$$

Therefore,

$$\omega = \frac{8M}{L^2}$$



Based on the bending moment diagram:

$$M_{\max} = \frac{\omega L^2}{8}$$

Therefore,

$$\omega = \frac{8M}{L^2}$$

Substituting for M obtained previously,

$$\omega = \frac{8(2.58 \text{ k-ft.})}{(14')^2} = 0.105 \text{ k/ft.} = 105 \text{ \#/ft.}$$

But

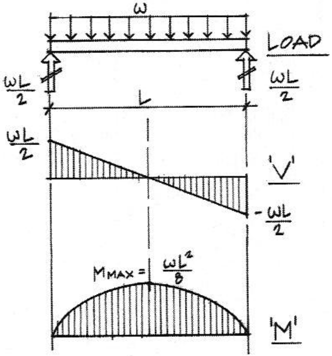
$$\omega = \text{\#/ft.}^2 \times \text{tributary width (joist spacing } s\text{)}$$

$$s = \frac{\omega}{50 \text{ psf}} = \frac{105 \text{ \#/ft.}}{50 \text{ \#/ft.}^2} = 2.1'$$

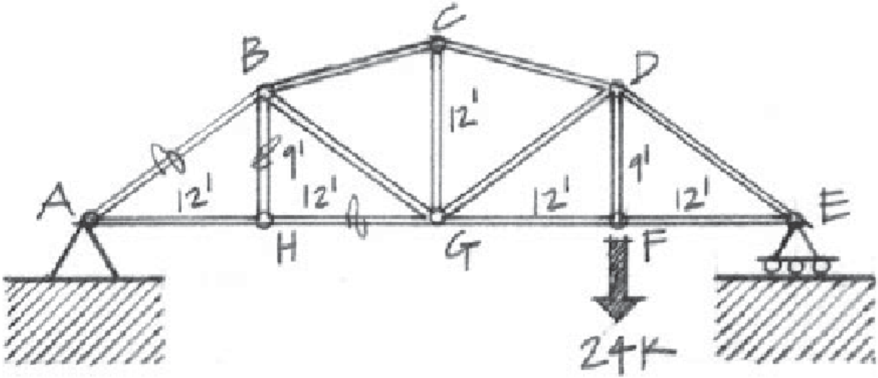
$s = 25''$ spacing

Use 24″ o.c. spacing.

Note: *Spacing is more practical for plywood subflooring, based on a 4 ft. module of the sheet.*

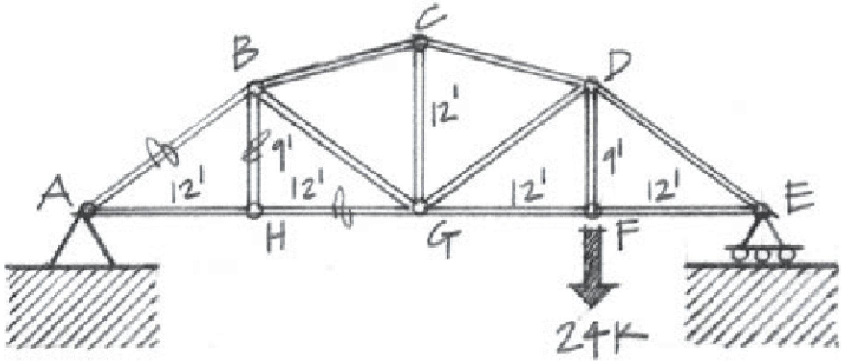


Using the method of sections, solve for the forces in members AB, BH, and HG in the truss shown below. Indicate whether each of these members is in tension or in compression. Use only one section cut through the truss.



If we have time left, let's solve this truss problem together. It's actually pretty quick, using method of sections. First solve for vertical support force at A, then analyze left side of section.

Using the method of sections, solve for the forces in members AB, BH, and HG in the truss shown below. Indicate whether each of these members is in tension or in compression. Use only one section cut through the truss.



XC2. **(I haven't checked this with anyone else yet.)** For the truss as a whole $\sum F_x = 0$ gives $\boxed{R_{Ax} = 0}$. Then $\sum M_A = 0 = R_{Ey}(48') - 24\text{k}(36')$ gives $\boxed{R_{Ey} = 18\text{k}}$. Then $\sum F_y = 0 = R_{Ay} + R_{Ey} - 24\text{k}$ gives $\boxed{R_{Ay} = 6\text{k}}$. Now section the truss through members AB, BH, and GH and analyze the left-hand side. Then $\sum M_A = 0 = T_{BH}(12')$ gives $\boxed{T_{BH} = 0}$, which one can see by inspection of the vertical forces at joint H: bar BH is a "zero-force member." Then $\sum F_y = 0 = +6\text{k} + (3/5)T_{AB} + T_{BH}$ gives $\boxed{T_{AB} = -10\text{k}}$ (i.e. compression). Finally, $\sum F_x = 0 = (4/5)T_{AB} + T_{GH}$ gives $\boxed{T_{GH} = 8\text{k}}$.

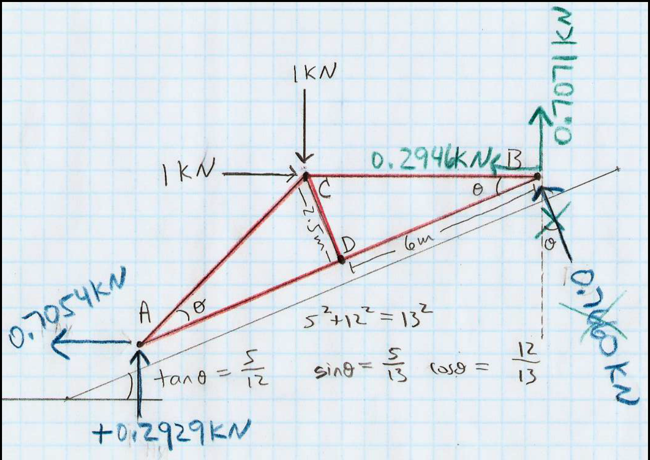
Physics 8 — Monday, November 18, 2019

- HW11 is “due” on Friday, but you can turn it in on Monday, Nov 25, just in case it takes us an extra day to get through the material on beams.
- Last week, you read Ch6 (cross-sectional properties) and Ch7 (simple beams). This week, read Ch8 (more about beams).

Physics 8 — Wednesday, November 20, 2019

- HW11 is "due" on Friday, but you can turn it in on Monday, Nov 25, as it will probably take us an extra day to get through the material on beams.
- HW help: (Bill) Wed 4-6:30pm DRL 3C4, (Brooke/Grace) Thu 6-8pm DRL 2C4.
- This week, read/skim O/K Ch8 (more about beams).
- You may find my "equation sheet" to be a helpful summary of the key results from the Onouye/Kane reading:

`http://positron.hep.upenn.edu/p8/files/equations.pdf#page=12`



Here's what puzzled me about this statics problem last week: You can evaluate the moment (about A) due to the horizontal 1 kN force in two ways — with the same result, of course:

- how we did it: $-1\,\mathrm{kN} \times 6.5\,\mathrm{m}\sin(2\theta) = 4.615\,\mathrm{kN}\cdot\mathrm{m}$
- clever, tricker way: $-1\,\mathrm{kN} \times 12\,\mathrm{m}\sin\theta = 4.615\,\mathrm{kN}\cdot\mathrm{m}$

We discovered a geometrical proof that $\sin(2\theta) = 2\cos\theta\sin\theta$.

Last time we worked out $V(x)$ and $M(x)$ for this cantilever beam.

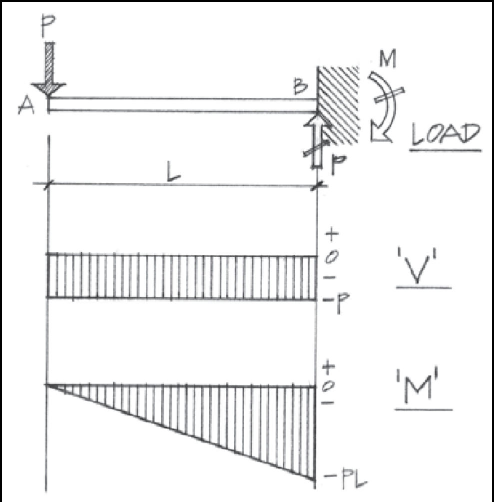
Sign conventions: $V(x) > 0$ when beam left of x is pulling up on beam right of x. $M(x) > 0$ when beam is smiling.

Transverse shear $V(x)$ is the running sum of forces on beam, from $0 \ldots x$, where upward $=$ positive.

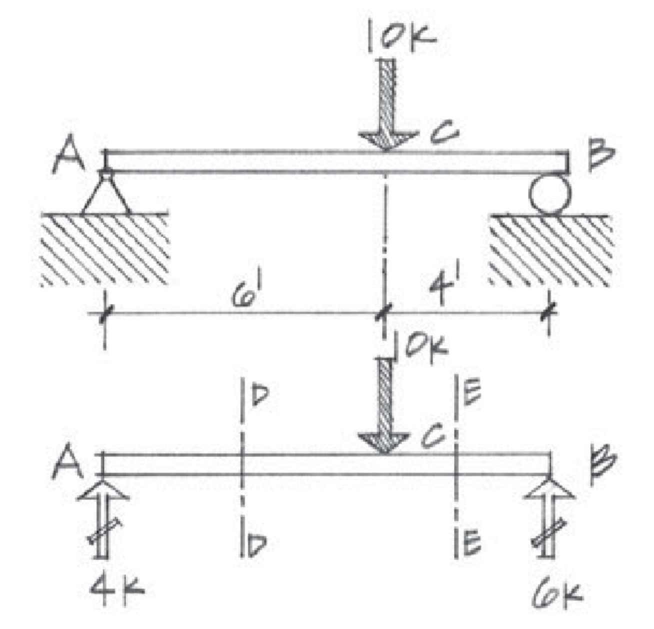
Bending moment $M(x)$ is the torque exerted by each side of the beam, cut at x, on the other side; but beware of sign convention.

$$V(x) = \frac{\mathrm{d}}{\mathrm{d}x} M(x)$$

The V diagram graphs the slope of the M diagram.



Draw V and M for this "simply supported" beam: $V(x)$ is running sum (up − down) of forces on beam. $M > 0$ when beam smiles.

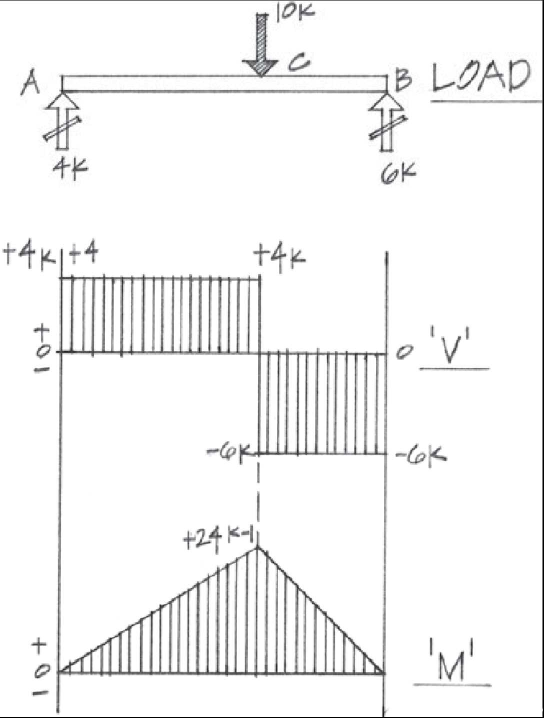


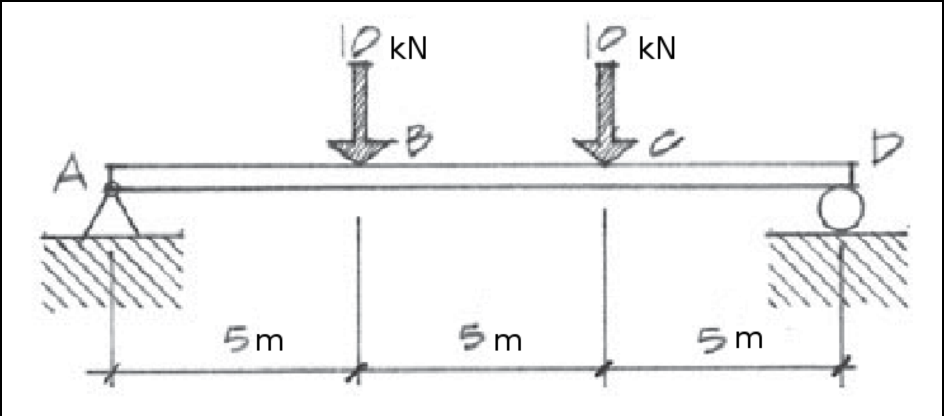
$$V(x) = \frac{d}{dx} M(x)$$

The shear (V) diagram equals the slope of the moment (M) diagram.

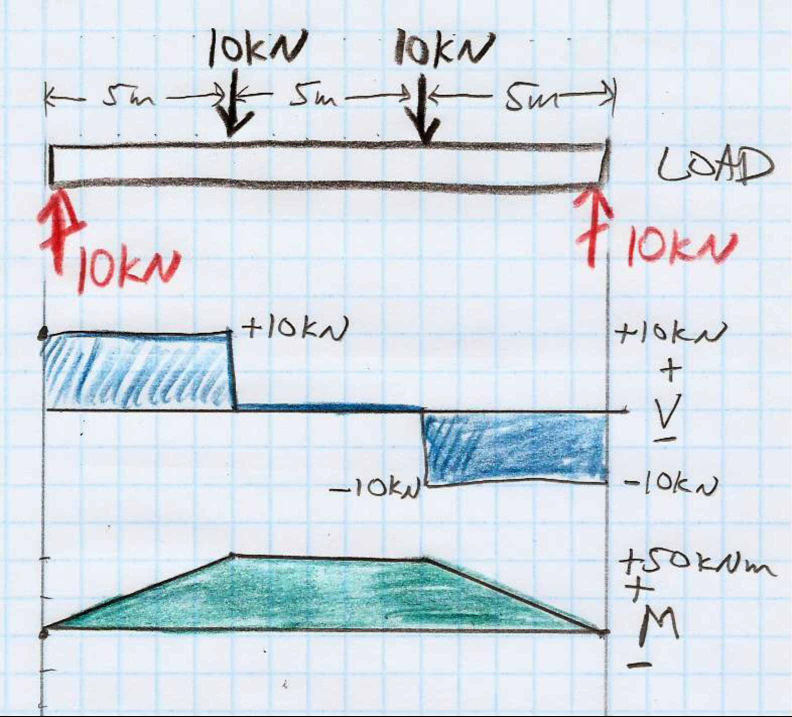
$$M(x) = \int V(x)\,dx$$

But be careful about the M values at the ends — depends how the beam is supported. A free, hinged, or roller-supported end has $M = 0$: support exerts no torque on that end. Fixed end of cantilever has $M \neq 0$.





Let's try drawing load, V, and M diagrams for this simply-supported beam. Pretend the units are meters and kilonewtons rather than the original drawing's feet and kilopounds ("kips").


10kN
10kN
5m
5m
5m
LOAD
10kN
10kN
+10kN
+10kN
+
V
−
−10kN
−10kN
+50kNm
+
M
−

Shear (V) and moment (M) diagrams:

- First draw a "load diagram," which is an EFBD that shows all of the vertical forces acting on the beam.
- The "shear diagram" $V(x)$ graphs the running sum of all vertical forces (both supports and loads) acting on the beam, from the left side up to x, where upward = positive, downward = negative.
- To draw the "moment diagram" $M(x)$, note that V is the slope of M:

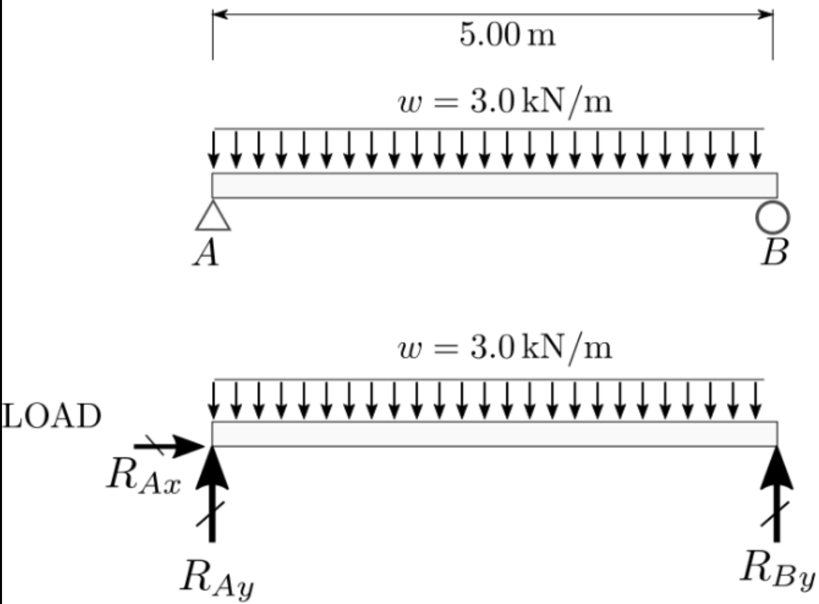
$$V(x) = \frac{\mathrm{d}}{\mathrm{d}x} M(x)$$

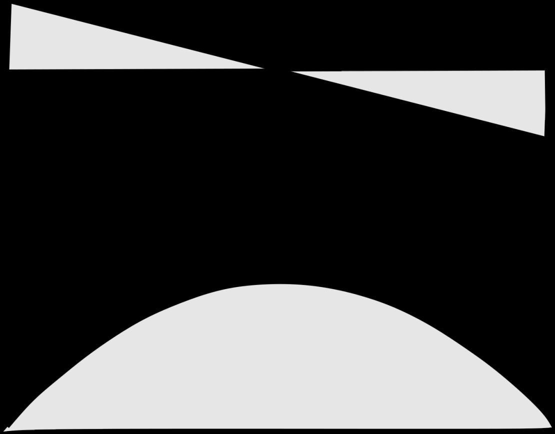
- The change in M from x_1 to x_2 is given by

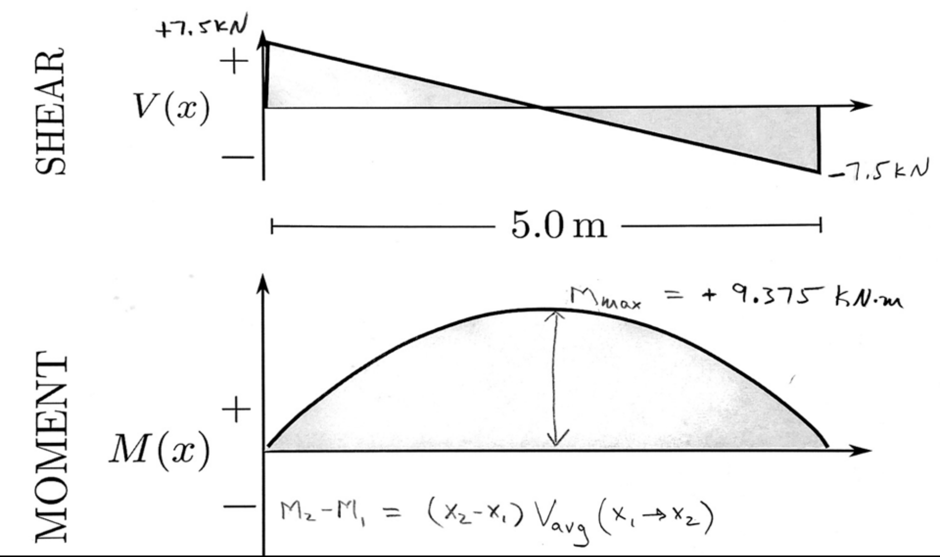
$$M_2 - M_1 = (x_2 - x_1)\, V_{1\to 2}^{\text{average}}$$

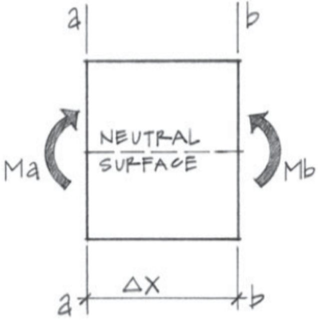
- If an end of a beam is unsupported ("free"), is hinge/pin supported, or is roller supported, then $M = 0$ at that end. You can only have $M \neq 0$ at an end if the support at that end is capable of exerting a torque on the beam — for example, the fixed end of a cantilever has $M \neq 0$.

Let's try drawing $V(x)$ and $M(x)$ diagrams for a simply supported beam with uniform distributed load, as shown.

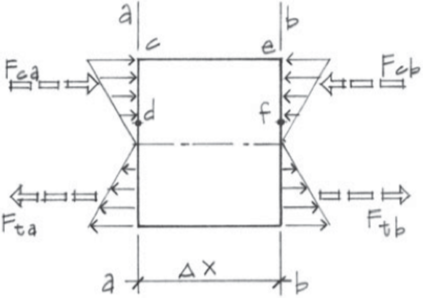





SHEAR
+7.5kN
+
V(x)
−
-7.5kN
5.0 m
MOMENT
Mmax = + 9.375 kN·m
+
M(x)
−
M2 - M1 = (x2 - x1) Vavg (x1 → x2)

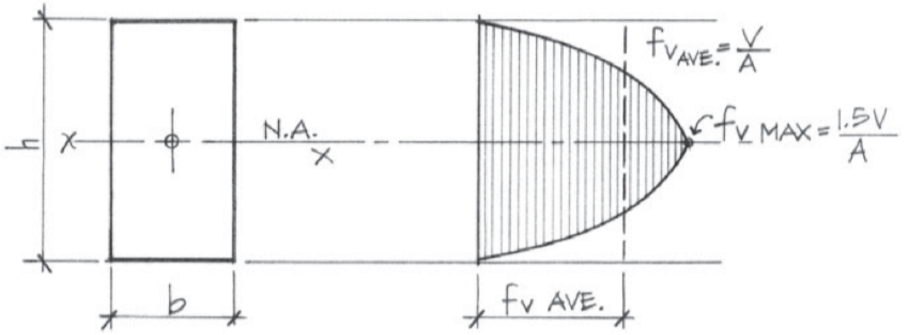


(a) Beam section between sections a *and* b.



(b) Bending stresses on the beam section a-b.

Figure 8.19 Bending stress on a beam section.



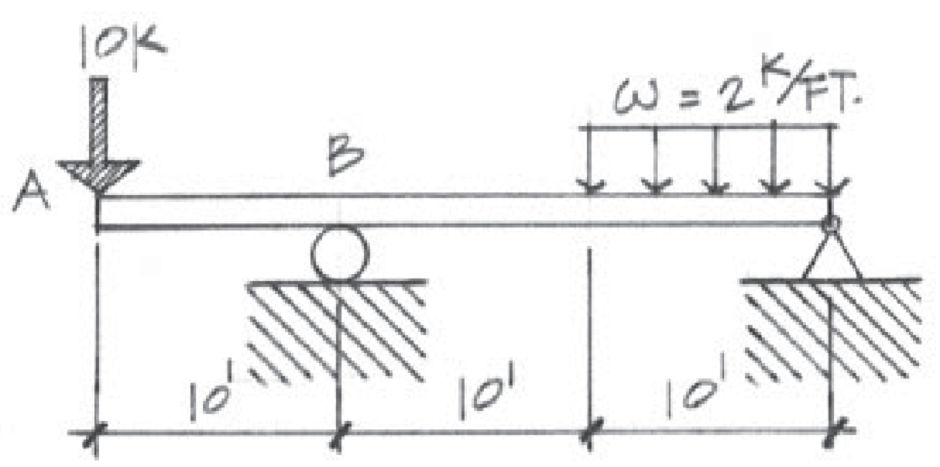
Cross Section.

Shear stress graph of a rectangular cross section.

Figure 8.26 Shear stress distribution—key points.

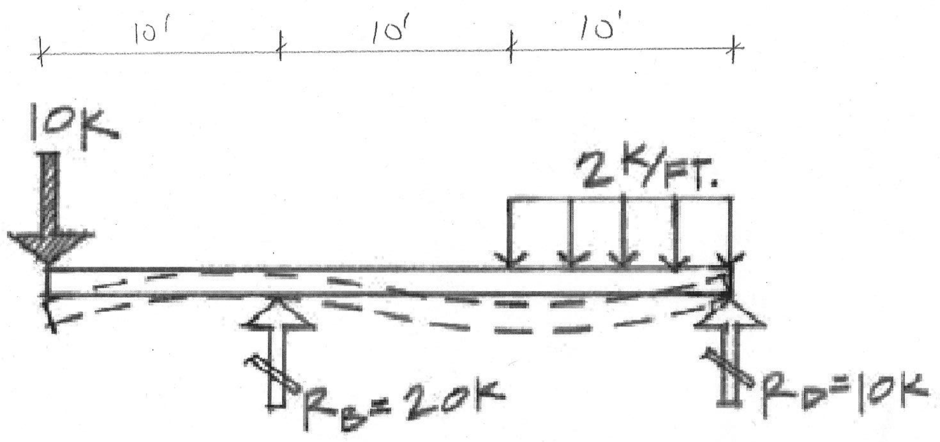
Questions for Prof. Farley!

- How do we explain the variation of shear stress across the cross-section of a beam — for example: where is shear stress largest for a simply supported beam with uniform distributed load, rectangular cross-section?
- Should we add to this course some physics of masonry structures, e.g. a classic Roman arch?
- For design criteria of a structure (O/K ch1), what is meant by redundancy and continuity?
- Z.E. question: how to study moments in complex shapes?
- Any others?!

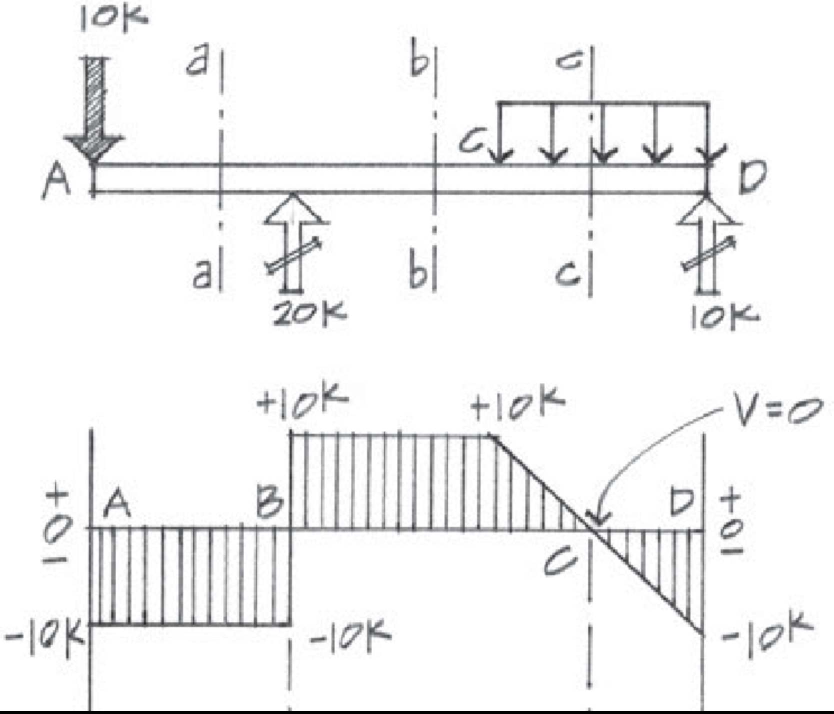


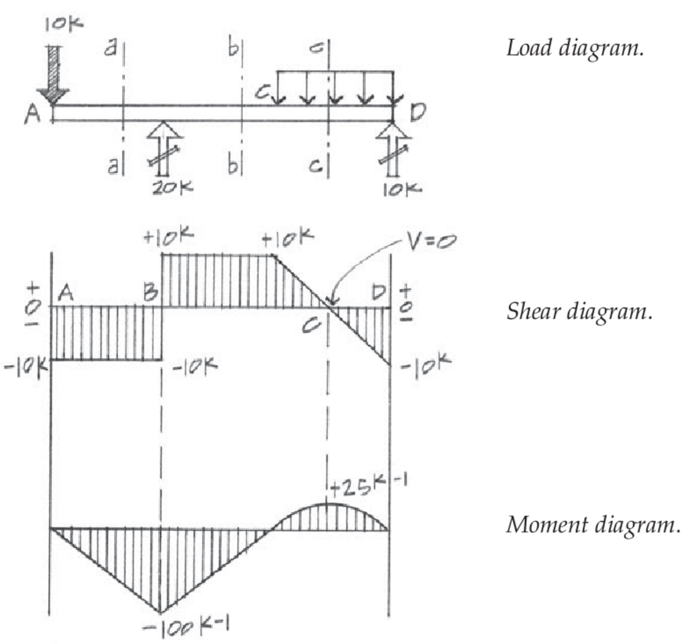
Draw shear (V) and moment (M) diagrams for this beam! Tricky! First one needs to solve for the support ("reaction") forces.

Note: in solving for the support forces, you replace distributed load w with equivalent point load. But when you draw the load diagram to find V and M, you need to keep w in its original form.



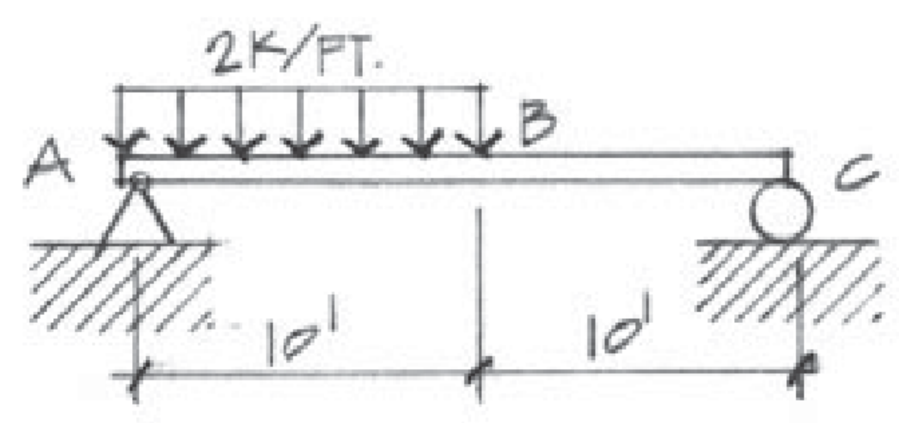
Remember that $V(x)$ is the running sum, from LHS to x, of vertical forces acting on the beam, with upward=positive.

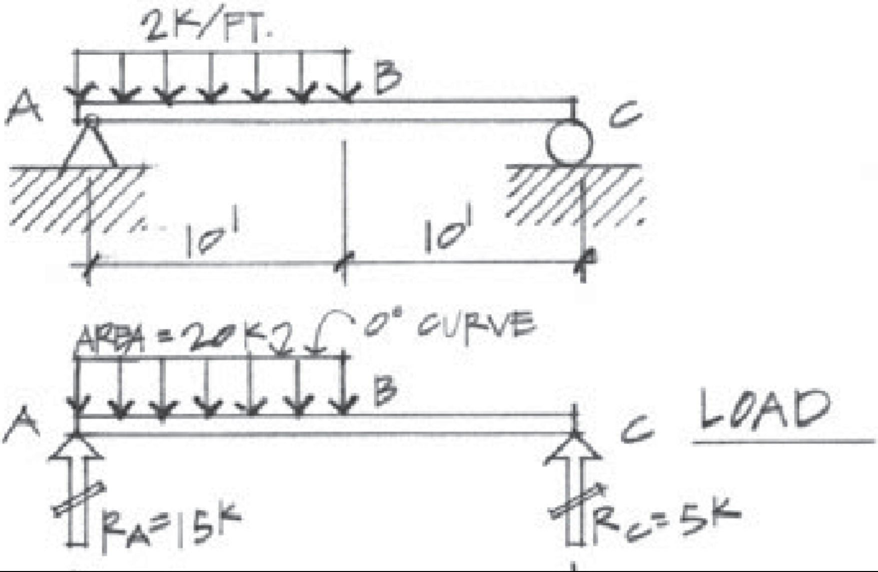

10K
a
b
c
A
D
a
b
c
20K
10K
+10K
+10K
V=0
+
0
−
A
B
D
C
−10K
−10K
−10K



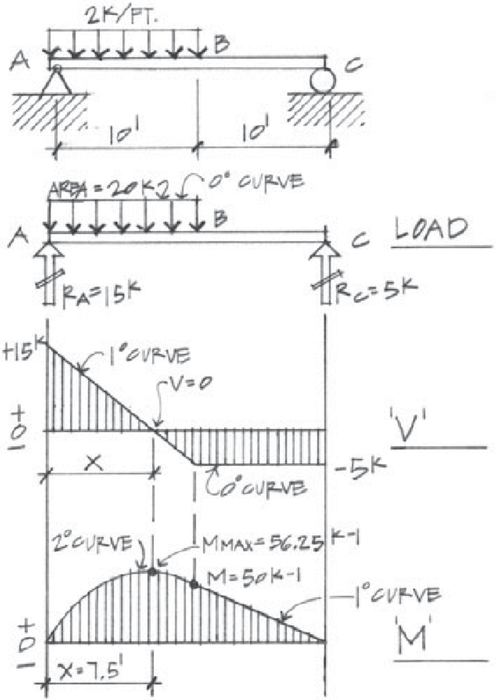
Neat trick: $M_2 - M_1 = (V_{1\to 2}^{\text{average}})(x_2 - x_1)$

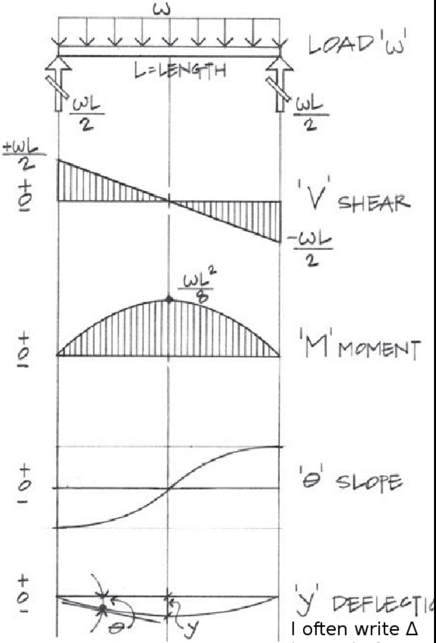
Draw load, V, and M diagrams for this simply supported beam with a partial uniform load.





2K/FT.
A
B
C
10'
10'
AREA = 20K
0° CURVE
A
B
C
LOAD
R_A=15K
R_C=5K

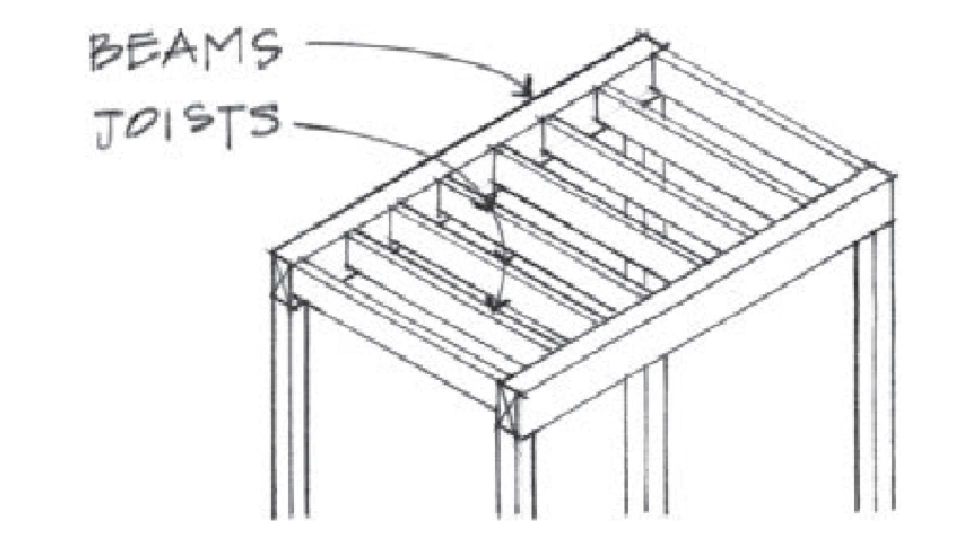

2K/FT.
A
B
C
10'
10'
AREA = 20K
2° CURVE
LOAD
RA=15K
RC=5K
+15K
1° CURVE
V=0
'V'
X
-5K
0° CURVE
2° CURVE
MMAX=56.25 K-1
M=50K-1
1° CURVE
'M'
X=7.5'



I often write Δ instead of y

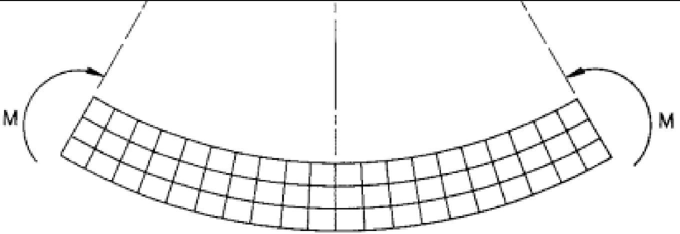
Figure 7.11 Relationship of load, shear, moment, slope, and deflection diagrams.

Why do we care about these beam diagrams, anyway? Usually the floor of a structure must carry a specified weight per unit area. The beams (beams, girders, joists, etc.) must be strong enough to support this load without failing and must be stiff enough to support this load without excessive deflection.



Beam criteria:

- Normal stress in the extreme fibers of the beam (farthest from neutral surface) must be smaller than the allowable bending stress, F_b, which depends on the material (wood, steel, etc.).
- This happens where $M(x)$ has largest magnitude.
- Shear stress (in both y ("transverse") and x ("longitudinal") must be smaller than the allowable shear stress, F_v, which is also a property of the material (wood, steel, etc.).
- This happens where $V(x)$ has largest magnitude, and (surprisingly) is largest near the neutral surface.
- The above two are "strength" criteria. The third one is a "stiffness" criterion:
- The maximum deflection under load must satisfy the building code: typically $\Delta y_{\max} < L/360$.
- For a uniform load, this happens farthest away from the supports. If deflection is too large, plaster ceilings develop cracks, floors feel uncomfortably bouncy or sloped.
- The book also notes buckling as a beam failure mode.

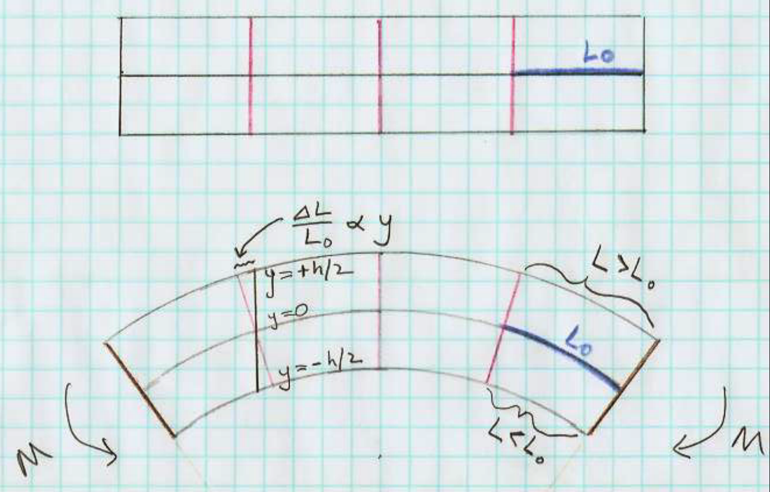


(b)

Fig. 6.5.
Navier's assumption. Originally plane and parallel sections (a) remain plane after bending (b), but converge onto a common center of curvature. This assumption can be illustrated with a rubber beam.

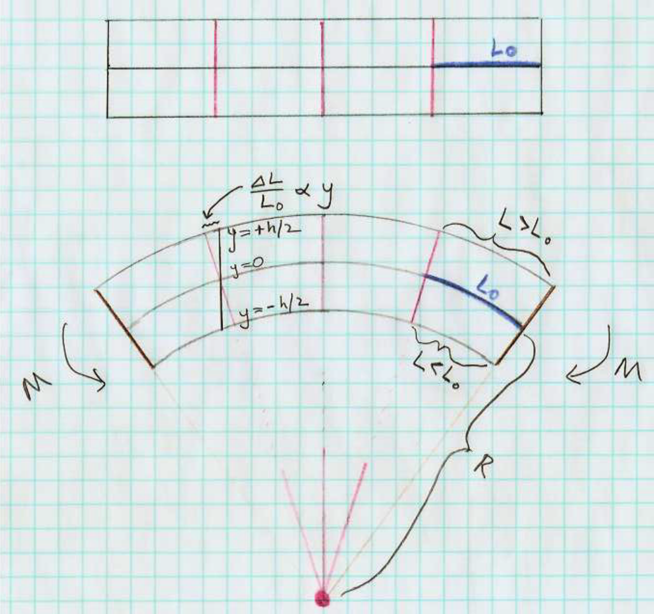
(In this illustration, bottom is in tension, top is in compression, as in a "simply supported" beam.)

A big topic from this week's reading was to see how an initially horizontal beam responds to the bending moment $M(x)$ by deforming into a curved shape.



(In this illustration, top is in tension, as in a cantilever.)

Key idea: bending moment $M \propto \frac{1}{R}$, where R is the radius of curvature of the beam. For constant M, vertical lines converge toward common center of curvature.



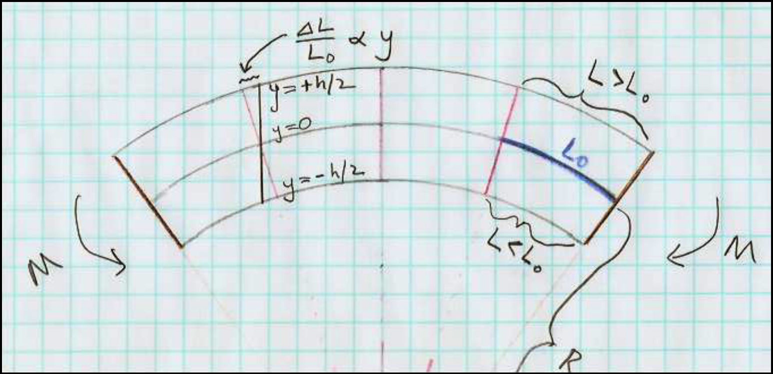
$$\text{strain} = \frac{\Delta L}{L_0} = \frac{y}{R}$$

where $y = 0$ is the neutral surface.

So in this case $y > 0$ is in tension and $y < 0$ is in compression.

If you think of wood fibers running along the beam's axis, then the fibers above the neutral surface ($y > 0$) are stretched in proportion to y, and the fibers below the neutral surface ($y < 0$) are compressed in proportion to $|y|$.

$$\text{strain} = \frac{\Delta L}{L} = \frac{y}{R}$$



Now remember that $\frac{\Delta L}{L}$ is called (axial) *strain*, and force per unit area is called *stress*. For an elastic material, strain $(e) \propto$ stress (f).

$$\frac{\Delta L}{L} = \frac{1}{E} \times \frac{\text{Force}}{\text{Area}} = \frac{1}{E} \times f \qquad e = \frac{1}{E} \times f$$

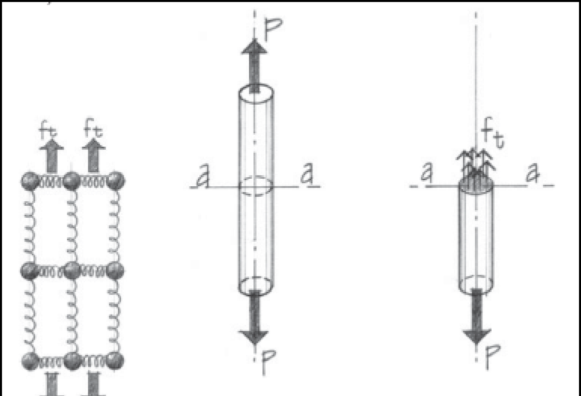
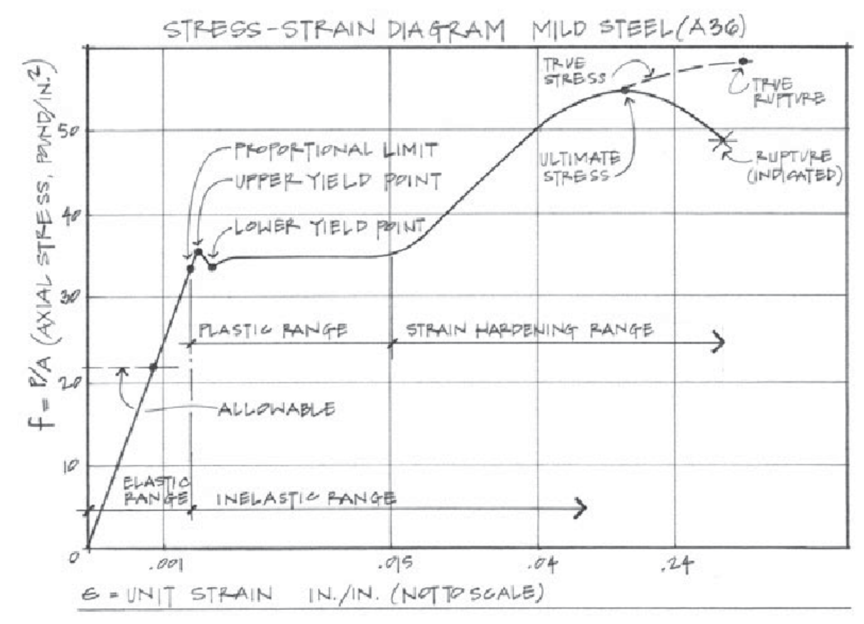


Figure 5.9 *Normal tensile stress through section a-a.*

In the elastic region, strain ($e = \Delta L/L$) is proportional to stress ($f = F/A$). $\boxed{f = Ee}$. The slope E is Young's modulus.

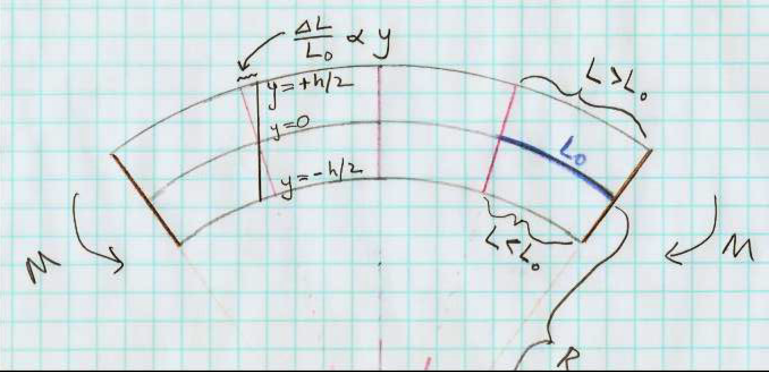


Plugging in $f = Ee$ to the bending-beam diagram:

$$\frac{y}{R} = \frac{\Delta L}{L} = e = \frac{f}{E}$$

we find the force-per-unit area (stress) exerted by the fibers is

$$f = \frac{Ey}{R}$$

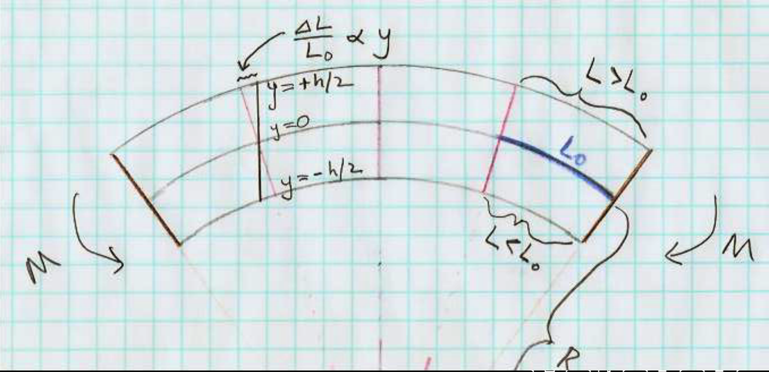


The force-per-unit area (stress) exerted by the fibers is

$$f = \frac{Ey}{R}$$

while the torque (bending moment $\mathrm{d}M$, pivot about N.A.) exerted by each tiny fiber of area $\mathrm{d}A$ is proportional to its lever arm y

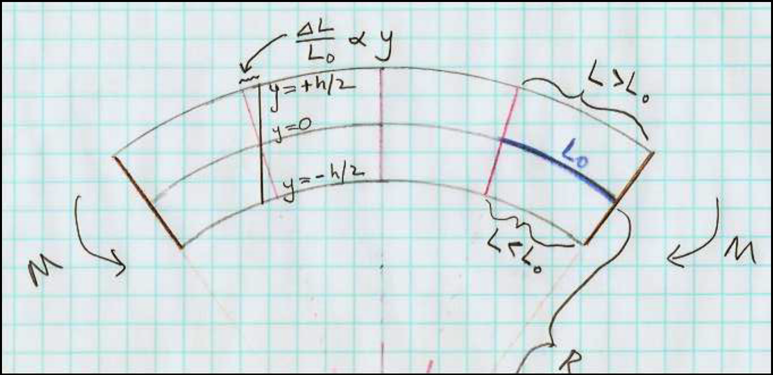
$$\mathrm{d}M = y\ \mathrm{d}F = y\ f\mathrm{d}A = y\left(\frac{Ey}{R}\right)\mathrm{d}A = \frac{E}{R}\ y^2\ \mathrm{d}A$$



So the bending moment M exerted by a curved beam is

$$M = \frac{E}{R} \int y^2 \, \mathrm{d}A = \frac{EI}{R}$$

where R is the curved beam's radius of curvature and $I = \int y^2 \, \mathrm{d}A$ is the "second moment of area" a.k.a. "area moment of inertia."



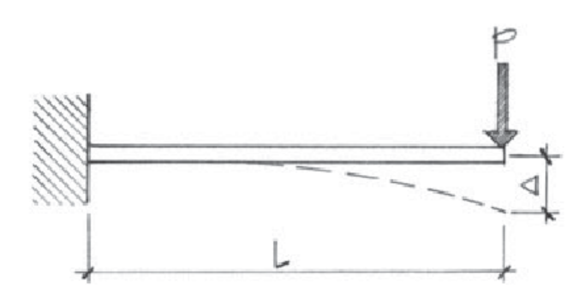
$$M = \frac{E}{R} \int y^2 \mathrm{d}A = \frac{EI}{R}$$

Meanwhile, the vertical *deflection* Δ of a point along the beam is related to its curvature by (in limit where $\Delta \ll R$)

$$\frac{1}{R} \approx \frac{\mathrm{d}^2\Delta}{\mathrm{d}x^2}$$

so you can integrate the $M(x)$ curve twice to get deflection

$$\frac{\mathrm{d}^2\Delta}{\mathrm{d}x^2} = \frac{M}{EI} \quad \Rightarrow \quad \Delta(x) = \frac{1}{EI} \int \mathrm{d}x \int M(x)\, \mathrm{d}x$$



Calculus digression (not important — but you may be curious):

You may have seen in calculus that the "curvature" (which means $1/R$, where R is the radius of curvature) of a function $y = f(x)$ is

$$\frac{1}{R} = \frac{y''}{(1+(y')^2)^{3/2}}$$

We are working in the limit $y' \ll 1$, so

$$\frac{1}{R} \approx y''$$

That's how we arrived at

$$\frac{1}{R} \approx \frac{\mathrm{d}^2\Delta}{\mathrm{d}x^2}$$

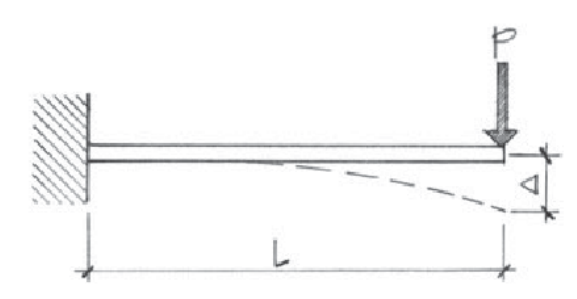
$$M = \frac{E}{R} \int y^2 \mathrm{d}A = \frac{EI}{R}$$

Meanwhile, the vertical *deflection* Δ of a point along the beam is related to its curvature by (in limit where $\Delta \ll R$)

$$\frac{1}{R} \approx \frac{\mathrm{d}^2\Delta}{\mathrm{d}x^2}$$

so you can **integrate the $M(x)$ curve twice to get deflection**

$$\frac{\mathrm{d}^2\Delta}{\mathrm{d}x^2} = \frac{M}{EI} \quad \Rightarrow \quad \boxed{\Delta(x) = \frac{1}{EI} \int \mathrm{d}x \int M(x)\,\mathrm{d}x}$$



This Onouye/Kane figure writes "y" here for deflection, but I wrote "Δ" for deflection on the preceding pages (and they usually do, too), because we were already using y for "distance above the neutral surface."

So you integrate $M(x)/EI$ twice w.r.t. x to get the deflection $\Delta(x)$.

The bending moment $M(x) = EI\ \mathrm{d}^2\Delta/\mathrm{d}x^2$, where E is Young's modulus and I is second moment of area.

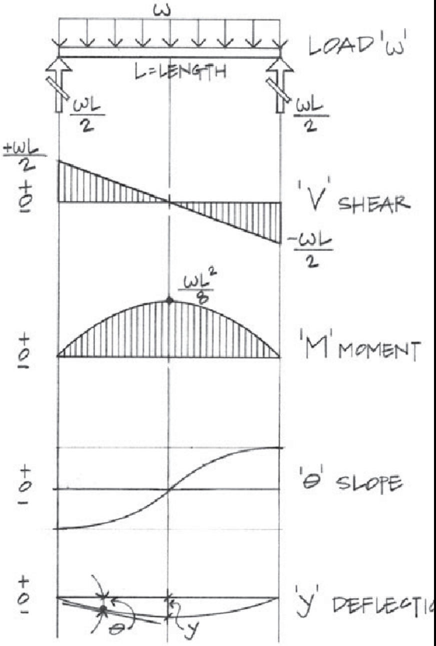
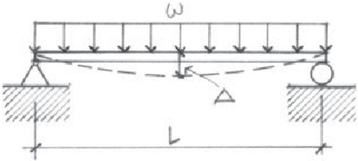
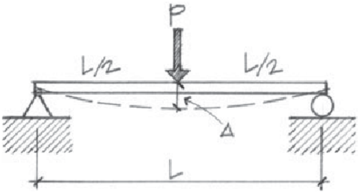
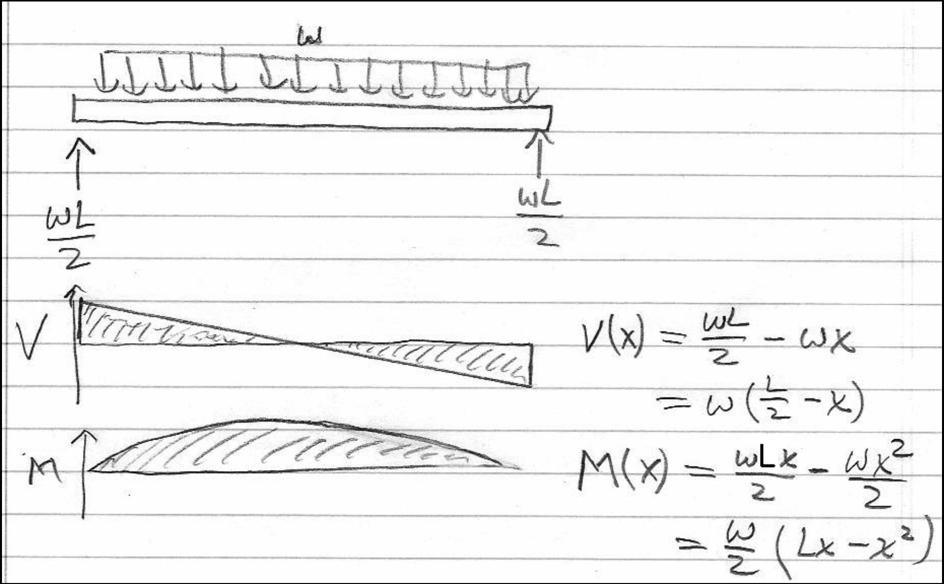


Figure 7.11 Relationship of load, shear, moment, slope, and deflection diagrams.

The most common deflection results can be found in tables.

Beam Load and Support	Actual Deflection*
 (a) Uniform load, simple span	$\Delta_{max} = \frac{5\omega L^4}{384EI}$ (at the centerline)
 (b) Concentrated load at midspan	$\Delta_{max} = \frac{PL^3}{48EI}$ (at the centerline)

FYI, here's where that crazy $(5wL^4)/(384EI)$ comes from!



(continued on next page)

Here's where that crazy $(5wL^4)/(384EI)$ comes from!

$$\Delta(x) = -\frac{1}{EI}\int dx \int M(x)\,dx$$

$$= -\frac{w}{2EI}\int dx \int (Lx - x^2)\,dx = -\frac{w}{2EI}\int dx\left(\frac{Lx^2}{2} - \frac{x^3}{3} + C_1\right)$$

$$= -\frac{w}{2EI}\left(\frac{Lx^3}{6} - \frac{x^4}{12} + C_1 x + C_2\right)$$

$$\Delta(0) = 0 \Rightarrow \boxed{C_2 = 0}$$

$$\Delta(L) = 0 \Rightarrow \frac{L^4}{6} - \frac{L^4}{12} + C_1 L = 0 \Rightarrow \boxed{C_1 = -\frac{L^3}{12}}$$

$$\Delta(x) = -\frac{w}{2EI}\left(\frac{Lx^3}{6} - \frac{x^4}{12} - \frac{L^3 x}{12}\right)$$

$$\Delta_{max} = \Delta\left(\frac{L}{2}\right) = -\frac{w}{2EI}\left(\frac{L(L/2)^3}{6} - \frac{(L/2)^4}{12} - \frac{L^3(L/2)}{12}\right)$$

$$= -\frac{wL^4}{2EI}\left(\frac{1}{48} - \frac{1}{192} - \frac{1}{24}\right) = -\frac{wL^4}{2EI}\left(-\frac{5}{192}\right) = \boxed{\frac{5wL^4}{384EI}}$$

The 2 integration constants can be tricky. Simply supported: $\Delta(0) = \Delta(L) = 0$. (For cantilever, $\Delta(0) = \Delta'(0) = 0$ instead.)

Maximum deflection is one of several beam-design criteria. You can see now how it relates to the load and $M(x)$ diagrams: Δ_{max} comes from integrating $M(x)/(EI)$ twice w.r.t. x to get $\Delta(x)$.

For point load P at the end of a cantilever (for example), you get

$$\Delta_{\mathrm{max}} = \frac{PL^3}{3EI}$$

For uniform load w on simply-supported beam, you get

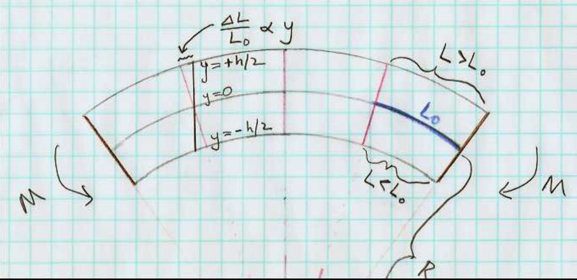
$$\Delta_{\mathrm{max}} = \frac{5wL^4}{384EI}$$

You just look these results up, or use a computer to calculate them.

Deflection is proportional to load, and inversely proportional to Young's modulus and to the second moment of area.

- More load $\rightarrow$ more deflection
- Stiffer material $\rightarrow$ less deflection
- Cross-section with larger $I = \int y^2 \, \mathrm{d}A$ $\rightarrow$ less deflection

Another beam-design criterion is maximum bending stress: the fibers farthest from the neutral surface experience the largest tension or compression, hence largest bending stress.

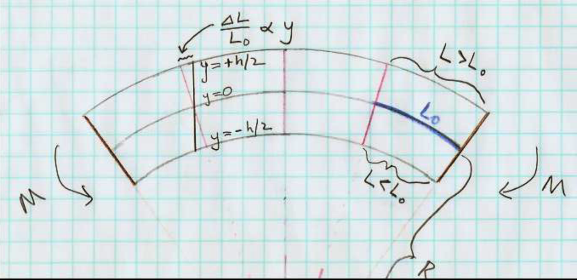


When we section the beam at x, bending moment $M(x)$ is

$$M = \frac{EI}{R}$$

which we can solve for the radius of curvature $R = EI/M$. Then the stress a distance y above the neutral surface is

$$f = Ee = E\,\frac{y}{R} = \frac{E\,y}{(EI/M)} = \frac{M\,y}{I}$$



The bending stress a distance y above the neutral surface is

$$f = \frac{M\,y}{I}$$

The largest bending stress happens in the fibers farthest above or below the neutral surface. Call this largest distance $y_{\text{max}} \equiv c$.

$$f_{\text{max}} = \frac{M\,c}{I} = \frac{M}{(I/c)} = \frac{M}{S}$$

The ratio $S = I/c$ is called "section modulus."

Bending stress in fibers farthest from neutral surface:

$$f_{\max} = \frac{M}{(I/c)} = \frac{M}{S}$$

So you sketch your load, V, and M diagrams, and you find $M_{\max}$, i.e. the largest magnitude of $M(x)$.

Then, the material you are using for beams (wood, steel, etc.) has a maximum allowable bending stress, F_b.

So then you look in your table of beam cross-sections and choose

$$S \geq S_{\text{required}} = \frac{M_{\max}}{F_b}$$

Beam criteria:

- Normal stress in the extreme fibers of the beam (farthest from neutral surface) must be smaller than the allowable bending stress, F_b, which depends on the material (wood, steel, etc.).
- This happens where $M(x)$ has largest magnitude.
- Shear stress (in both y ("transverse") and x ("longitudinal") must be smaller than the allowable shear stress, F_v, which is also a property of the material (wood, steel, etc.).
- This happens where $V(x)$ has largest magnitude, and (surprisingly) is largest near the neutral surface.
- The above two are "strength" criteria. The third one is a "stiffness" criterion:
- The maximum deflection under load must satisfy the building code: typically $\Delta y_{\max} < L/360$.
- For a uniform load, this happens farthest away from the supports. If deflection is too large, plaster ceilings develop cracks, floors feel uncomfortably bouncy or sloped.
- The book also notes longitudinal buckling as a failure mode.

Maximum deflection is one of several beam-design criteria. $\Delta_{\max}$ comes from integrating $M(x)/(EI)$ twice w.r.t. x to get $\Delta(x)$.

For uniform load w on simply-supported beam, you get

$$\Delta_{\max} = \frac{5wL^4}{384EI}$$

You just look these results up, or use a computer to calculate them. But I had great fun calculating the $5/384$ myself!

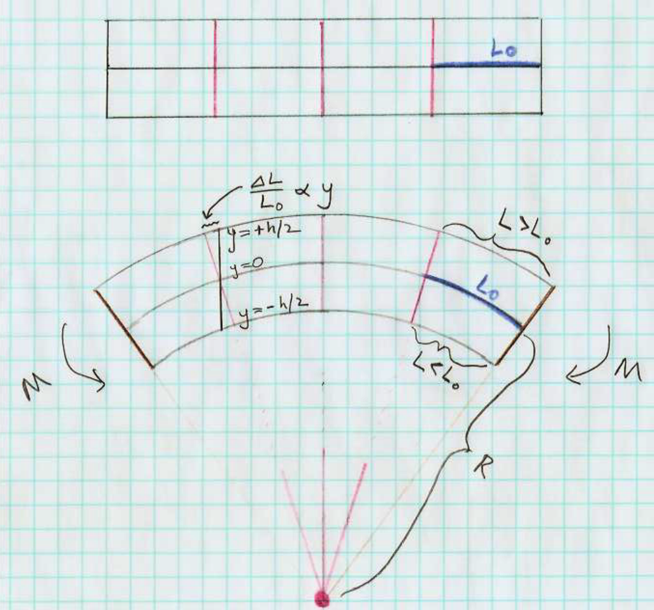
Deflection is proportional to load, and inversely proportional to Young's modulus and to the second moment of area.

- More load $\rightarrow$ more deflection
- Stiffer material $\rightarrow$ less deflection
- Cross-section with larger $I = \int y^2 \, \mathrm{d}A$ $\rightarrow$ less deflection

Notice that putting a column in the middle of a long, uniformly loaded beam reduces $\Delta_{\max}$ by a factor of $2^4 = 16$. Alternatively, if you want to span a large, open space without intermediate columns or bearing walls, you need beams with large I.

Bending beam into circular arc of radius R gives $\boxed{e = \frac{\Delta L}{L_0} = \frac{y}{R}}$,

strain e vs. distance y above the neutral surface.



Hooke's Law $\boxed{f = Ee}$

gives stress $\boxed{f = \frac{E\,y}{R}}$

Torque exerted by fibers of beam is

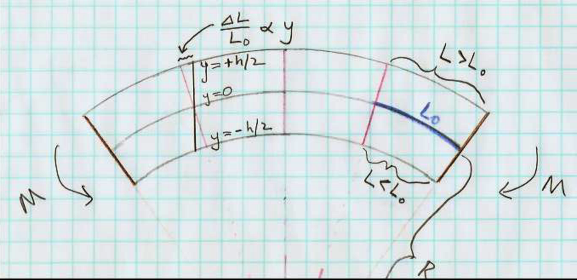
$$M = \int y\,(f\,\mathrm{d}A) =$$

$$y\,\frac{E\,y}{R}\,\mathrm{d}A = \frac{E}{R}\,y^2\,\mathrm{d}A$$

$$\boxed{M = \frac{EI}{R}}$$

Eliminate R $\Rightarrow$

$$\boxed{f = \frac{M\,y}{I} = \frac{M}{I/y}}$$



The bending stress a distance y above the neutral surface is

$$f = \frac{M\,y}{I}$$

The largest bending stress happens in the fibers farthest above or below the neutral surface. Call this largest distance $y_{\max} \equiv c$.

$$f_{\max} = \frac{M_{\max}\,c}{I} = \frac{M_{\max}}{(I/c)} = \frac{M_{\max}}{S}$$

The ratio $S = I/c$ is called "section modulus." The load diagram gives you $M_{\max}$. Each material (wood, steel, etc.) has allowed bending stress $f_{\max}$. Then $S_{\min}$ tells you how big a beam you need.

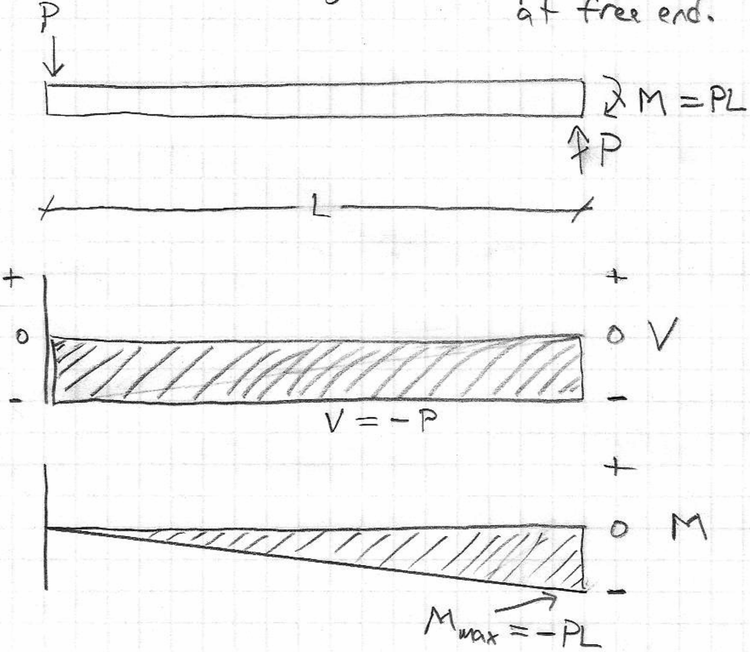
Example (using metric units!): A cantilever beam has a span of $3.0\ \mathrm{m}$ with a single concentrated load of $100\ \mathrm{kg}$ at its unsupported end. If the beam is made of timber having allowable bending stress $F_b = 1.1 \times 10^7\ \mathrm{N/m^2}$ (was 1600 psi in US units), what minimum section modulus is required?

What is the smallest "2×" dimensional lumber (width = 1.5 inch = 0.038 m) whose cross-section satisfies this strength criterion?

Would this beam also satisfy a $\Delta_{\mathrm{max}} < L/240$ (maximum deflection) stiffness criterion? If not, what standard "2×" cross-section is needed instead?

$\Delta_{\mathrm{max}} = PL^3/(3EI)$ for a cantilever with concentrated load at end. Use Young's modulus $E = 1.1 \times 10^{10}\ \mathrm{N/m^2}$ for southern pine.

Cantilever of length L with point load P at free end.



$$S_{\min} = \frac{|M_{\max}|}{f_{\text{allowed}}} = \frac{PL}{F_b} = \frac{(980\,\mathrm{N})(3\,\mathrm{m})}{1.1\times 10^{7}\,\mathrm{N/m^2}} = 26.7\times 10^{-5}\,\mathrm{m}^3$$

$$\Delta_{\text{allowed}} = \frac{L}{240} = \frac{3.0\,\mathrm{m}}{240} = 0.0125\ \mathrm{m}$$

$$\Delta_{\max} = \frac{PL^3}{3EI} \quad\Rightarrow\quad I_{\min} = \frac{PL^3}{3E\Delta_{\text{allowed}}} = 64.2\times 10^{-6}\,\mathrm{m}^4$$

I worked out b, h, I, and $S = I/c$ values in metric units for standard "2×" dimensional lumber.

	b	b	h	h	$I = bh^3/12$	$S = bh^2/6$
2×4	1.5 in	.038 m	3.5 in	.089 m	$2.23 \times 10^{-6}\,\mathrm{m}^4$	$5.02 \times 10^{-5}\,\mathrm{m}^3$
2×6	1.5 in	.038 m	5.5 in	.140 m	$8.66 \times 10^{-6}\,\mathrm{m}^4$	$12.4 \times 10^{-5}\,\mathrm{m}^3$
2×8	1.5 in	.038 m	7.5 in	.191 m	$21.9 \times 10^{-6}\,\mathrm{m}^4$	$23.0 \times 10^{-5}\,\mathrm{m}^3$
2×10	1.5 in	.038 m	9.5 in	.241 m	$44.6 \times 10^{-6}\,\mathrm{m}^4$	$37.0 \times 10^{-5}\,\mathrm{m}^3$
2×12	1.5 in	.038 m	11.5 in	.292 m	$79.1 \times 10^{-6}\,\mathrm{m}^4$	$54.2 \times 10^{-5}\,\mathrm{m}^3$

The numbers are nicer if you use centimeters instead of meters, but then you have the added hassle of remembering to convert back to meters in calculations.

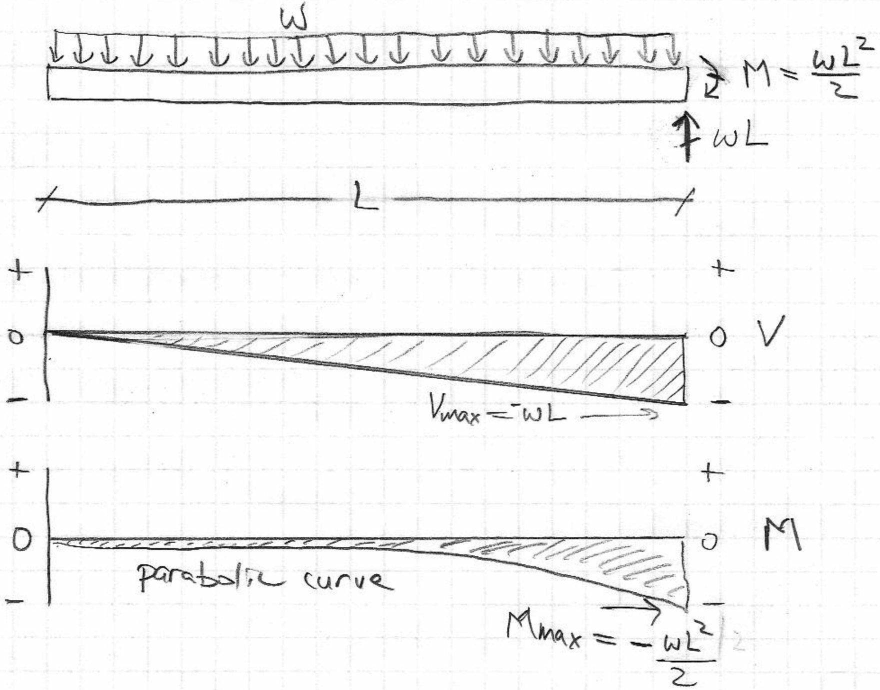
	b	b	h	h	$I = bh^3/12$	$S = bh^2/6$
2×4	1.5 in	3.8 cm	3.5 in	8.9 cm	$223\,\mathrm{cm}^4$	$50.2\,\mathrm{cm}^3$
2×6	1.5 in	3.8 cm	5.5 in	14.0 cm	$866\,\mathrm{cm}^4$	$124\,\mathrm{cm}^3$
2×8	1.5 in	3.8 cm	7.5 in	19.1 cm	$2195\,\mathrm{cm}^4$	$230\,\mathrm{cm}^3$
2×10	1.5 in	3.8 cm	9.5 in	24.1 cm	$4461\,\mathrm{cm}^4$	$370\,\mathrm{cm}^3$
2×12	1.5 in	3.8 cm	11.5 in	29.2 cm	$7913\,\mathrm{cm}^4$	$542\,\mathrm{cm}^3$

Minor variation on same problem: A cantilever beam has a span of $3.0\ \mathrm{m}$ with a uniform distributed load of $33.3\ \mathrm{kg/m}$ along its entire length. If we use timber with allowable bending stress $F_b = 1.1 \times 10^7\ \mathrm{N/m^2}$, what minimum section modulus is required?

What is the smallest "2×" dimensional lumber (width $=$ 1.5 inch $=$ 0.038 m) whose cross-section satisfies this strength criterion?

Would this beam also satisfy a $\Delta_{\mathrm{max}} < L/240$ (maximum deflection) stiffness criterion? If not, what standard "2×" cross-section is needed instead?

$\Delta_{\mathrm{max}} = wL^4/(8EI)$ for a cantilever with uniform load. Use Young's modulus $E = 1.1 \times 10^{10}\ \mathrm{N/m^2}$ for southern pine.


w
M = wL²/2
wL
L
+
0
−
V
Vmax = −wL
+
0
−
M
parabolic curve
Mmax = −wL²/2

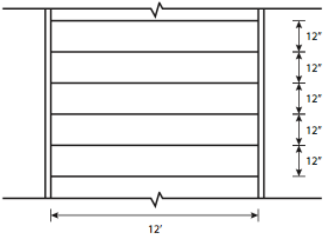
$$S_{\mathrm{min}} = \frac{|M_{\mathrm{max}}|}{f_{\mathrm{allowed}}} = \frac{wL^2/2}{F_b} = \frac{(326\,\mathrm{N/m})(3\,\mathrm{m})^2/2}{1.1\times 10^7\,\mathrm{N/m^2}} = 13.3\times 10^{-5}\,\mathrm{m}^3$$

$$\Delta_{\mathrm{allowed}} = \frac{L}{240} = \frac{3.0\,\mathrm{m}}{240} = 0.0125\ \mathrm{m}$$

$$\Delta_{\mathrm{max}} = \frac{wL^4}{8EI} \quad \Rightarrow \quad I_{\mathrm{min}} = \frac{wL^4}{8E\Delta_{\mathrm{allowed}}} = 24.0\times 10^{-6}\,\mathrm{m}^4$$

2) Size a wood joist for a row house floor which spans 12 feet. Joists are spaced at 16 inches on center.

f = 1,300 psi
f = 85 psi
E = 1.7 x 10 psi
LL = 60 psf
DL =30 psf



Plan View

Hint: remember that a "2 x 4" wood joist is only nominal; its true dimensions are "1.5 x 3.5" inches.
(4 = 1.5, 6 = 5.5, 8 = 7.25, 10 = 9.25 inches)

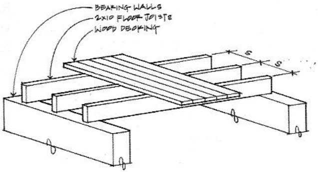
(Here's a homework problem from ARCH 435.)

Actually, Home Depot's 2×10 really is 9.5 inches deep, not 9.25 inches, and 2×12 really is 11.5 inches deep.

A timber floor system uses joists made of "2×10" dimensional lumber. Each joist spans a length of 4.27 m (simply supported). The floor carries a load of 2400 $\mathrm{N/m^2}$. At what spacing should the joists be placed, in order not to exceed allowable bending stress $F_b = 10000\ \mathrm{kN/m^2}$ $(1.0 \times 10^7\ \mathrm{N/m^2})$?

(We should get an answer around 24 inches $=$ 0.61 meters.)

8.6 A timber floor system utilizing 2 × 10 S4S joists spans a length of 14′ (simply supported). The floor carries a load of 50 psf (DL + LL). At what spacing should the joists be placed? Assume Douglas Fir–Larch No. 2 (F_b = 1,450 psi).



Solution:

Based on the allowable stress criteria:

$$f = \frac{Mc}{I} = \frac{M}{S}$$

$$M_{max} = S \times f_b = (21.4 \text{ in.}^3)(1.45 \text{ k/in.}^3) = 31 \text{ k-in.}$$

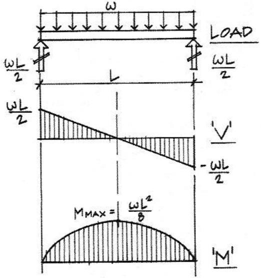
$$M = \frac{31 \text{ k-in.}}{12 \text{ in./ft.}} = 2.58 \text{ k-ft.}$$

Based on the bending moment diagram:

$$M_{max} = \frac{\omega L^2}{8}$$

Therefore,

$$\omega = \frac{8M}{L^2}$$



Substituting for M obtained previously,

$$\omega = \frac{8(2.58 \text{ k-ft.})}{(14')^2} = 0.105 \text{ k/ft.} = 105 \text{ \#/ft.}$$

But

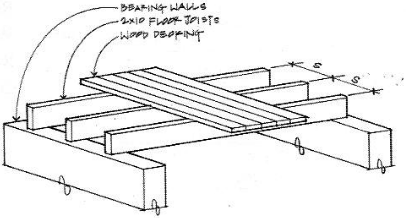
$$\omega = \text{\#/ft.}^2 \times \text{tributary width (joist spacing } s)$$

$$s = \frac{\omega}{50 \text{ psf}} = \frac{105 \text{ \#/ft.}}{50 \text{ \#/ft.}^2} = 2.1'$$

$$s = 25'' \text{ spacing}$$

Use 24″ o.c. spacing.

8.6 A timber floor system utilizing 2 × 10 S4S joists spans a length of 14′ (simply supported). The floor carries a load of 50 psf (DL + LL). At what spacing should the joists be placed? Assume Douglas Fir–Larch No. 2 (F_b = 1,450 psi).



Solution:

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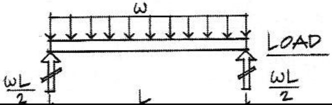

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But

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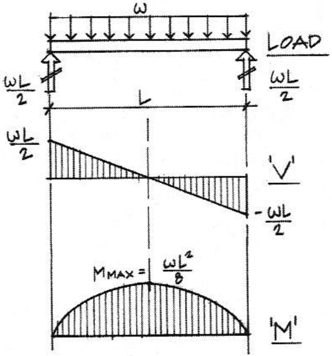
$$s = \frac{\omega}{50 \text{ psf}} = \frac{105 \text{ \#/ft.}}{50 \text{ \#/ft.}^2} = 2.1'$$

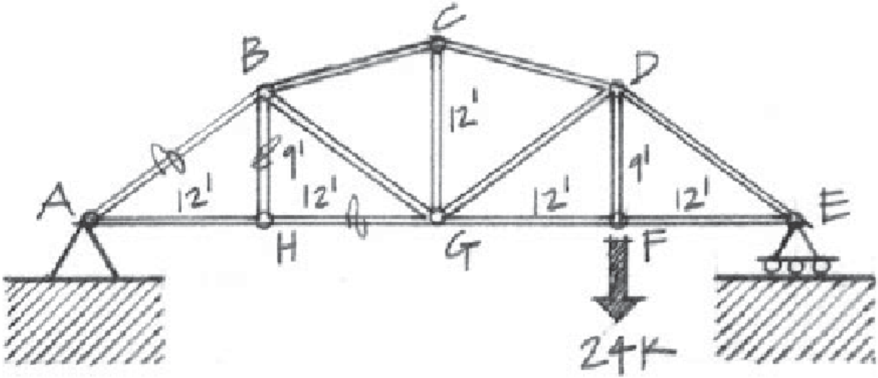


$$s = 25'' \text{ spacing}$$

Use 24″ o.c. spacing.

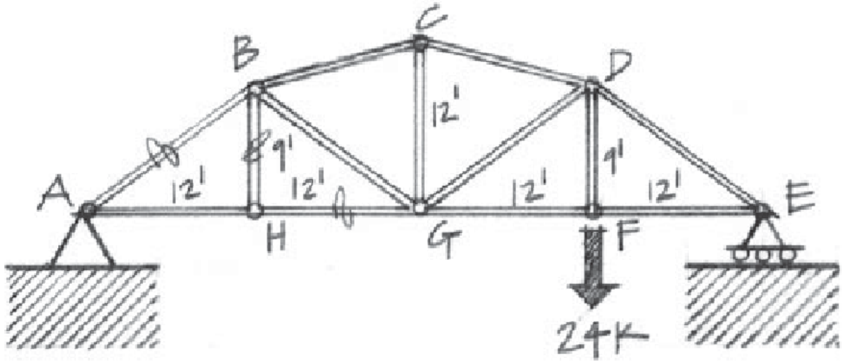
Note: *Spacing is more practical for plywood subflooring, based on a 4 ft. module of the sheet.*

Using the method of sections, solve for the forces in members AB, BH, and HG in the truss shown below. Indicate whether each of these members is in tension or in compression. Use only one section cut through the truss.



If we have time left, let's solve this truss problem together. It's actually pretty quick, using method of sections. First solve for vertical support force at A, then analyze left side of section.

Using the method of sections, solve for the forces in members AB, BH, and HG in the truss shown below. Indicate whether each of these members is in tension or in compression. Use only one section cut through the truss.



XC2. **(I haven't checked this with anyone else yet.)** For the truss as a whole $\sum F_x = 0$ gives $\boxed{R_{Ax} = 0}$. Then $\sum M_A = 0 = R_{Ey}(48') - 24\text{k}(36')$ gives $\boxed{R_{Ey} = 18\text{k}}$. Then $\sum F_y = 0 = R_{Ay} + R_{Ey} - 24\text{k}$ gives $\boxed{R_{Ay} = 6\text{k}}$. Now section the truss through members AB, BH, and GH and analyze the left-hand side. Then $\sum M_A = 0 = T_{BH}(12')$ gives $\boxed{T_{BH} = 0}$, which one can see by inspection of the vertical forces at joint H: bar BH is a "zero-force member." Then $\sum F_y = 0 = +6\text{k} + (3/5)T_{AB} + T_{BH}$ gives $\boxed{T_{AB} = -10\text{k}}$ (i.e. compression). Finally, $\sum F_x = 0 = (4/5)T_{AB} + T_{GH}$ gives $\boxed{T_{GH} = 8\text{k}}$.

Physics 8 — Wednesday, November 20, 2019

- HW11 is "due" on Friday, but you can turn it in on Monday, Nov 25, as it will probably take us an extra day to get through the material on beams.
- HW help: (Bill) Wed 4-6:30pm DRL 3C4, (Brooke/Grace) Thu 6-8pm DRL 2C4.
- This week, read/skim O/K Ch8 (more about beams).
- You may find my "equation sheet" to be a helpful summary of the key results from the Onouye/Kane reading:

`http://positron.hep.upenn.edu/p8/files/equations.pdf#page=12`

Physics 8 — Friday, November 22, 2019

- Turn in HW11 either today or Monday, as you prefer.
- I just added to my "equation sheet" the math behind the shear-stress results Prof. Farley illustrated for us on Wednesday:

`http://positron.hep.upenn.edu/p8/files/equations.pdf#page=22`

- Today and Monday are our last two days on beams. Then we'll spend a week on oscillation / vibration / periodic motion.
- I hope to hand out the take-home practice exam (due Dec 6) on Monday (Nov 25), but if I don't get it done in time, I will put the PDF online before Thanksgiving. I intend for it to be comparable in length and coverage to the in-class final exam (Dec 12, noon, A1).
- I plan to do a review session on Dec 11, time/place TBD.
- Prof. Farley will join us Monday via FaceTime. He'd like us to send him in advance questions you'd like him to answer.
- Next Wednesday, you don't have to show up, but if you do, you'll get a bonus point. I will spend the hour introducing "Python Mode for Processing" (`py.processing.org`) — a visual-arts-oriented approach to writing code to do drawing and animation.

Shear (V) and moment (M) diagrams:

- First draw a "load diagram," which is an EFBD that shows all of the vertical forces acting on the beam.
- The "shear diagram" $V(x)$ graphs the running sum of all vertical forces (both supports and loads) acting on the beam, from the left side up to x, where upward = positive, downward = negative.
- To draw the "moment diagram" $M(x)$, note that V is the slope of M:

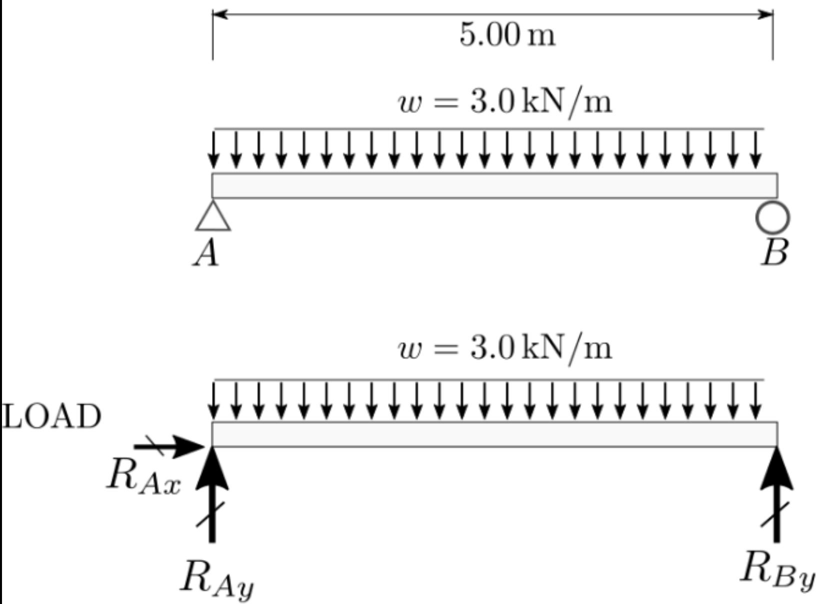
$$V(x) = \frac{\mathrm{d}}{\mathrm{d}x} M(x)$$

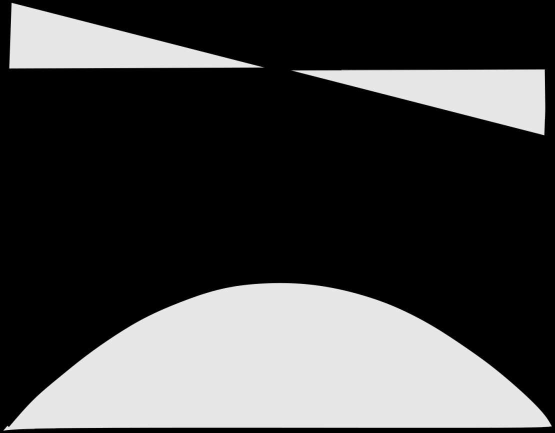
- The change in M from x_1 to x_2 is given by

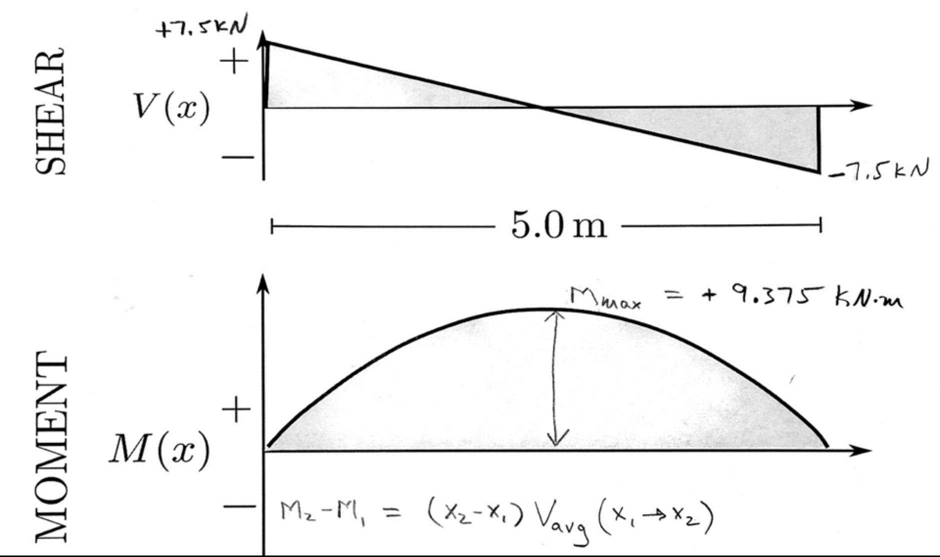
$$M_2 - M_1 = (x_2 - x_1)\, V_{1\to2}^{\text{average}}$$

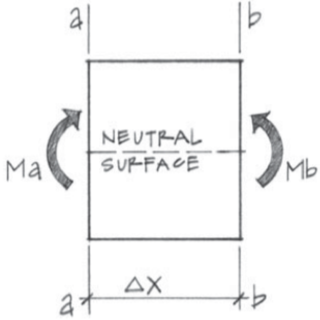
- If an end of a beam is unsupported ("free"), is hinge/pin supported, or is roller supported, then $M = 0$ at that end. You can only have $M \neq 0$ at an end if the support at that end is capable of exerting a torque on the beam — for example, the fixed end of a cantilever has $M \neq 0$.

On Wednesday, we drew $V(x)$ and $M(x)$ diagrams for this simply supported beam with uniform distributed load:

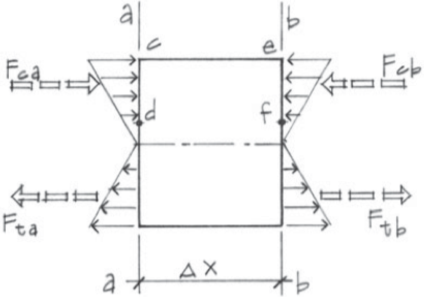





SHEAR
+7.5kN
+
V(x)
−
−7.5kN
5.0 m
MOMENT
Mmax = + 9.375 kN·m
+
M(x)
−
M2 − M1 = (x2 − x1) Vavg (x1 → x2)



(a) Beam section between sections a *and* b.

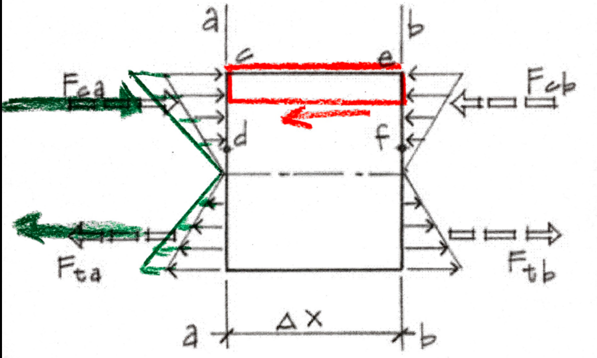


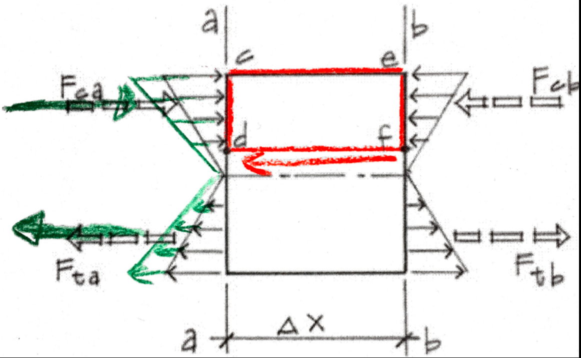
(b) Bending stresses on the beam section a-b.

Figure 8.19 Bending stress on a beam section.

Notice (on next slide):

- For most load/support conditions, bending moment $M(x)$ varies with x, and bending stress is proportional to bending moment.
- The shear stress (exerted between parallel fibers) along the bottom edge of the red rectangle must make up the difference between the left and right total bending forces.
- The left and right total bending forces depend on how much area we add up in drawing the red rectangle.
- The total reaches a maximum at the neutral surface, then decreases, since the direction of the bending stress reverses at the neutral surface.


a
b
c
e
d
f
Fca
Fcb
Fta
Ftb
ΔX
a
b


a
b
c
e
d
f
Fca
Fcb
Fta
Ftb
ΔX
a
b

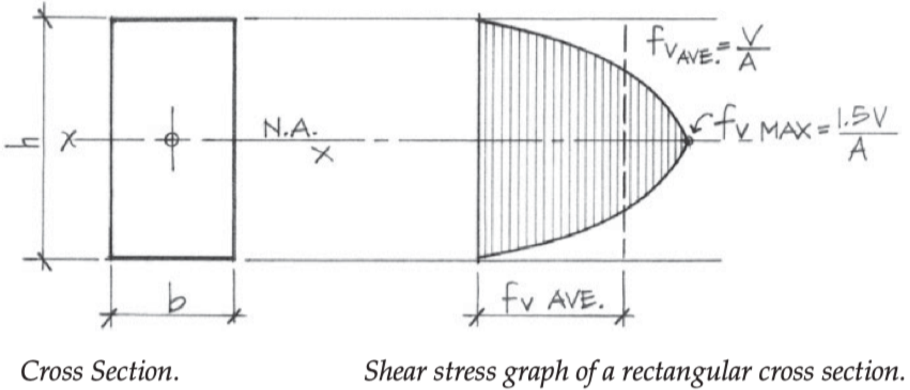
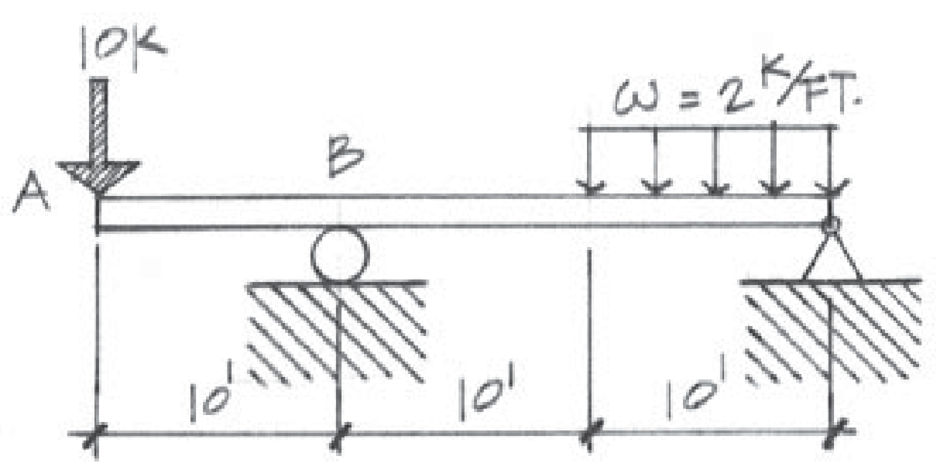


Figure 8.26 *Shear stress distribution—key points.*

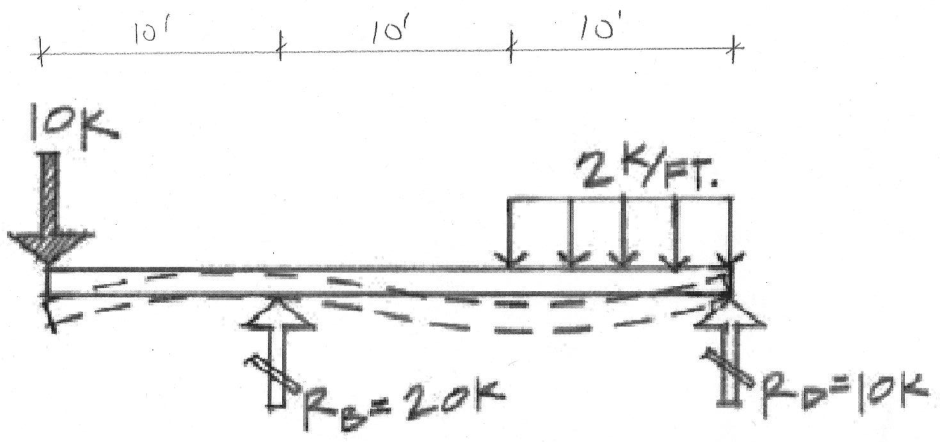
If you find this confusing:
(a) You don't really need to know it for this course. If you're an architect, you'll learn it again when you study structures.
(b) You might look at the explanation I wrote up in the Onouye/Kane chapter 8 pages of
`http://positron.hep.upenn.edu/p8/files/equations.pdf#page=22`

(Let's skip ahead to slide 17.)

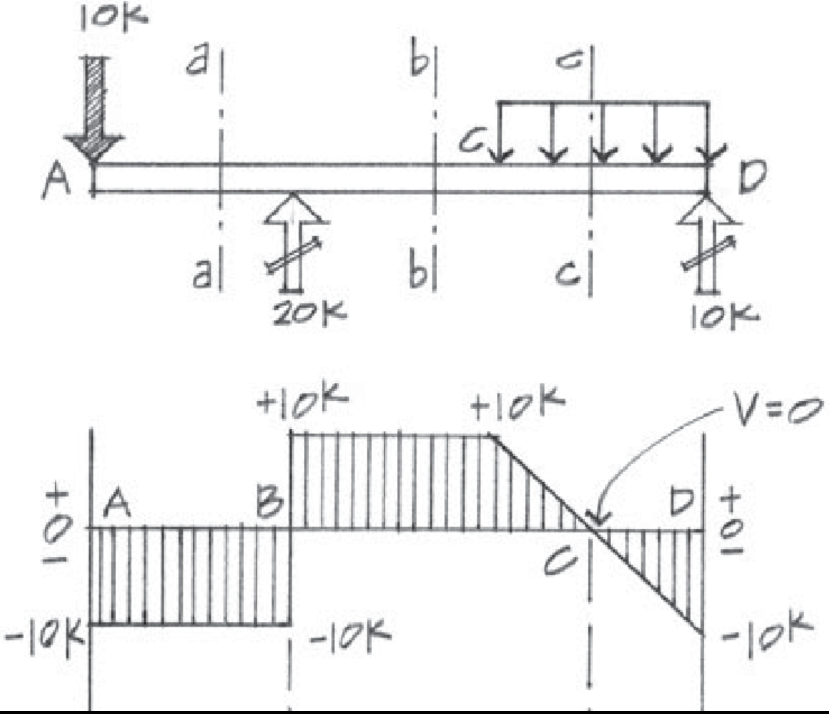


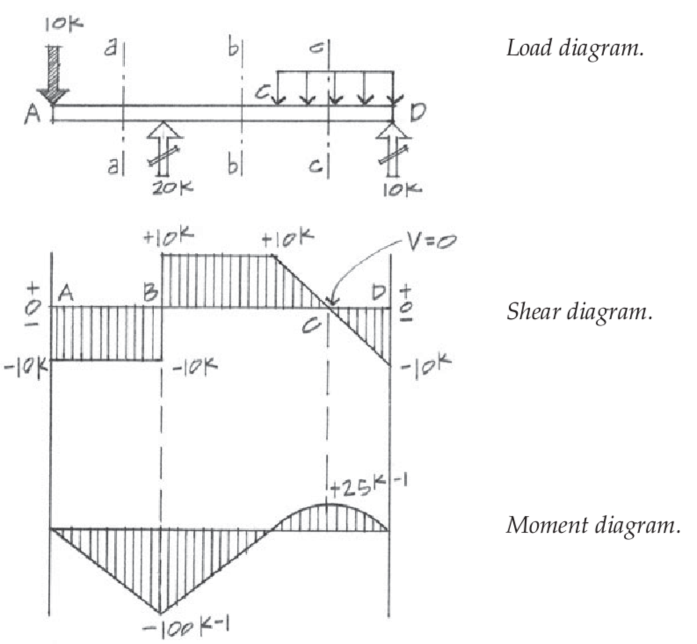
Draw shear (V) and moment (M) diagrams for this beam! Tricky! First one needs to solve for the support ("reaction") forces.

Note: in solving for the support forces, you replace distributed load w with equivalent point load. But when you draw the load diagram to find V and M, you need to keep w in its original form.



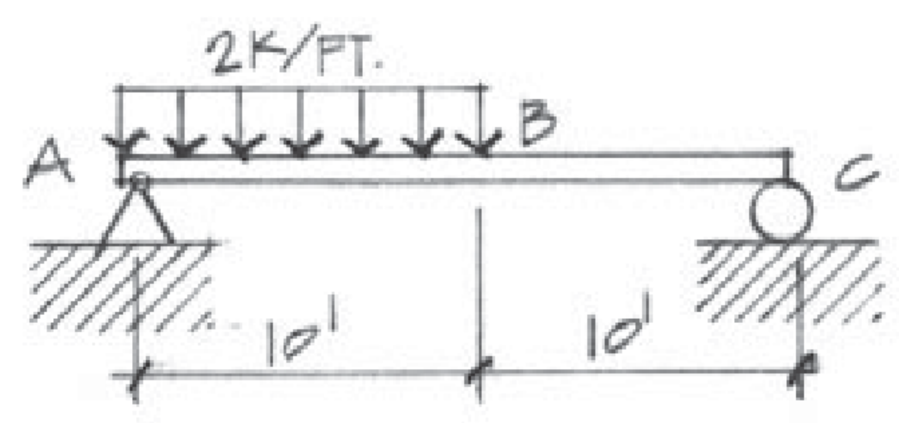
Remember that $V(x)$ is the running sum, from LHS to x, of vertical forces acting on the beam, with upward=positive.

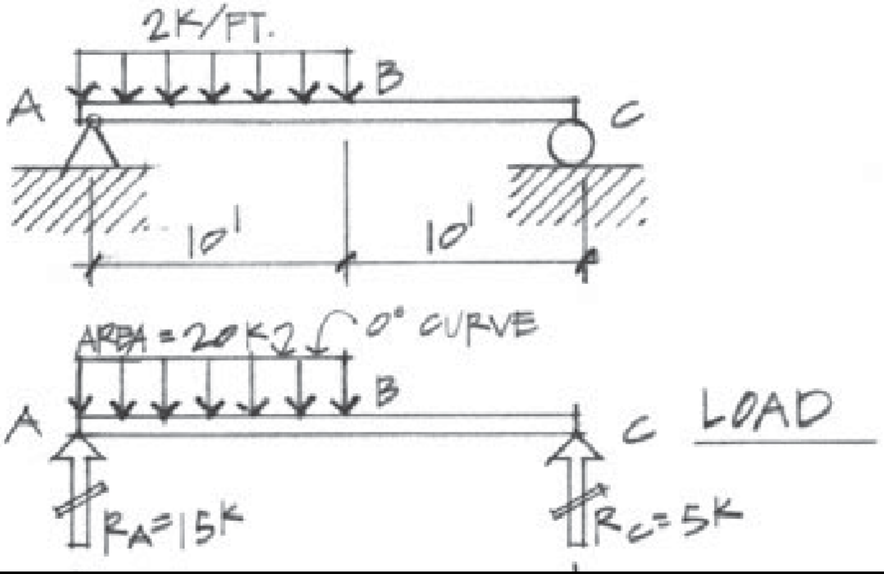

10K
a
b
c
A
D
a
b
c
20K
10K
+10K
+10K
V=0
+
0
−
A
B
D
+
0
−
C
−10K
−10K
−10K



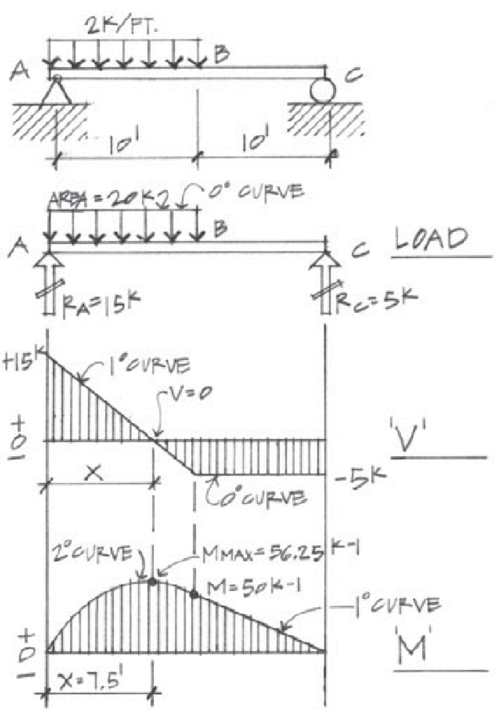
Neat trick: $M_2 - M_1 = (V_{1 \to 2}^{\text{average}})(x_2 - x_1)$

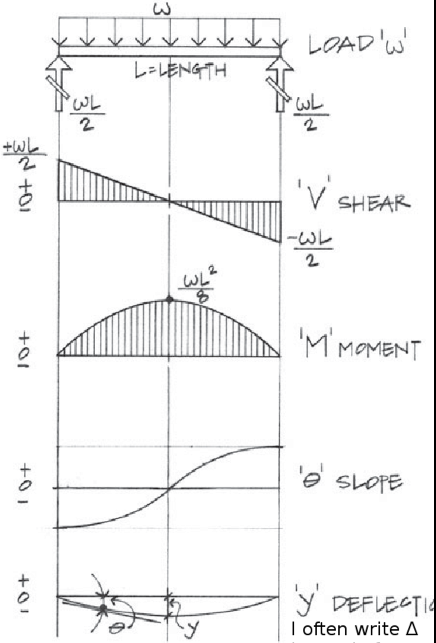
Draw load, V, and M diagrams for this simply supported beam with a partial uniform load.





2K/FT.
A
B
C
10'
10'
AREA = 20K
0° CURVE
A
B
C
LOAD
R_A=15K
R_C=5K

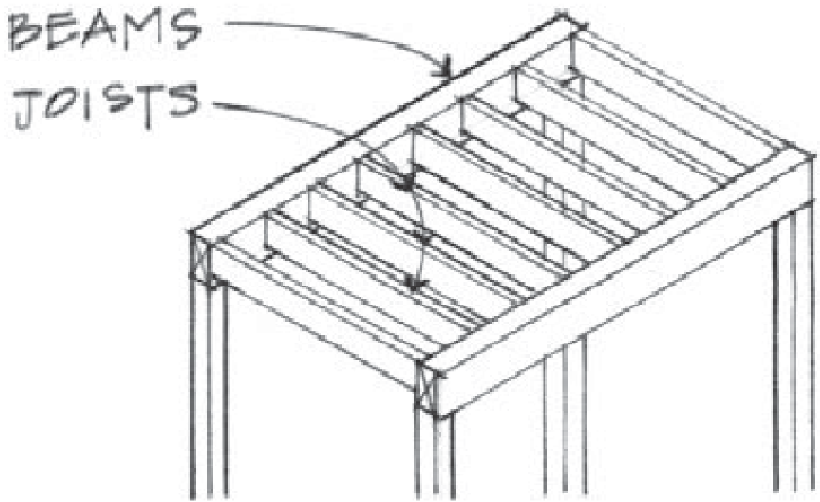

2K/FT.
A
B
C
10'
10'
AREA = 20K
0° CURVE
LOAD
R_A=15K
R_C=5K
+15K
1° CURVE
V=0
'V'
X
-5K
0° CURVE
2° CURVE
MMAX=56.25 K-1
M=50K-1
1° CURVE
'M'
X=7.5'



I often write Δ instead of y

Figure 7.11 Relationship of load, shear, moment, slope, and deflection diagrams.

Why do we care about these beam diagrams, anyway? Usually the floor of a structure must carry a specified weight per unit area. The beams (beams, girders, joists, etc.) must be strong enough to support this load without failing and must be stiff enough to support this load without excessive deflection.



Beam criteria:

- Normal stress in the extreme fibers of the beam (farthest from neutral surface) must be smaller than the allowable bending stress, F_b, which depends on the material (wood, steel, etc.).
- This happens where $M(x)$ has largest magnitude.
- Shear stress (in both y ("transverse") and x ("longitudinal") must be smaller than the allowable shear stress, F_v, which is also a property of the material (wood, steel, etc.).
- This happens where $V(x)$ has largest magnitude, and (surprisingly) is largest near the neutral surface.
- The above two are "strength" criteria. The third one is a "stiffness" criterion:
- The maximum deflection under load must satisfy the building code: typically $\Delta y_{\max} < L/360$.
- For a uniform load, this happens farthest away from the supports. If deflection is too large, plaster ceilings develop cracks, floors feel uncomfortably bouncy or sloped.
- The book also notes buckling as a beam failure mode.

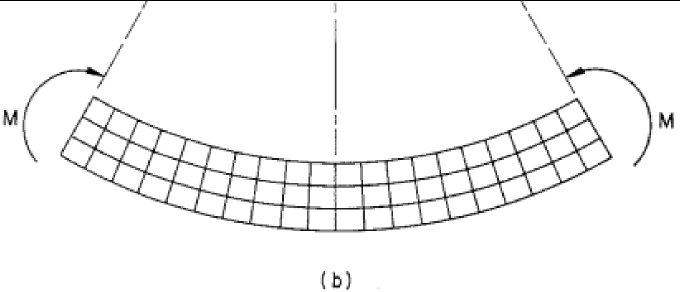
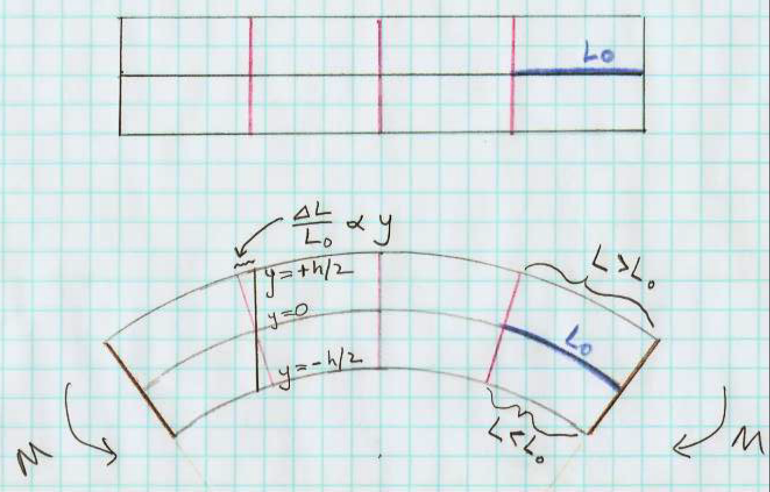


Fig. 6.5.
Navier's assumption. Originally plane and parallel sections (a) remain plane after bending (b), but converge onto a common center of curvature. This assumption can be illustrated with a rubber beam.

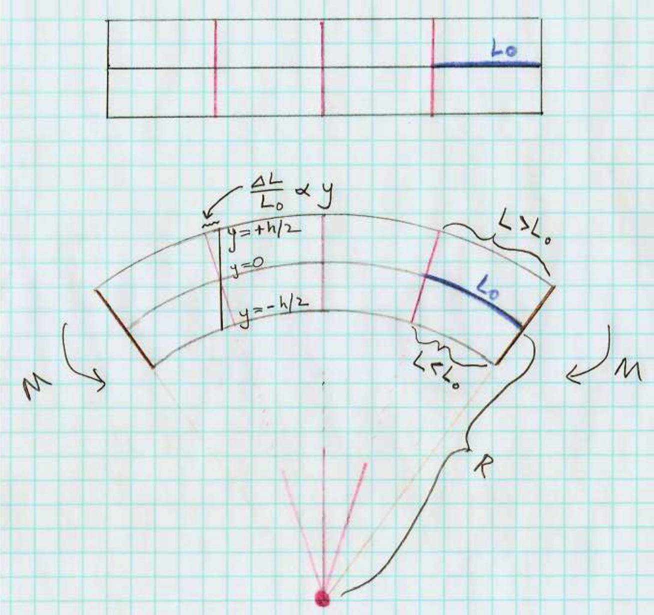
(In this illustration, bottom is in tension, top is in compression, as in a "simply supported" beam.)

A big topic from this week's reading was to see how an initially horizontal beam responds to the bending moment $M(x)$ by deforming into a curved shape.



(In this illustration, top is in tension, as in a cantilever.)

Key idea: bending moment $M \propto \frac{1}{R}$, where R is the radius of curvature of the beam. For constant M, vertical lines converge toward common center of curvature.



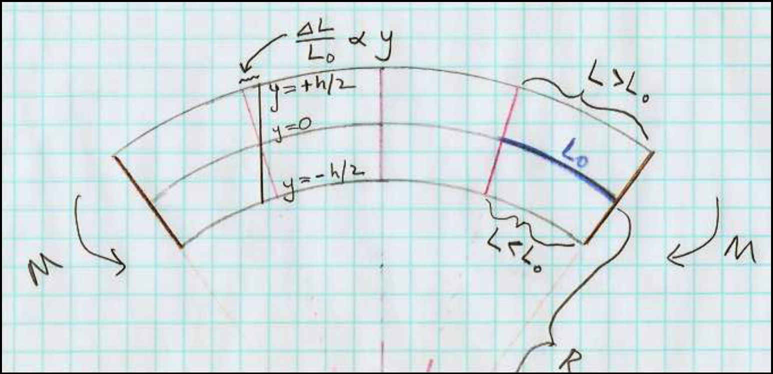
$$\text{strain} = \frac{\Delta L}{L_0} = \frac{y}{R}$$

where $y = 0$ is the neutral surface.

So in this case $y > 0$ is in tension and $y < 0$ is in compression.

If you think of wood fibers running along the beam's axis, then the fibers above the neutral surface ($y > 0$) are stretched in proportion to y, and the fibers below the neutral surface ($y < 0$) are compressed in proportion to $|y|$.

$$\text{strain} = \frac{\Delta L}{L} = \frac{y}{R}$$



Now remember that $\frac{\Delta L}{L}$ is called (axial) *strain*, and force per unit area is called *stress*. For an elastic material, strain $(e) \propto$ stress (f).

$$\frac{\Delta L}{L} = \frac{1}{E} \times \frac{\text{Force}}{\text{Area}} = \frac{1}{E} \times f \qquad e = \frac{1}{E} \times f$$

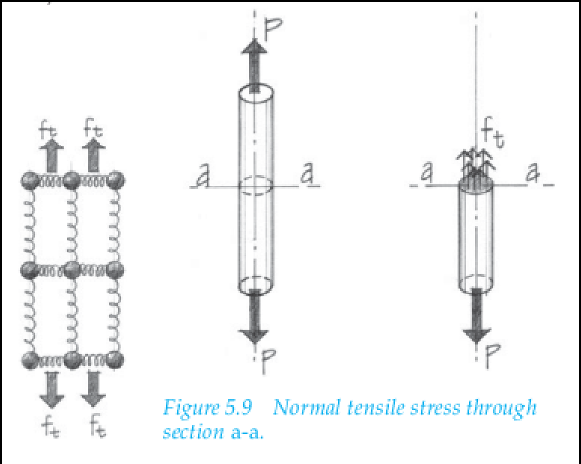
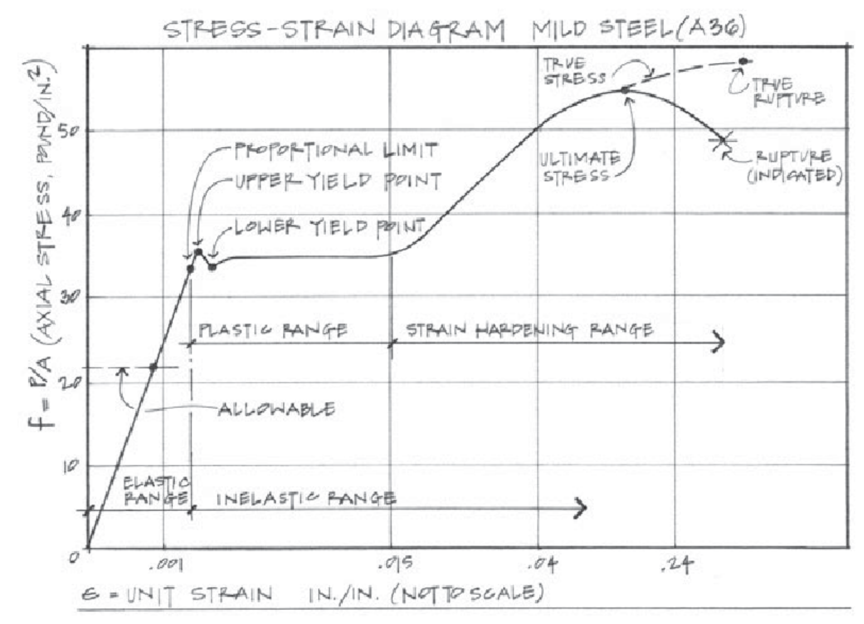


Figure 5.9 Normal tensile stress through section a-a.

In the elastic region, strain ($e = \Delta L/L$) is proportional to stress ($f = F/A$). $\boxed{f = Ee}$. The slope E is Young's modulus.



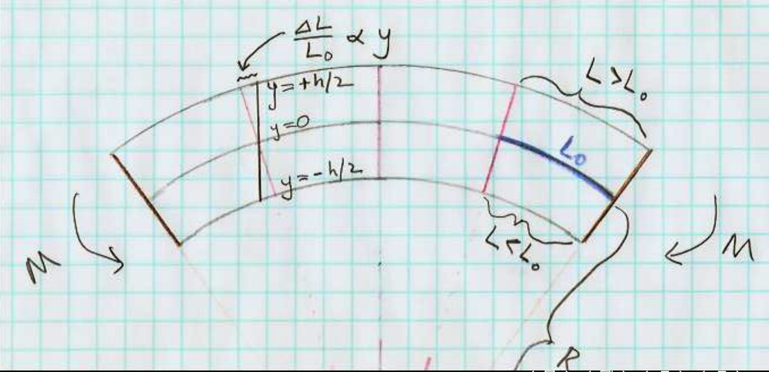
(Now skip ahead to slide 31.)

Plugging in $f = Ee$ to the bending-beam diagram:

$$\frac{y}{R} = \frac{\Delta L}{L} = e = \frac{f}{E}$$

we find the force-per-unit area (stress) exerted by the fibers is

$$f = \frac{Ey}{R}$$

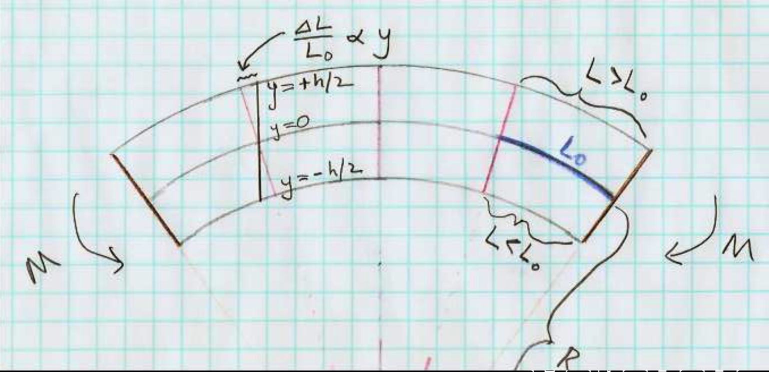


The force-per-unit area (stress) exerted by the fibers is

$$f = \frac{Ey}{R}$$

while the torque (bending moment $\mathrm{d}M$, pivot about N.A.) exerted by each tiny fiber of area $\mathrm{d}A$ is proportional to its lever arm y

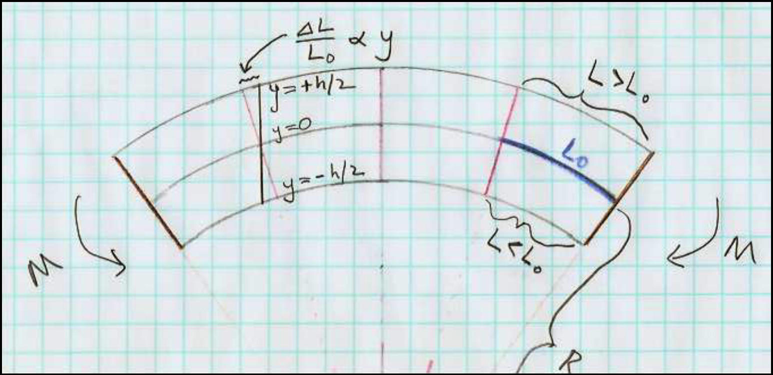
$$\mathrm{d}M = y\ \mathrm{d}F = y\ f\mathrm{d}A = y\left(\frac{Ey}{R}\right)\mathrm{d}A = \frac{E}{R}\ y^2\ \mathrm{d}A$$



So the bending moment M exerted by a curved beam is

$$M = \frac{E}{R} \int y^2 \, \mathrm{d}A = \frac{EI}{R}$$

where R is the curved beam's radius of curvature and $I = \int y^2 \, \mathrm{d}A$ is the "second moment of area" a.k.a. "area moment of inertia."



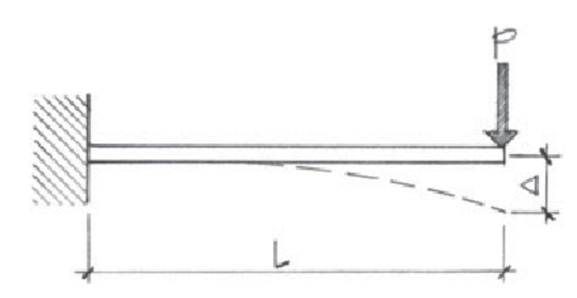
$$M = \frac{E}{R} \int y^2 \mathrm{d}A = \frac{EI}{R}$$

Meanwhile, the vertical *deflection* Δ of a point along the beam is related to its curvature by (in limit where $\Delta \ll R$)

$$\frac{1}{R} \approx \frac{\mathrm{d}^2\Delta}{\mathrm{d}x^2}$$

so you can integrate the $M(x)$ curve twice to get deflection

$$\frac{\mathrm{d}^2\Delta}{\mathrm{d}x^2} = \frac{M}{EI} \quad \Rightarrow \quad \Delta(x) = \frac{1}{EI} \int \mathrm{d}x \int M(x)\, \mathrm{d}x$$



Calculus digression (not important — but you may be curious):

You may have seen in calculus that the "curvature" (which means $1/R$, where R is the radius of curvature) of a function $y = f(x)$ is

$$\frac{1}{R} = \frac{y''}{(1+(y')^2)^{3/2}}$$

We are working in the limit $y' \ll 1$, so

$$\frac{1}{R} \approx y''$$

That's how we arrived at

$$\frac{1}{R} \approx \frac{\mathrm{d}^2\Delta}{\mathrm{d}x^2}$$

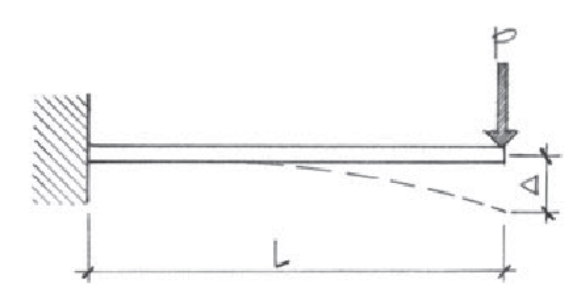
$$M = \frac{E}{R} \int y^2 \mathrm{d}A = \frac{EI}{R}$$

Meanwhile, the vertical *deflection* Δ of a point along the beam is related to its curvature by (in limit where $\Delta \ll R$)

$$\frac{1}{R} \approx \frac{\mathrm{d}^2\Delta}{\mathrm{d}x^2}$$

so you can **integrate the $M(x)$ curve twice to get deflection**

$$\frac{\mathrm{d}^2\Delta}{\mathrm{d}x^2} = \frac{M}{EI} \quad \Rightarrow \quad \boxed{\Delta(x) = \frac{1}{EI} \int \mathrm{d}x \int M(x)\, \mathrm{d}x}$$



This Onouye/Kane figure writes "y" here for deflection, but I wrote "Δ" for deflection on the preceding pages (and they usually do, too), because we were already using y for "distance above the neutral surface."

So you integrate $M(x)/EI$ twice w.r.t. x to get the deflection $\Delta(x)$.

The bending moment $M(x) = EI\ \mathrm{d}^2\Delta/\mathrm{d}x^2$, where E is Young's modulus and I is second moment of area.

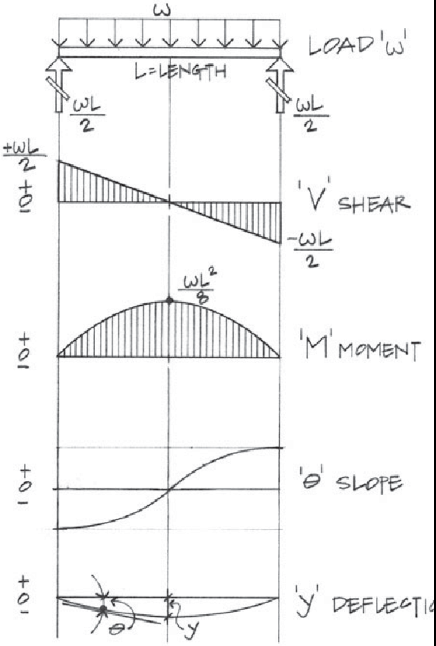
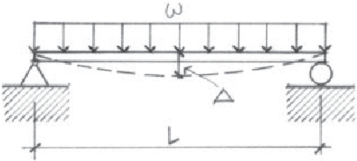
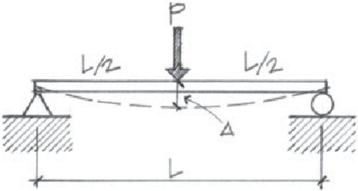


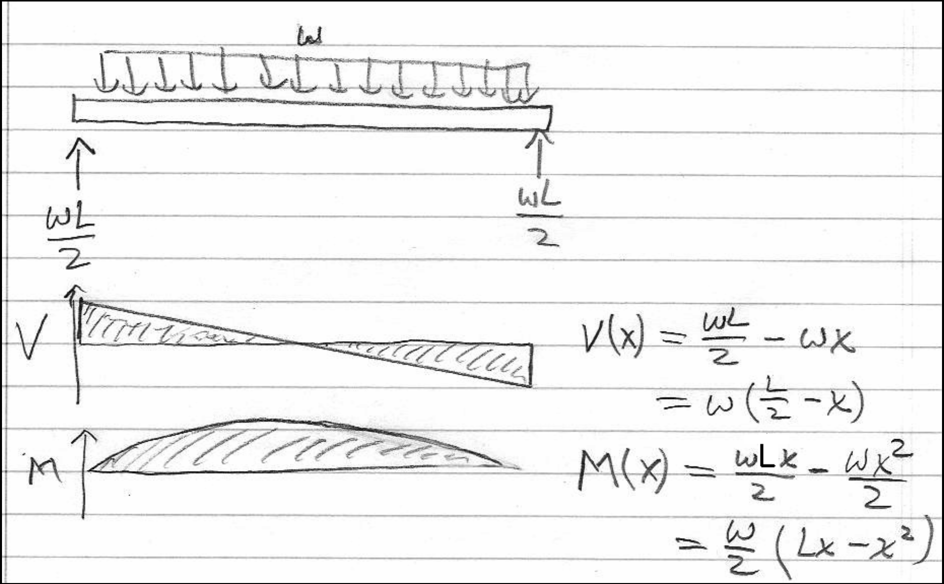
Figure 7.11 Relationship of load, shear, moment, slope, and deflection diagrams.

The most common deflection results can be found in tables.

Beam Load and Support	Actual Deflection*
 (a) Uniform load, simple span	$\Delta_{max} = \dfrac{5\omega L^4}{384EI}$ (at the centerline)
 (b) Concentrated load at midspan	$\Delta_{max} = \dfrac{PL^3}{48EI}$ (at the centerline)

(After this, skip ahead to slide 36.)

FYI, here's where that crazy $(5wL^4)/(384EI)$ comes from!



(continued on next page)

Here's where that crazy $(5wL^4)/(384EI)$ comes from!

$$\Delta(x) = -\frac{1}{EI}\int dx \int M(x)\,dx$$

$$= -\frac{w}{2EI}\int dx \int (Lx - x^2)\,dx = -\frac{w}{2EI}\int dx \left(\frac{Lx^2}{2} - \frac{x^3}{3} + C_1\right)$$

$$= -\frac{w}{2EI}\left(\frac{Lx^3}{6} - \frac{x^4}{12} + C_1 x + C_2\right)$$

$$\Delta(0) = 0 \Rightarrow \boxed{C_2 = 0}$$

$$\Delta(L) = 0 \Rightarrow \frac{L^4}{6} - \frac{L^4}{12} + C_1 L = 0 \Rightarrow \boxed{C_1 = -\frac{L^3}{12}}$$

$$\Delta(x) = -\frac{w}{2EI}\left(\frac{Lx^3}{6} - \frac{x^4}{12} - \frac{L^3 x}{12}\right)$$

$$\Delta_{max} = \Delta\left(\frac{L}{2}\right) = -\frac{w}{2EI}\left(\frac{L(L/2)^3}{6} - \frac{(L/2)^4}{12} - \frac{L^3(L/2)}{12}\right)$$

$$= -\frac{wL^4}{2EI}\left(\frac{1}{48} - \frac{1}{192} - \frac{1}{24}\right) = -\frac{wL^4}{2EI}\left(-\frac{5}{192}\right) = \boxed{\frac{5wL^4}{384EI}}$$

The 2 integration constants can be tricky. Simply supported: $\Delta(0) = \Delta(L) = 0$. (For cantilever, $\Delta(0) = \Delta'(0) = 0$ instead.)

Maximum deflection is one of several beam-design criteria. You can see now how it relates to the load and $M(x)$ diagrams: $\Delta_{\max}$ comes from integrating $M(x)/(EI)$ twice w.r.t. x to get $\Delta(x)$.

For point load P at the end of a cantilever (for example), you get

$$\Delta_{\max} = \frac{PL^3}{3EI}$$

For uniform load w on simply-supported beam, you get

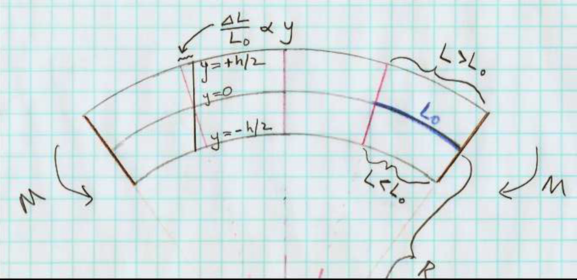
$$\Delta_{\max} = \frac{5wL^4}{384EI}$$

You just look these results up, or use a computer to calculate them.

Deflection is proportional to load, and inversely proportional to Young's modulus and to the second moment of area.

- More load $\rightarrow$ more deflection
- Stiffer material $\rightarrow$ less deflection
- Cross-section with larger $I = \int y^2 \, \mathrm{d}A$ $\rightarrow$ less deflection

Another beam-design criterion is maximum bending stress: the fibers farthest from the neutral surface experience the largest tension or compression, hence largest bending stress.

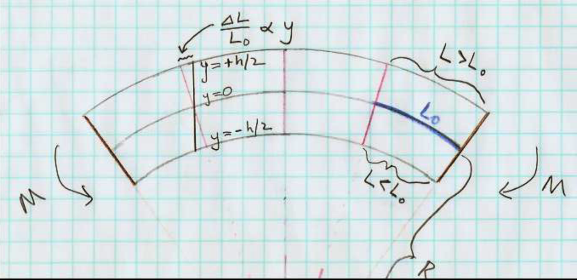


When we section the beam at x, bending moment $M(x)$ is

$$M = \frac{EI}{R}$$

which we can solve for the radius of curvature $R = EI/M$. Then the stress a distance y above the neutral surface is

$$f = Ee = E\,\frac{y}{R} = \frac{E\,y}{(EI/M)} = \frac{M\,y}{I}$$



The bending stress a distance y above the neutral surface is

$$f = \frac{M\,y}{I}$$

The largest bending stress happens in the fibers farthest above or below the neutral surface. Call this largest distance $y_{\mathrm{max}} \equiv c$.

$$f_{\mathrm{max}} = \frac{M\,c}{I} = \frac{M}{(I/c)} = \frac{M}{S}$$

The ratio $S = I/c$ is called "section modulus."

Bending stress in fibers farthest from neutral surface:

$$f_{\max} = \frac{M}{(I/c)} = \frac{M}{S}$$

So you sketch your load, V, and M diagrams, and you find $M_{\max}$, i.e. the largest magnitude of $M(x)$.

Then, the material you are using for beams (wood, steel, etc.) has a maximum allowable bending stress, F_b.

So then you look in your table of beam cross-sections and choose

$$S \geq S_{\text{required}} = \frac{M_{\max}}{F_b}$$

Beam criteria:

- Normal stress in the extreme fibers of the beam (farthest from neutral surface) must be smaller than the allowable bending stress, F_b, which depends on the material (wood, steel, etc.).
- This happens where $M(x)$ has largest magnitude.
- Shear stress (in both y ("transverse") and x ("longitudinal") must be smaller than the allowable shear stress, F_v, which is also a property of the material (wood, steel, etc.).
- This happens where $V(x)$ has largest magnitude, and (surprisingly) is largest near the neutral surface.
- The above two are "strength" criteria. The third one is a "stiffness" criterion:
- The maximum deflection under load must satisfy the building code: typically $\Delta y_{\max} < L/360$.
- For a uniform load, this happens farthest away from the supports. If deflection is too large, plaster ceilings develop cracks, floors feel uncomfortably bouncy or sloped.
- The book also notes buckling as a beam failure mode.

Maximum deflection is one of several beam-design criteria. $\Delta_{\max}$ comes from integrating $M(x)/(EI)$ twice w.r.t. x to get $\Delta(x)$.

For uniform load w on simply-supported beam, you get

$$\Delta_{\max} = \frac{5wL^4}{384EI}$$

You just look these results up, or use a computer to calculate them. But I had great fun calculating the 5/384 myself!

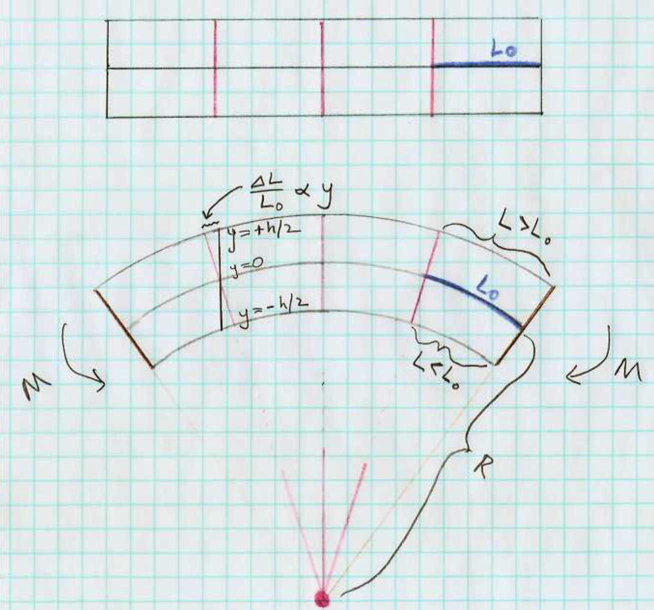
Deflection is proportional to load, and inversely proportional to Young's modulus and to the second moment of area.

- More load $\rightarrow$ more deflection
- Stiffer material $\rightarrow$ less deflection
- Cross-section with larger $I = \int y^2 \, \mathrm{d}A$ $\rightarrow$ less deflection

Notice that putting a column in the middle of a long, uniformly loaded beam reduces $\Delta_{\max}$ by a factor of $2^4 = 16$. Alternatively, if you want to span a large, open space without intermediate columns or bearing walls, you need beams with large I.

Bending beam into circular arc of radius R gives $\boxed{e = \frac{\Delta L}{L_0} = \frac{y}{R}}$,

strain e vs. distance y above the neutral surface.



Hooke's Law $\boxed{f = Ee}$

gives stress $\boxed{f = \frac{E\,y}{R}}$

Torque exerted by fibers of beam is

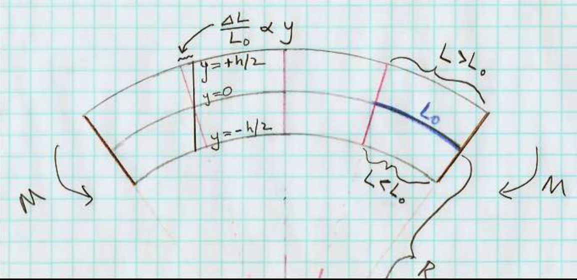
$$M = \int y\,(f\,\mathrm{d}A) =$$

$$y\,\frac{E\,y}{R}\,\mathrm{d}A = \frac{E}{R}\,y^2\,\mathrm{d}A$$

$$\boxed{M = \frac{EI}{R}}$$

Eliminate R $\Rightarrow$

$$\boxed{f = \frac{M\,y}{I} = \frac{M}{I/y}}$$



The bending stress a distance y above the neutral surface is

$$f = \frac{M\,y}{I}$$

The largest bending stress happens in the fibers farthest above or below the neutral surface. Call this largest distance $y_{\max} \equiv c$.

$$f_{\max} = \frac{M_{\max}\,c}{I} = \frac{M_{\max}}{(I/c)} = \frac{M_{\max}}{S}$$

The ratio $S = I/c$ is called "section modulus." The load diagram gives you $M_{\max}$. Each material (wood, steel, etc.) has allowed bending stress $f_{\max}$. Then $S_{\min}$ tells you how big a beam you need.

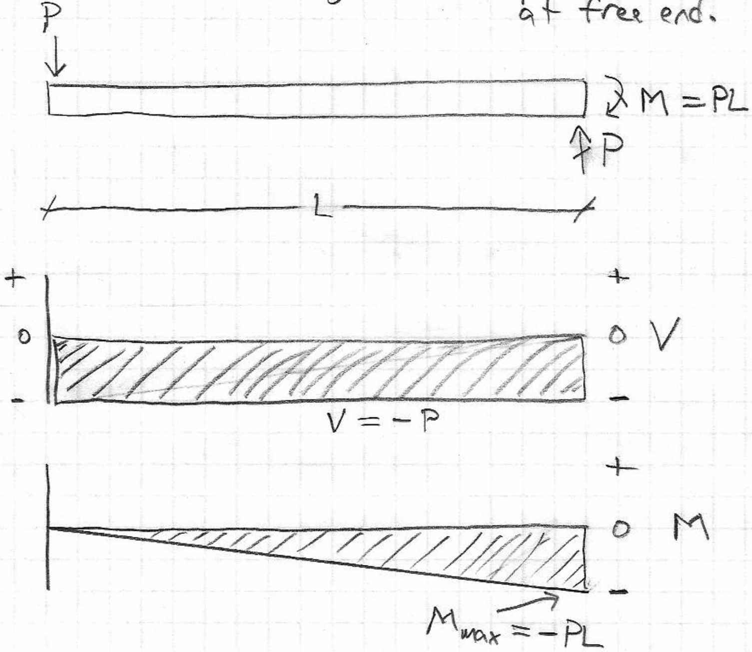
Example (using metric units!): A cantilever beam has a span of 3.0 m with a single concentrated load of 100 kg at its unsupported end. If the beam is made of timber having allowable bending stress $F_b = 1.1 \times 10^7\ \mathrm{N/m^2}$ (was 1600 psi in US units), what minimum section modulus is required?

What is the smallest "2×" dimensional lumber (width = 1.5 inch = 0.038 m) whose cross-section satisfies this strength criterion?

Would this beam also satisfy a $\Delta_{\mathrm{max}} < L/240$ (maximum deflection) stiffness criterion? If not, what standard "2×" cross-section is needed instead?

$\Delta_{\mathrm{max}} = PL^3/(3EI)$ for a cantilever with concentrated load at end. Use Young's modulus $E = 1.1 \times 10^{10}\ \mathrm{N/m^2}$ for southern pine.

Cantilever of length L with point load P at free end.



$$S_{\min} = \frac{|M_{\max}|}{f_{\text{allowed}}} = \frac{PL}{F_b} = \frac{(980\,\text{N})(3\,\text{m})}{1.1 \times 10^7\,\text{N/m}^2} = 26.7 \times 10^{-5}\,\text{m}^3$$

$$\Delta_{\text{allowed}} = \frac{L}{240} = \frac{3.0\,\text{m}}{240} = 0.0125\ \text{m}$$

$$\Delta_{\max} = \frac{PL^3}{3EI} \quad \Rightarrow \quad I_{\min} = \frac{PL^3}{3E\Delta_{\text{allowed}}} = 64.2 \times 10^{-6}\,\text{m}^4$$

I worked out b, h, I, and $S = I/c$ values in metric units for standard "2×" dimensional lumber.

	b	b	h	h	$I = bh^3/12$	$S = bh^2/6$
2×4	1.5 in	.038 m	3.5 in	.089 m	$2.23 \times 10^{-6}\ \mathrm{m}^4$	$5.02 \times 10^{-5}\ \mathrm{m}^3$
2×6	1.5 in	.038 m	5.5 in	.140 m	$8.66 \times 10^{-6}\ \mathrm{m}^4$	$12.4 \times 10^{-5}\ \mathrm{m}^3$
2×8	1.5 in	.038 m	7.5 in	.191 m	$21.9 \times 10^{-6}\ \mathrm{m}^4$	$23.0 \times 10^{-5}\ \mathrm{m}^3$
2×10	1.5 in	.038 m	9.5 in	.241 m	$44.6 \times 10^{-6}\ \mathrm{m}^4$	$37.0 \times 10^{-5}\ \mathrm{m}^3$
2×12	1.5 in	.038 m	11.5 in	.292 m	$79.1 \times 10^{-6}\ \mathrm{m}^4$	$54.2 \times 10^{-5}\ \mathrm{m}^3$

The numbers are nicer if you use centimeters instead of meters, but then you have the added hassle of remembering to convert back to meters in calculations.

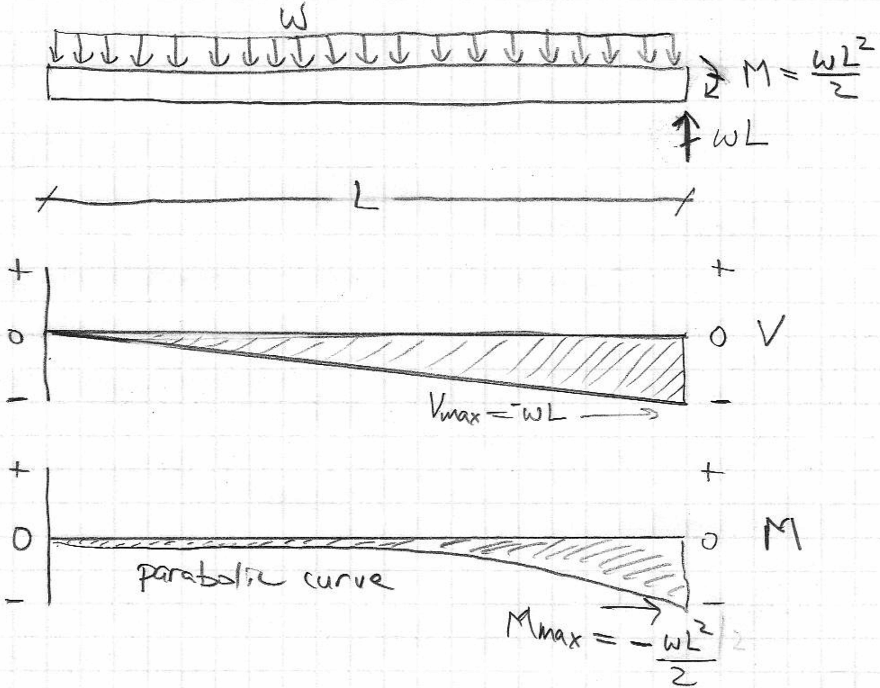
	b	b	h	h	$I = bh^3/12$	$S = bh^2/6$
2×4	1.5 in	3.8 cm	3.5 in	8.9 cm	$223\ \mathrm{cm}^4$	$50.2\ \mathrm{cm}^3$
2×6	1.5 in	3.8 cm	5.5 in	14.0 cm	$866\ \mathrm{cm}^4$	$124\ \mathrm{cm}^3$
2×8	1.5 in	3.8 cm	7.5 in	19.1 cm	$2195\ \mathrm{cm}^4$	$230\ \mathrm{cm}^3$
2×10	1.5 in	3.8 cm	9.5 in	24.1 cm	$4461\ \mathrm{cm}^4$	$370\ \mathrm{cm}^3$
2×12	1.5 in	3.8 cm	11.5 in	29.2 cm	$7913\ \mathrm{cm}^4$	$542\ \mathrm{cm}^3$

Minor variation on same problem: A cantilever beam has a span of 3.0 m with a uniform distributed load of 33.3 $\mathrm{kg/m}$ along its entire length. If we use timber with allowable bending stress $F_b = 1.1 \times 10^7\ \mathrm{N/m^2}$, what minimum section modulus is required?

What is the smallest "2×" dimensional lumber (width $=$ 1.5 inch $=$ 0.038 m) whose cross-section satisfies this strength criterion?

Would this beam also satisfy a $\Delta_{\max} < L/240$ (maximum deflection) stiffness criterion? If not, what standard "2×" cross-section is needed instead?

$\Delta_{\max} = wL^4/(8EI)$ for a cantilever with uniform load. Use Young's modulus $E = 1.1 \times 10^{10}\ \mathrm{N/m^2}$ for southern pine.


w
$M = \frac{wL^2}{2}$
wL
L
+
0
−
+
0
−
V
$V_{max} = -wL$
+
0
−
+
0
−
M
parabolic curve
$M_{max} = -\frac{wL^2}{2}$

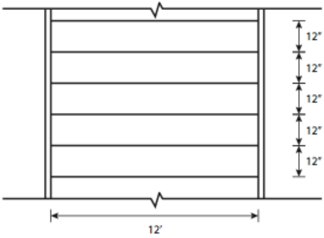
$$S_{\min} = \frac{|M_{\max}|}{f_{\text{allowed}}} = \frac{wL^2/2}{F_b} = \frac{(326\,\mathrm{N/m})(3\,\mathrm{m})^2/2}{1.1 \times 10^7\,\mathrm{N/m^2}} = 13.3 \times 10^{-5}\,\mathrm{m}^3$$

$$\Delta_{\text{allowed}} = \frac{L}{240} = \frac{3.0\,\mathrm{m}}{240} = 0.0125\ \mathrm{m}$$

$$\Delta_{\max} = \frac{wL^4}{8EI} \quad \Rightarrow \quad I_{\min} = \frac{wL^4}{8E\Delta_{\text{allowed}}} = 24.0 \times 10^{-6}\,\mathrm{m}^4$$

2) Size a wood joist for a row house floor which spans 12 feet. Joists are spaced at 16 inches on center.

f = 1,300 psi
f = 85 psi
E = 1.7 x 10 psi
LL = 60 psf
DL = 30 psf



Plan View

Hint: remember that a "2 x 4" wood joist is only nominal; its true dimensions are "1.5 x 3.5" inches.
(4 = 1.5, 6 = 5.5, 8 = 7.25, 10 = 9.25 inches)

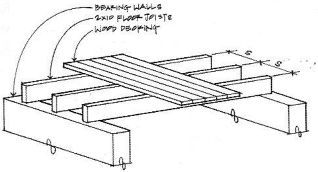
(Here's a homework problem from ARCH 435.)

Actually, Home Depot's 2×10 really is 9.5 inches deep, not 9.25 inches, and 2×12 really is 11.5 inches deep.

A timber floor system uses joists made of “2×10” dimensional lumber. Each joist spans a length of 4.27 m (simply supported). The floor carries a load of 2400 $\mathrm{N/m^2}$. At what spacing should the joists be placed, in order not to exceed allowable bending stress $F_b = 10000\ \mathrm{kN/m^2}\ (1.0 \times 10^7\ \mathrm{N/m^2})$?

(We should get an answer around 24 inches $=$ 0.61 meters.)

8.6 A timber floor system utilizing 2 × 10 S4S joists spans a length of 14′ (simply supported). The floor carries a load of 50 psf (DL + LL). At what spacing should the joists be placed? Assume Douglas Fir–Larch No. 2 (F_b = 1,450 psi).



Solution:

Based on the allowable stress criteria:

$$f = \frac{Mc}{I} = \frac{M}{S}$$

$$M_{max} = S \times f_b = (21.4 \text{ in.}^3)(1.45 \text{ k/in.}^3) = 31 \text{ k-in.}$$

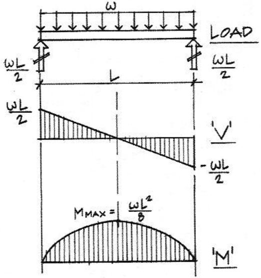
$$M = \frac{31 \text{ k-in.}}{12 \text{ in./ft.}} = 2.58 \text{ k-ft.}$$

Based on the bending moment diagram:

$$M_{max} = \frac{\omega L^2}{8}$$

Therefore,

$$\omega = \frac{8M}{L^2}$$



Substituting for M obtained previously,

$$\omega = \frac{8(2.58 \text{ k-ft.})}{(14')^2} = 0.105 \text{ k/ft.} = 105 \text{ #/ft.}$$

But

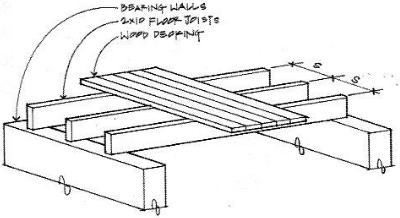
$$\omega = \text{#/ft.}^2 \times \text{tributary width (joist spacing } s)$$

$$s = \frac{\omega}{50 \text{ psf}} = \frac{105 \text{ #/ft.}}{50 \text{ #/ft.}^2} = 2.1'$$

$$s = 25'' \text{ spacing}$$

Use 24″ o.c. spacing.

8.6 A timber floor system utilizing 2 × 10 S4S joists spans a length of 14′ (simply supported). The floor carries a load of 50 psf (DL + LL). At what spacing should the joists be placed? Assume Douglas Fir–Larch No. 2 (F_b = 1,450 psi).



Solution:

Based on the allowable stress criteria:

$$f = \frac{Mc}{I} = \frac{M}{S}$$

$$M_{max} = S \times f_b = (21.4 \text{ in.}^3)(1.45 \text{ k/in.}^3) = 31 \text{ k-in.}$$

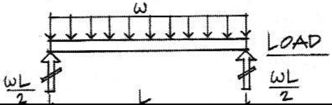
$$M = \frac{31 \text{ k-in.}}{12 \text{ in./ft.}} = 2.58 \text{ k-ft.}$$

Based on the bending moment diagram:

$$M_{max} = \frac{\omega L^2}{8}$$

Therefore,

$$\omega = \frac{8M}{L^2}$$

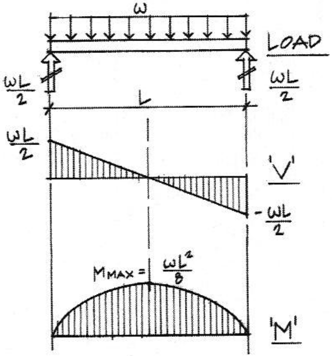


Based on the bending moment diagram:

$$M_{\max} = \frac{\omega L^2}{8}$$

Therefore,

$$\omega = \frac{8M}{L^2}$$



Substituting for M obtained previously,

$$\omega = \frac{8(2.58 \text{ k-ft.})}{(14')^2} = 0.105 \text{ k/ft.} = 105 \text{ \#/ft.}$$

But

$$\omega = \text{\#/ft.}^2 \times \text{tributary width (joist spacing } s)$$

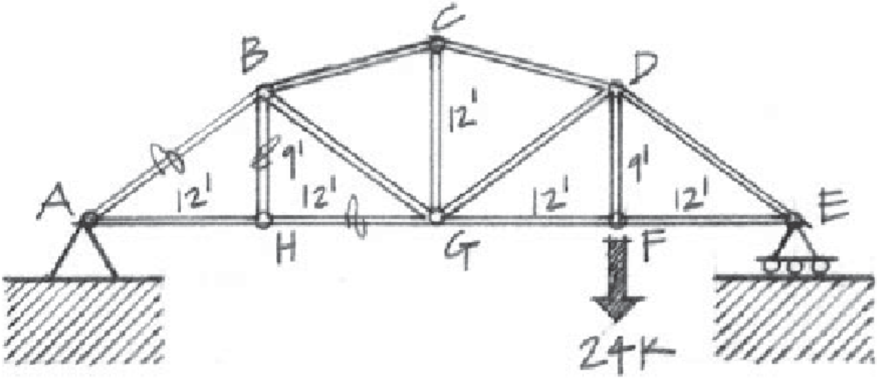
$$s = \frac{\omega}{50 \text{ psf}} = \frac{105 \text{ \#/ft.}}{50 \text{ \#/ft.}^2} = 2.1'$$

$$s = 25'' \text{ spacing}$$

Use 24″ o.c. spacing.

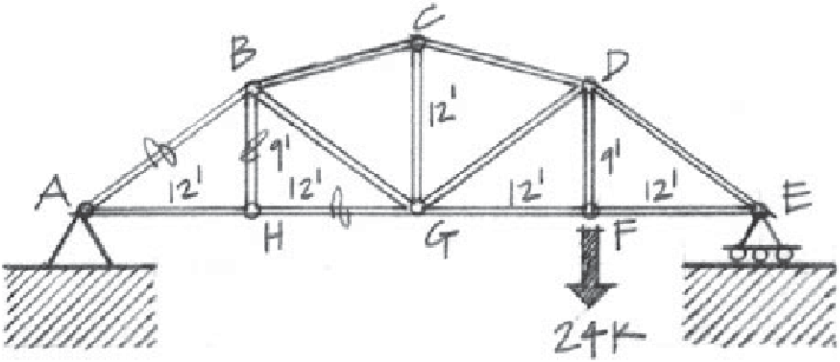
Note: *Spacing is more practical for plywood subflooring, based on a 4 ft. module of the sheet.*

Using the method of sections, solve for the forces in members AB, BH, and HG in the truss shown below. Indicate whether each of these members is in tension or in compression. Use only one section cut through the truss.



If we have time left, let's solve this truss problem together. It's actually pretty quick, using method of sections. First solve for vertical support force at A, then analyze left side of section.

Using the method of sections, solve for the forces in members AB, BH, and HG in the truss shown below. Indicate whether each of these members is in tension or in compression. Use only one section cut through the truss.



XC2. **(I haven't checked this with anyone else yet.)** For the truss as a whole $\sum F_x = 0$ gives $\boxed{R_{Ax} = 0}$. Then $\sum M_A = 0 = R_{Ey}(48') - 24\text{k}(36')$ gives $\boxed{R_{Ey} = 18\text{k}}$. Then $\sum F_y = 0 = R_{Ay} + R_{Ey} - 24\text{k}$ gives $\boxed{R_{Ay} = 6\text{k}}$. Now section the truss through members AB, BH, and GH and analyze the left-hand side. Then $\sum M_A = 0 = T_{BH}(12')$ gives $\boxed{T_{BH} = 0}$, which one can see by inspection of the vertical forces at joint H: bar BH is a "zero-force member." Then $\sum F_y = 0 = +6\text{k} + (3/5)T_{AB} + T_{BH}$ gives $\boxed{T_{AB} = -10\text{k}}$ (i.e. compression). Finally, $\sum F_x = 0 = (4/5)T_{AB} + T_{GH}$ gives $\boxed{T_{GH} = 8\text{k}}$.

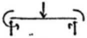
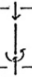
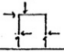
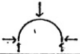
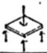
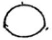
Physics 8 — Friday, November 22, 2019

- Turn in HW11 either today or Monday, as you prefer.
- I just added to my "equation sheet" the math behind the shear-stress results Prof. Farley illustrated for us on Wednesday:
`http://positron.hep.upenn.edu/p8/files/equations.pdf#page=22`
- Today and Monday are our last two days on beams. Then we'll spend a week on oscillation / vibration / periodic motion.
- I hope to hand out the take-home practice exam (due Dec 6) on Monday (Nov 25), but if I don't get it done in time, I will put the PDF online before Thanksgiving. I intend for it to be comparable in length and coverage to the in-class final exam (Dec 12, noon, A1).
- I plan to do a review session on Dec 11, time/place TBD.
- Prof. Farley will join us Monday via FaceTime. He'd like us to send him in advance questions you'd like him to answer.
- Next Wednesday, you don't have to show up, but if you do, you'll get a bonus point. I will spend the hour introducing "Python Mode for Processing" (`py.processing.org`) — a visual-arts-oriented approach to writing code to do drawing and animation.

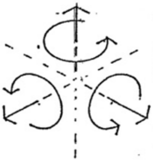
Physics 8 — Monday, November 25, 2019

- Turn in HW11.
- Today is our last day on beams. Then we'll spend a week on oscillation / vibration / periodic motion.
- I'll put the PDF of the take-home practice exam online before Thanksgiving. I intend for it to be similar in coverage to the in-class final exam (Dec 12, noon, A1), though the in-class exam will be shorter than the take-home. If you turn it in on Friday, Dec 6, then I will email it back to you after class on Monday, Dec 9. If you turn it in on Monday, Dec 9, then I will give it back to you at the Wednesday (Dec 11) review session.
- I plan to do a review session on Dec 11, time/place TBD.
- Prof. Farley plans to join us today. Stop me before it gets too late!
- Wednesday, you don't have to show up, but if you do, you'll get a bonus point. I will spend the hour introducing "Python Mode for Processing" (`py.processing.org`) — a visual-arts-oriented approach to writing code to do drawing and animation. Last time we coded a simplified "breakout" video game. This year we may code a highly simplified "asteroids" video game, if all goes well.

ELEMENTS OF STRUCTURE

	1D in 1D	Beam
	1D in 1D	Column
	1D in 2D	Frame
	1D in 2D	Truss
	1D in 2D	Arch
	2D in 2D	Slab Plate
	2D in 2D	Wall
	1D in 3D	Spaceframe
	2D in 3D	Vault Shell
	2D in 3D	Dome Membrane
	3D in 3D	Foundations Soils

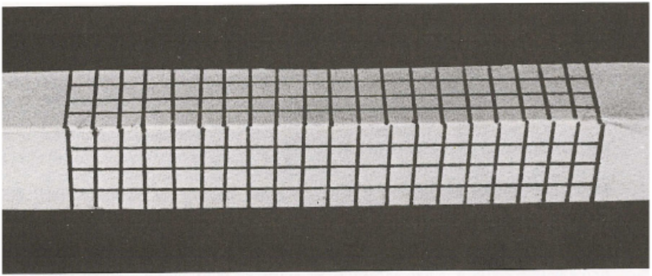
DEGREES OF FREEDOM



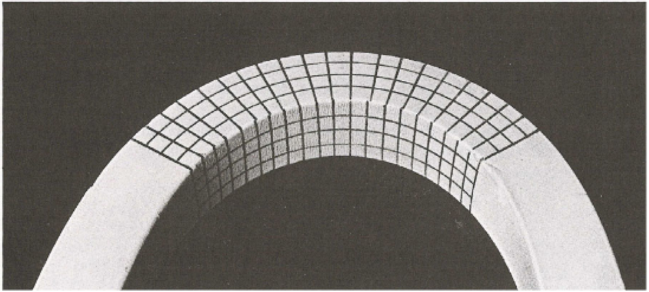
IDEAS OF STRUCTURE

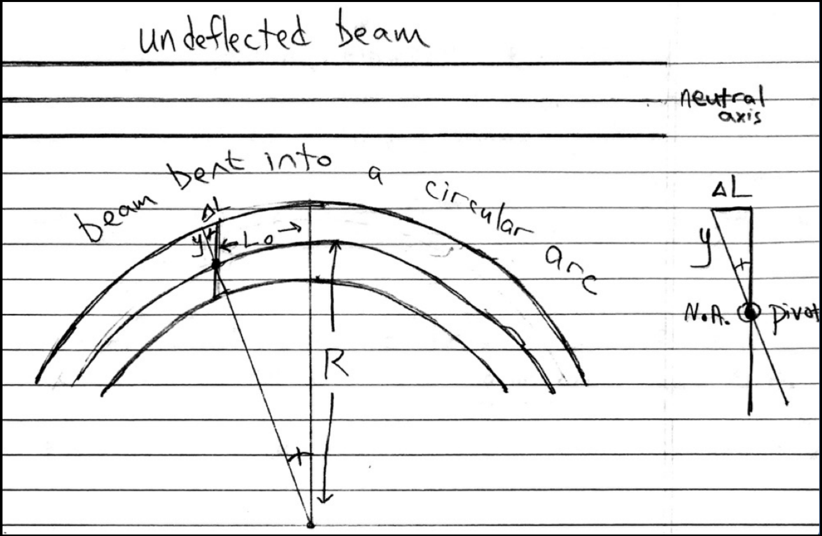
- Force
- Restraint
- Equilibrium
- Stress
- Deformation
- Elasticity
- Geometry
- Strength
- Stiffness
- Scale
- Continuity
- Stability
- Safety

rubber beam with grid lines subjected to pure bending.



(a) Before Loading





By similar triangles, $\frac{\Delta L}{y} = \frac{L_0}{R} \quad \Rightarrow \quad$ strain $e = \frac{\Delta L}{L_0} = \frac{y}{R}$

Hooke's law: $f = eE \quad \Rightarrow \quad \boxed{f = \frac{Ey}{R}}$ (1)

f = stress = force per unit area. E = Young's modulus.

Imagine a fiber running along the length of the bent beam. Let the fiber have cross-section area $\mathrm{d}A$ and height y above the neutral surface. The tension (force) in the fiber is

$$\mathrm{d}F = f\,\mathrm{d}A = \frac{E}{R}\,y\,\mathrm{d}A$$

Pivoting about the neutral axis, the moment (torque) exerted by this fiber is (since y is the lever arm from the pivot)

$$\mathrm{d}M = y\,\mathrm{d}F = \frac{E}{R}\,y^2\,\mathrm{d}A$$

To find the total bending moment exerted by this cross-section of beam, we add up all of the fibers over the entire cross-section:

$$M = \frac{E}{R}\int y^2\,\mathrm{d}A = \frac{EI}{R} \qquad \text{where} \qquad \boxed{I = \int y^2\,\mathrm{d}A} \quad (2)$$

- One factor of y comes from strain $\Delta L / L_0 \propto y$.
- The second factor of y is lever arm above the N.A.

So the beam's radius of curvature is $\boxed{R = \frac{EI}{M}}$ (3) (illustrate).

Combine (1) + (3) $\Rightarrow$ bending stress $f = \frac{Ey}{EI/M} = \boxed{\frac{My}{I} = f}$

The maximum bending stress is

$$f_{\max} = \frac{|M|_{\max}|y|_{\max}}{I} = \boxed{\frac{|M|_{\max}}{S} = f_{\max}}$$

where S is the "section modulus" $\boxed{S = \frac{I}{|y|_{\max}}}$

- know load & span $\rightarrow$ find $|M|_{\max}$
- know type of material $\rightarrow$ allowable $f_{\max}$

$$S_{\text{required}} \geq \frac{|M|_{\max}}{f_{\text{allowable}}}$$

tells you how "big" a beam cross-section you need for this load, span, & material, to meet the maximum-bending-stress criterion, which is a "strength" criterion (not a "stiffness" criterion).

In calculus, $\frac{1}{R}$ quantifies the "curvature" of a function $Y(x)$

$$\text{curvature} = \frac{1}{R} = \frac{Y''(x)}{[1 + Y'(x)^2]^{3/2}} \approx Y''(x)$$

The curvature of a function is closely related to its second derivative $Y''(x)$. If the slope $|Y'(x)| \ll 1$, as is true for beams used in structures, then $\frac{1}{R} = Y''(x)$.

For clarity, I'll write $Y(x)$ for the shape of the deflected beam, and reserve y to denote height above the neutral surface.

$$Y''(x) = \frac{1}{R} = \frac{M(x)}{EI}$$

$$\text{slope} \quad Y'(x) = \frac{1}{EI} \int M(x)\,\mathrm{d}x$$

$$Y(x) = \frac{1}{EI} \int \mathrm{d}x \int M(x)\,\mathrm{d}x$$

deflection under load $\Delta(x) = -Y(x)$. This is where you get $\Delta_{\max} = \frac{5wL^4}{384EI}$ for simply-supported beam with uniform load w, etc.

This Onouye/Kane figure writes "y" here for deflection, but I wrote "Y" for deflected beam shape, because we were already using y for "distance above the neutral surface."

So you integrate $M(x)/EI$ twice w.r.t. x to get the deflected beam shape $Y(x)$.

The bending moment $M(x) = EI\ \mathrm{d}^2Y/\mathrm{d}x^2$, where E is Young's modulus and I is second moment of area.

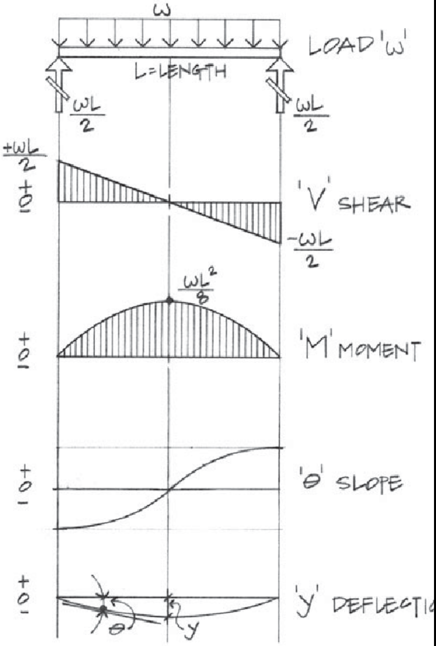
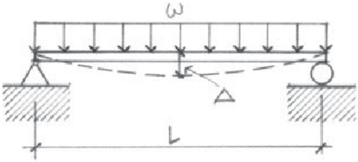
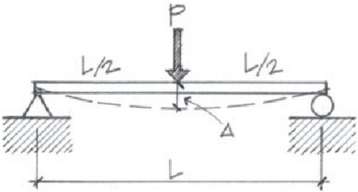
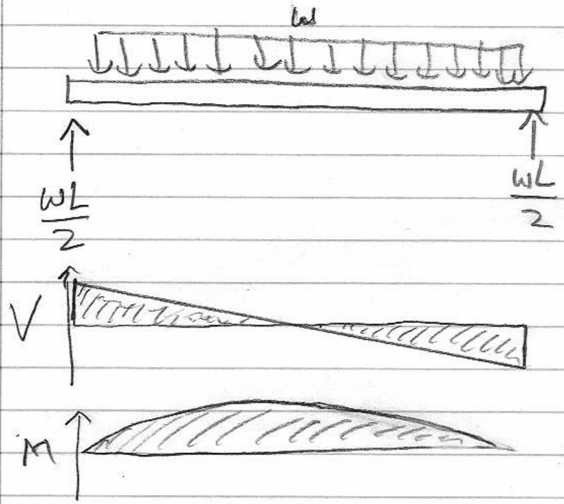


Figure 7.11 Relationship of load, shear, moment, slope, and deflection diagrams.

The most common deflection results can be found in tables.

Beam Load and Support	**Actual Deflection***
 (a) Uniform load, simple span	$\Delta_{max} = \frac{5\omega L^4}{384EI}$ (at the centerline)
 (b) Concentrated load at midspan	$\Delta_{max} = \frac{PL^3}{48EI}$ (at the centerline)

FYI, here's where that crazy $(5wL^4)/(384EI)$ comes from!



$$V(x) = \frac{wL}{2} - wx$$
$$= w\left(\frac{L}{2} - x\right)$$

$$M(x) = \frac{wLx}{2} - \frac{wx^2}{2}$$
$$= \frac{w}{2}\left(Lx - x^2\right)$$

(continued on next page)

Here's where that crazy $(5wL^4)/(384EI)$ comes from!

$$\Delta(x) = -\frac{1}{EI}\int dx \int M(x)\, dx$$

$$= -\frac{w}{2EI}\int dx \int (Lx - x^2)\, dx = -\frac{w}{2EI}\int dx \left(\frac{Lx^2}{2} - \frac{x^3}{3} + C_1\right)$$

$$= -\frac{w}{2EI}\left(\frac{Lx^3}{6} - \frac{x^4}{12} + C_1 x + C_2\right)$$

$$\Delta(0) = 0 \Rightarrow \boxed{C_2 = 0}$$

$$\Delta(L) = 0 \Rightarrow \frac{L^4}{6} - \frac{L^4}{12} + C_1 L = 0 \Rightarrow \boxed{C_1 = -\frac{L^3}{12}}$$

$$\Delta(x) = -\frac{w}{2EI}\left(\frac{Lx^3}{6} - \frac{x^4}{12} - \frac{L^3 x}{12}\right)$$

$$\Delta_{max} = \Delta\left(\frac{L}{2}\right) = -\frac{w}{2EI}\left(\frac{L(L/2)^3}{6} - \frac{(L/2)^4}{12} - \frac{L^3(L/2)}{12}\right)$$

$$= -\frac{wL^4}{2EI}\left(\frac{1}{48} - \frac{1}{192} - \frac{1}{24}\right) = -\frac{wL^4}{2EI}\left(-\frac{5}{192}\right) = \boxed{\frac{5wL^4}{384EI}}$$

The 2 integration constants can be tricky. Simply supported: $\Delta(0) = \Delta(L) = 0$. (For cantilever, $\Delta(0) = \Delta'(0) = 0$ instead.)

Maximum deflection is one of several beam-design criteria. $\Delta_{\max}$ comes from integrating $M(x)/(EI)$ twice w.r.t. x to get $\Delta(x)$.

For uniform load w on simply-supported beam, you get

$$\Delta_{\max} = \frac{5wL^4}{384EI}$$

You just look these results up, or use a computer to calculate them. But I had great fun calculating the 5/384 myself!

Deflection is proportional to load, and inversely proportional to Young's modulus and to the second moment of area.

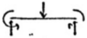
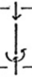
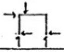
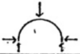
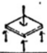
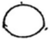
- More load $\rightarrow$ more deflection
- Stiffer material $\rightarrow$ less deflection
- Cross-section with larger $I = \int y^2 \, \mathrm{d}A$ $\rightarrow$ less deflection

Notice that putting a column in the middle of a long, uniformly loaded beam reduces $\Delta_{\max}$ by a factor of $2^4 = 16$. Alternatively, if you want to span a large, open space without intermediate columns or bearing walls, you need beams with large I.

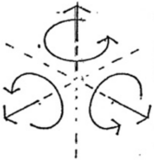
Beam criteria:

- Normal stress in the extreme fibers of the beam (farthest from neutral surface) must be smaller than the allowable bending stress, F_b, which depends on the material (wood, steel, etc.).
- This happens where $M(x)$ has largest magnitude.
- Shear stress (in both y ("transverse") and x ("longitudinal") must be smaller than the allowable shear stress, F_v, which is also a property of the material (wood, steel, etc.).
- This happens where $V(x)$ has largest magnitude, and (surprisingly) is largest near the neutral surface.
- The above two are "strength" criteria. The third one is a "stiffness" criterion:
- The maximum deflection under load must satisfy the building code: typically $\Delta y_{\max} < L/360$.
- For a uniform load, this happens farthest away from the supports. If deflection is too large, plaster ceilings develop cracks, floors feel uncomfortably bouncy or sloped.
- The book also notes buckling as a beam failure mode.

ELEMENTS OF STRUCTURE

	1D in 1D	Beam
	1D in 1D	Column
	1D in 2D	Frame
	1D in 2D	Truss
	1D in 2D	Arch
	2D in 2D	Slab Plate
	2D in 2D	Wall
	1D in 3D	Spaceframe
	2D in 3D	Vault Shell
	2D in 3D	Dome Membrane
	3D in 3D	Foundations Soils

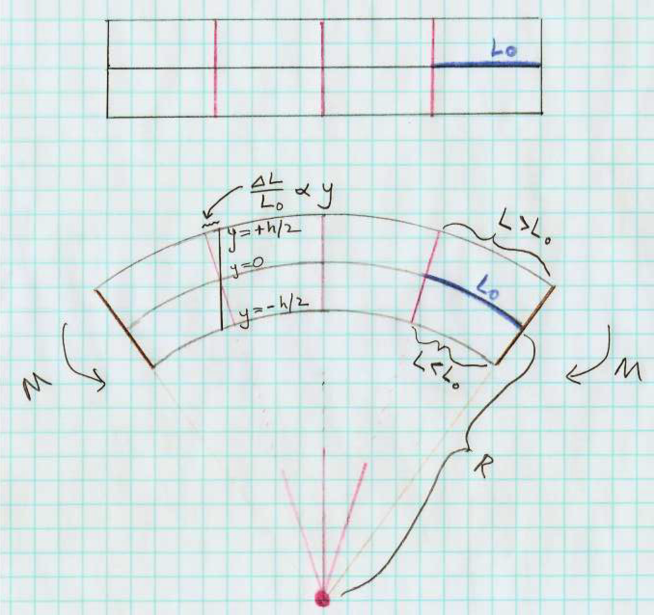
DEGREES OF FREEDOM



IDEAS OF STRUCTURE

- Force
- Restraint
- Equilibrium
- Stress
- Deformation
- Elasticity
- Geometry
- Strength
- Stiffness
- Scale
- Continuity
- Stability
- Safety

Key idea: bending moment $M \propto \frac{1}{R}$, where R is the radius of curvature of the beam. For constant M, vertical lines converge toward common center of curvature.



$$\text{strain} = \frac{\Delta L}{L_0} = \frac{y}{R}$$

where $y = 0$ is the neutral surface.

So in this case $y > 0$ is in tension and $y < 0$ is in compression.

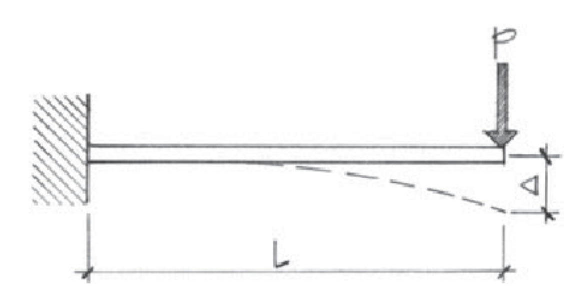
$$M = \frac{E}{R} \int y^2 \mathrm{d}A = \frac{EI}{R}$$

Meanwhile, the vertical *deflection* Δ of a point along the beam is related to its curvature by (in limit where $\Delta \ll R$)

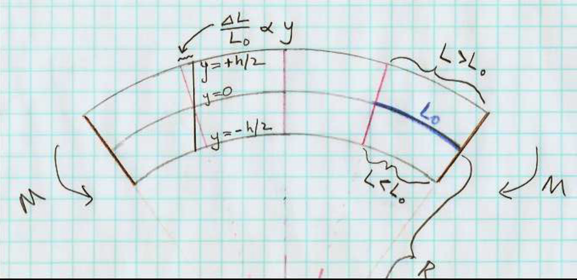
$$\frac{1}{R} \approx -\frac{\mathrm{d}^2\Delta}{\mathrm{d}x^2}$$

so you can integrate the $M(x)$ curve twice to get deflection

$$\frac{\mathrm{d}^2\Delta}{\mathrm{d}x^2} = -\frac{M}{EI} \quad \Rightarrow \quad \Delta(x) = -\frac{1}{EI}\int \mathrm{d}x \int M(x)\,\mathrm{d}x$$



Another beam-design criterion is maximum bending stress: the fibers farthest from the neutral surface experience the largest tension or compression, hence largest bending stress.

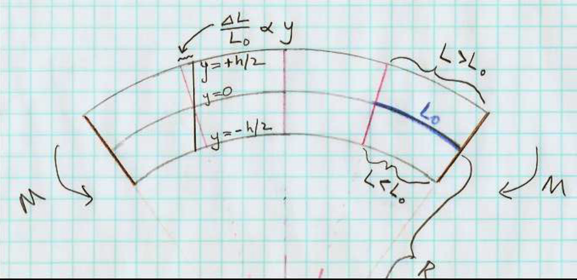


When we section the beam at x, bending moment $M(x)$ is

$$M = \frac{EI}{R}$$

which we can solve for the radius of curvature $R = EI/M$. Then the stress a distance y above the neutral surface is

$$f = Ee = E\,\frac{y}{R} = \frac{E\,y}{(EI/M)} = \frac{M\,y}{I}$$



The bending stress a distance y above the neutral surface is

$$f = \frac{M\,y}{I}$$

The largest bending stress happens in the fibers farthest above or below the neutral surface. Call this largest distance $y_{\max} \equiv c$.

$$f_{\max} = \frac{M\,c}{I} = \frac{M}{(I/c)} = \frac{M}{S}$$

The ratio $S = I/c$ is called "section modulus."

Bending stress in fibers farthest from neutral surface:

$$f_{\max} = \frac{M}{(I/c)} = \frac{M}{S}$$

So you sketch your load, V, and M diagrams, and you find $M_{\max}$, i.e. the largest magnitude of $M(x)$.

Then, the material you are using for beams (wood, steel, etc.) has a maximum allowable bending stress, F_b.

So then you look in your table of beam cross-sections and choose

$$S \geq S_{\text{required}} = \frac{M_{\max}}{F_b}$$

Maximum deflection is one of several beam-design criteria. $\Delta_{\max}$ comes from integrating $M(x)/(EI)$ twice w.r.t. x to get $\Delta(x)$.

For uniform load w on simply-supported beam, you get

$$\Delta_{\max} = \frac{5wL^4}{384EI}$$

You just look these results up, or use a computer to calculate them. But I had great fun calculating the 5/384 myself!

Deflection is proportional to load, and inversely proportional to Young's modulus and to the second moment of area.

- More load $\rightarrow$ more deflection
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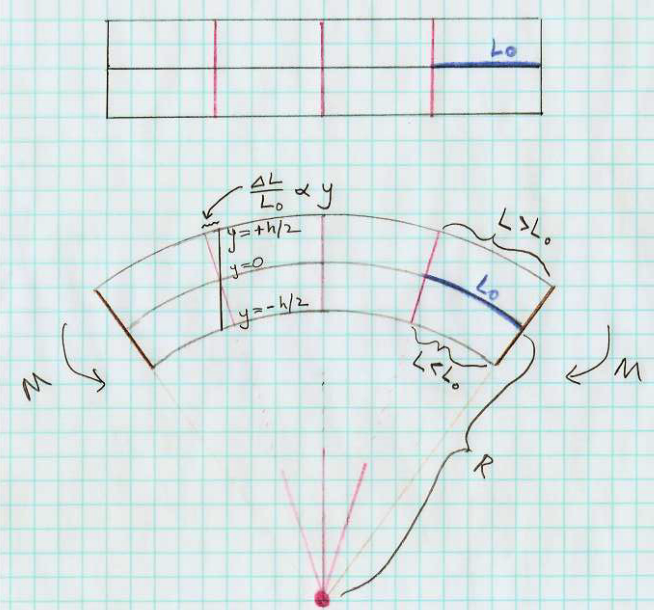
- Cross-section with larger $I = \int y^2 \, \mathrm{d}A$ $\rightarrow$ less deflection

Notice that putting a column in the middle of a long, uniformly loaded beam reduces $\Delta_{\max}$ by a factor of $2^4 = 16$. Alternatively, if you want to span a large, open space without intermediate columns or bearing walls, you need beams with large I.

Bending beam into circular arc of radius R gives $\boxed{e = \frac{\Delta L}{L_0} = \frac{y}{R}}$,

strain e vs. distance y above the neutral surface.



Hooke's Law $\boxed{f = Ee}$

gives stress $\boxed{f = \frac{E\,y}{R}}$

Torque exerted by fibers of beam is

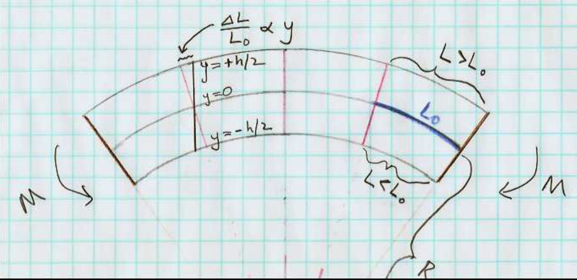
$$M = \int y\,(f\,\mathrm{d}A) =$$

$$y\,\frac{E\,y}{R}\,\mathrm{d}A = \frac{E}{R}\,y^2\,\mathrm{d}A$$

$$\boxed{M = \frac{EI}{R}}$$

Eliminate R $\Rightarrow$

$$\boxed{f = \frac{M\,y}{I} = \frac{M}{I/y}}$$



The bending stress a distance y above the neutral surface is

$$f = \frac{M\,y}{I}$$

The largest bending stress happens in the fibers farthest above or below the neutral surface. Call this largest distance $y_{\mathrm{max}} \equiv c$.

$$f_{\mathrm{max}} = \frac{M_{\mathrm{max}}\,c}{I} = \frac{M_{\mathrm{max}}}{(I/c)} = \frac{M_{\mathrm{max}}}{S}$$

The ratio $S = I/c$ is called "section modulus." The load diagram gives you M_{max}. Each material (wood, steel, etc.) has allowed bending stress f_{max}. Then S_{min} tells you how big a beam you need.

(The next few slides contain beam-design examples. Let's skip ahead to slide 1038)

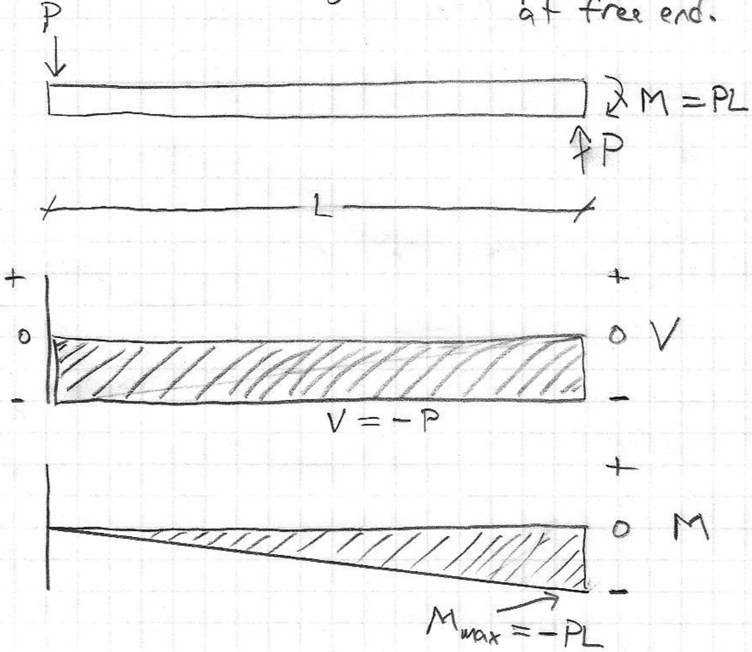
Example (using metric units!): A cantilever beam has a span of 3.0 m with a single concentrated load of 100 kg at its unsupported end. If the beam is made of timber having allowable bending stress $F_b = 1.1 \times 10^7\ \mathrm{N/m^2}$ (was 1600 psi in US units), what minimum section modulus is required?

What is the smallest "2×" dimensional lumber (width $=$ 1.5 inch $=$ 0.038 m) whose cross-section satisfies this strength criterion?

Would this beam also satisfy a $\Delta_{\mathrm{max}} < L/240$ (maximum deflection) stiffness criterion? If not, what standard "2×" cross-section is needed instead?

$\Delta_{\mathrm{max}} = PL^3/(3EI)$ for a cantilever with concentrated load at end. Use Young's modulus $E = 1.1 \times 10^{10}\ \mathrm{N/m^2}$ for southern pine.

Cantilever of length L with point load P at free end.



$$S_{\min} = \frac{|M_{\max}|}{f_{\text{allowed}}} = \frac{PL}{F_b} = \frac{(980\,\mathrm{N})(3\,\mathrm{m})}{1.1 \times 10^7\,\mathrm{N/m^2}} = 26.7 \times 10^{-5}\,\mathrm{m}^3$$

$$\Delta_{\text{allowed}} = \frac{L}{240} = \frac{3.0\,\mathrm{m}}{240} = 0.0125\ \mathrm{m}$$

$$\Delta_{\max} = \frac{PL^3}{3EI} \quad \Rightarrow \quad I_{\min} = \frac{PL^3}{3E\Delta_{\text{allowed}}} = 64.2 \times 10^{-6}\,\mathrm{m}^4$$

I worked out b, h, I, and $S = I/c$ values in metric units for standard "2×" dimensional lumber.

	b	b	h	h	$I = bh^3/12$	$S = bh^2/6$
2×4	1.5 in	.038 m	3.5 in	.089 m	$2.23 \times 10^{-6}\ \mathrm{m}^4$	$5.02 \times 10^{-5}\ \mathrm{m}^3$
2×6	1.5 in	.038 m	5.5 in	.140 m	$8.66 \times 10^{-6}\ \mathrm{m}^4$	$12.4 \times 10^{-5}\ \mathrm{m}^3$
2×8	1.5 in	.038 m	7.5 in	.191 m	$21.9 \times 10^{-6}\ \mathrm{m}^4$	$23.0 \times 10^{-5}\ \mathrm{m}^3$
2×10	1.5 in	.038 m	9.5 in	.241 m	$44.6 \times 10^{-6}\ \mathrm{m}^4$	$37.0 \times 10^{-5}\ \mathrm{m}^3$
2×12	1.5 in	.038 m	11.5 in	.292 m	$79.1 \times 10^{-6}\ \mathrm{m}^4$	$54.2 \times 10^{-5}\ \mathrm{m}^3$

The numbers are nicer if you use centimeters instead of meters, but then you have the added hassle of remembering to convert back to meters in calculations.

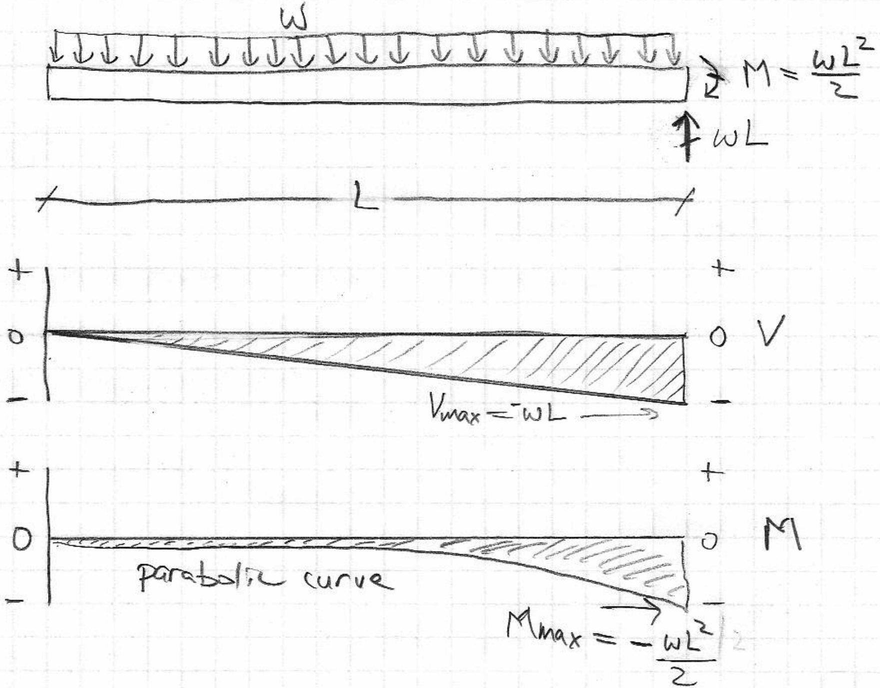
	b	b	h	h	$I = bh^3/12$	$S = bh^2/6$
2×4	1.5 in	3.8 cm	3.5 in	8.9 cm	$223\ \mathrm{cm}^4$	$50.2\ \mathrm{cm}^3$
2×6	1.5 in	3.8 cm	5.5 in	14.0 cm	$866\ \mathrm{cm}^4$	$124\ \mathrm{cm}^3$
2×8	1.5 in	3.8 cm	7.5 in	19.1 cm	$2195\ \mathrm{cm}^4$	$230\ \mathrm{cm}^3$
2×10	1.5 in	3.8 cm	9.5 in	24.1 cm	$4461\ \mathrm{cm}^4$	$370\ \mathrm{cm}^3$
2×12	1.5 in	3.8 cm	11.5 in	29.2 cm	$7913\ \mathrm{cm}^4$	$542\ \mathrm{cm}^3$

Minor variation on same problem: A cantilever beam has a span of 3.0 m with a uniform distributed load of 33.3 $\mathrm{kg/m}$ along its entire length. If we use timber with allowable bending stress $F_b = 1.1 \times 10^7\ \mathrm{N/m^2}$, what minimum section modulus is required?

What is the smallest "2×" dimensional lumber (width $= 1.5$ inch $= 0.038$ m) whose cross-section satisfies this strength criterion?

Would this beam also satisfy a $\Delta_{\mathrm{max}} < L/240$ (maximum deflection) stiffness criterion? If not, what standard "2×" cross-section is needed instead?

$\Delta_{\mathrm{max}} = wL^4/(8EI)$ for a cantilever with uniform load. Use Young's modulus $E = 1.1 \times 10^{10}\ \mathrm{N/m^2}$ for southern pine.


w
M = wL^2/2
wL
L
+
0
−
V
Vmax = −wL
+
0
−
M
parabolic curve
Mmax = −wL^2/2

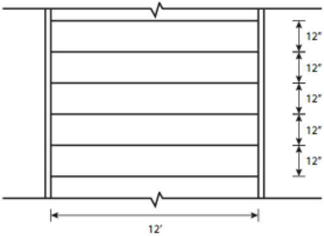
$$S_{\mathrm{min}} = \frac{|M_{\mathrm{max}}|}{f_{\mathrm{allowed}}} = \frac{wL^2/2}{F_b} = \frac{(326\,\mathrm{N/m})(3\,\mathrm{m})^2/2}{1.1 \times 10^7\,\mathrm{N/m^2}} = 13.3 \times 10^{-5}\,\mathrm{m}^3$$

$$\Delta_{\mathrm{allowed}} = \frac{L}{240} = \frac{3.0\,\mathrm{m}}{240} = 0.0125\ \mathrm{m}$$

$$\Delta_{\mathrm{max}} = \frac{wL^4}{8EI} \quad \Rightarrow \quad I_{\mathrm{min}} = \frac{wL^4}{8E\Delta_{\mathrm{allowed}}} = 24.0 \times 10^{-6}\,\mathrm{m}^4$$

2) Size a wood joist for a row house floor which spans 12 feet. Joists are spaced at 16 inches on center.

f = 1,300 psi
f = 85 psi
E = 1.7 x 10 psi
LL = 60 psf
DL =30 psf



Plan View

Hint: remember that a "2 x 4" wood joist is only nominal; its true dimensions are "1.5 x 3.5" inches. (4 = 1.5, 6 = 5.5, 8 = 7.25, 10 = 9.25 inches)

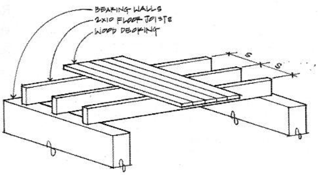
(Here's a homework problem from ARCH 435.)

Actually, Home Depot's 2×10 really is 9.5 inches deep, not 9.25 inches, and 2×12 really is 11.5 inches deep.

A timber floor system uses joists made of "2×10" dimensional lumber. Each joist spans a length of 4.27 m (simply supported). The floor carries a load of 2400 $\mathrm{N/m^2}$. At what spacing should the joists be placed, in order not to exceed allowable bending stress $F_b = 10000\ \mathrm{kN/m^2}\ (1.0 \times 10^7\ \mathrm{N/m^2})$?

(We should get an answer around 24 inches $=$ 0.61 meters.)

8.6 A timber floor system utilizing 2 × 10 S4S joists spans a length of 14′ (simply supported). The floor carries a load of 50 psf (DL + LL). At what spacing should the joists be placed? Assume Douglas Fir–Larch No. 2 (F_b = 1,450 psi).



Solution:

Based on the allowable stress criteria:

$$f = \frac{Mc}{I} = \frac{M}{S}$$

$$M_{max} = S \times f_b = (21.4 \text{ in.}^3)(1.45 \text{ k/in.}^3) = 31 \text{ k-in.}$$

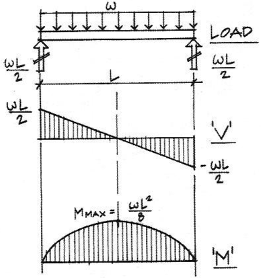
$$M = \frac{31 \text{ k-in.}}{12 \text{ in./ft.}} = 2.58 \text{ k-ft.}$$

Based on the bending moment diagram:

$$M_{max} = \frac{\omega L^2}{8}$$

Therefore,

$$\omega = \frac{8M}{L^2}$$



Substituting for M obtained previously,

$$\omega = \frac{8(2.58 \text{ k-ft.})}{(14')^2} = 0.105 \text{ k/ft.} = 105 \text{ \#/ft.}$$

But

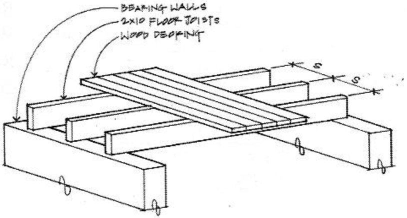
$$\omega = \text{\#/ft.}^2 \times \text{tributary width (joist spacing } s)$$

$$s = \frac{\omega}{50 \text{ psf}} = \frac{105 \text{ \#/ft.}}{50 \text{ \#/ft.}^2} = 2.1'$$

$$s = 25'' \text{ spacing}$$

Use 24″ o.c. spacing.

8.6 A timber floor system utilizing 2 × 10 S4S joists spans a length of 14′ (simply supported). The floor carries a load of 50 psf (DL + LL). At what spacing should the joists be placed? Assume Douglas Fir–Larch No. 2 (F_b = 1,450 psi).



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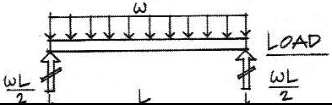

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But

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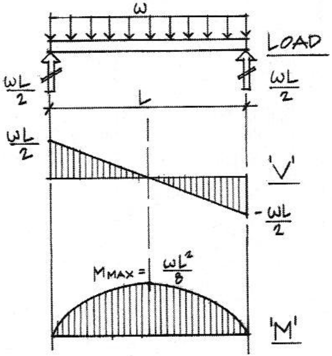
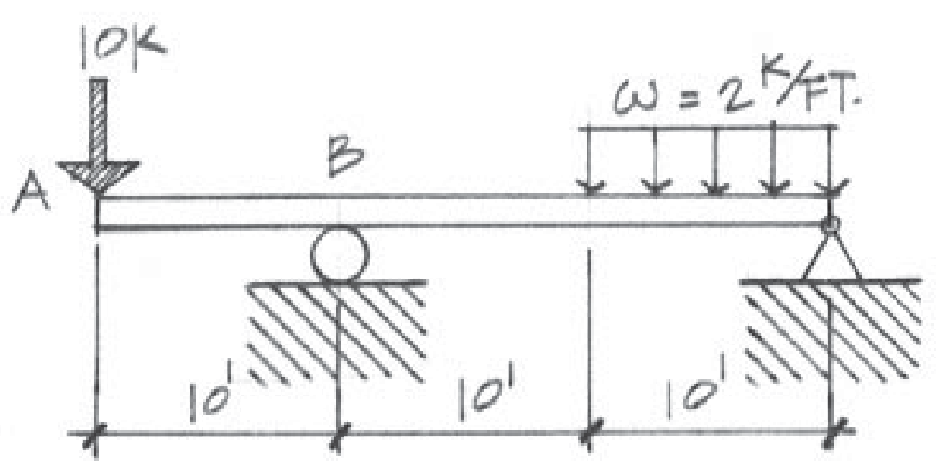


$$s = \frac{\omega}{50 \text{ psf}} = \frac{105 \text{ \#/ft.}}{50 \text{ \#/ft.}^2} = 2.1'$$

$s = 25''$ spacing

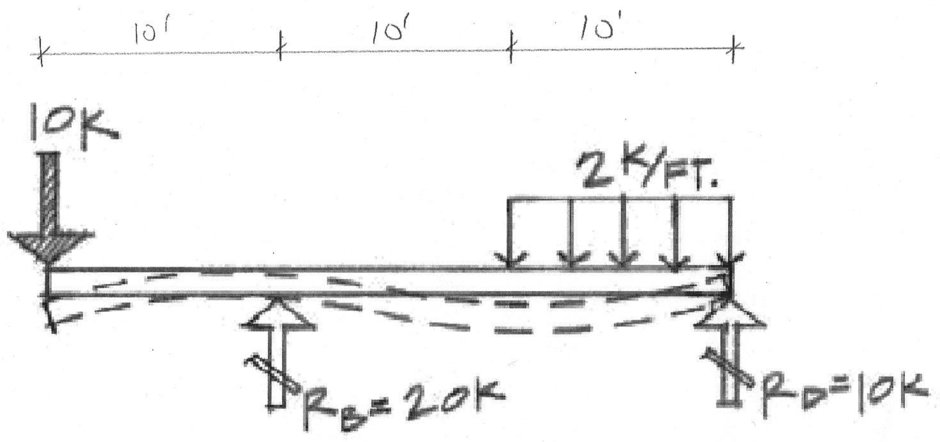
Use 24″ o.c. spacing.

Note: *Spacing is more practical for plywood subflooring, based on a 4 ft. module of the sheet.*

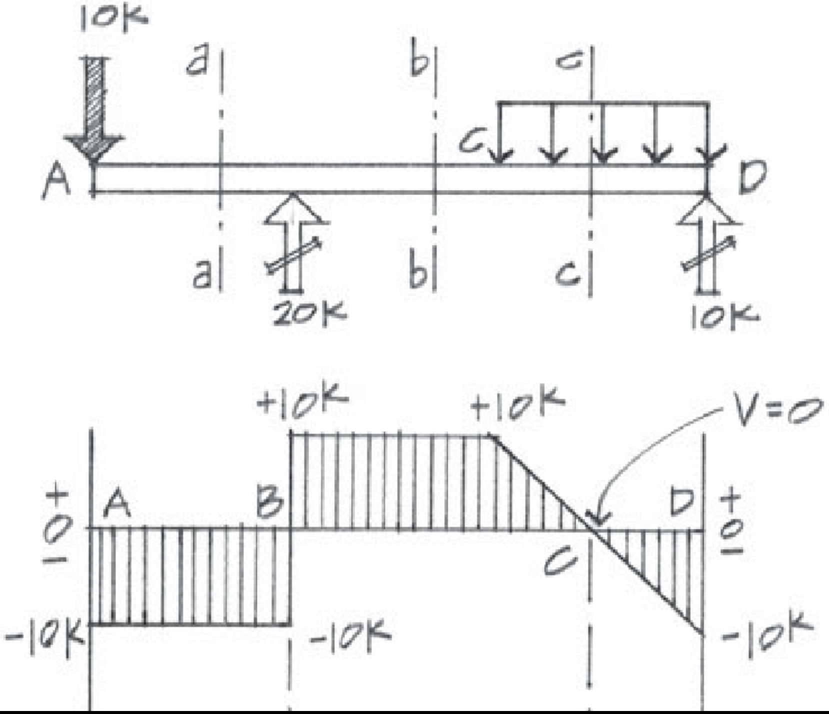


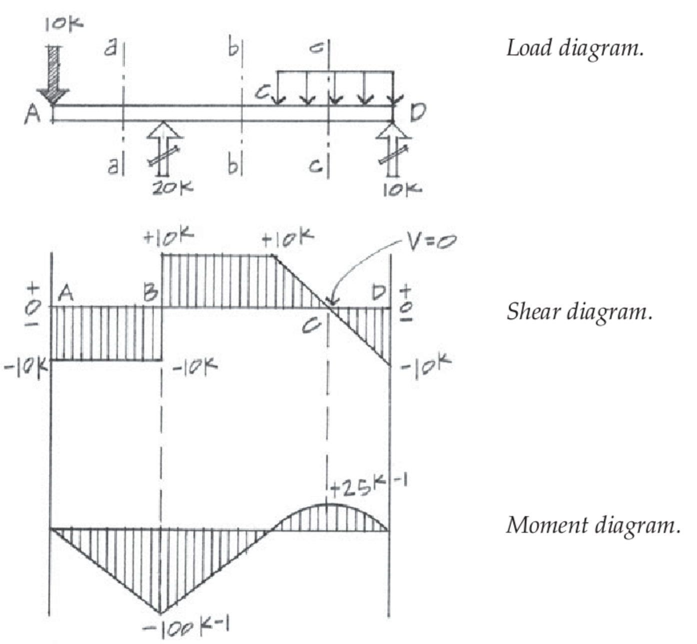
Draw shear (V) and moment (M) diagrams for this beam! Tricky! First one needs to solve for the support ("reaction") forces.

Note: in solving for the support forces, you replace distributed load w with equivalent point load. But when you draw the load diagram to find V and M, you need to keep w in its original form.



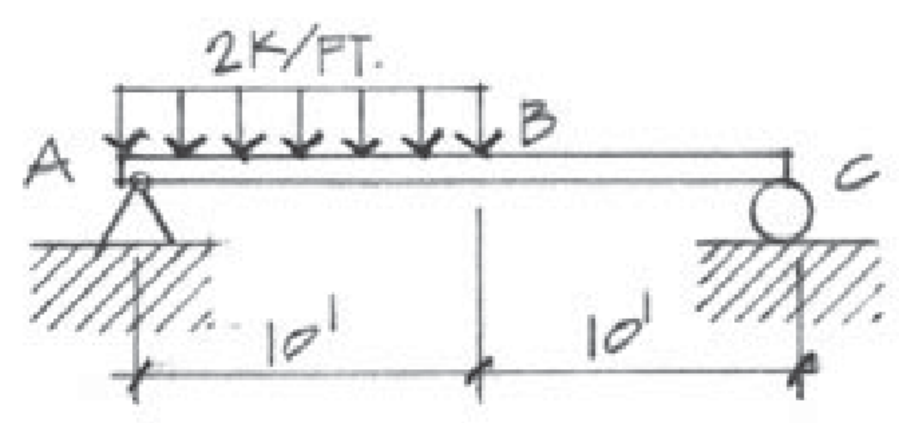
Remember that $V(x)$ is the running sum, from LHS to x, of vertical forces acting on the beam, with upward=positive.

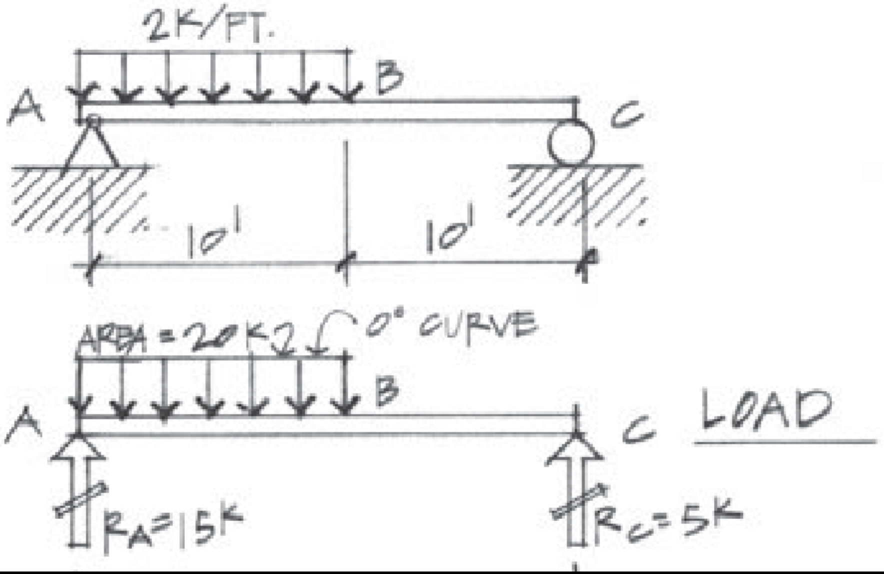

10K
a
b
c
A
D
a
b
c
20K
10K
+10K
+10K
V=0
+
0
−
A
B
D
C
−10K
−10K
−10K



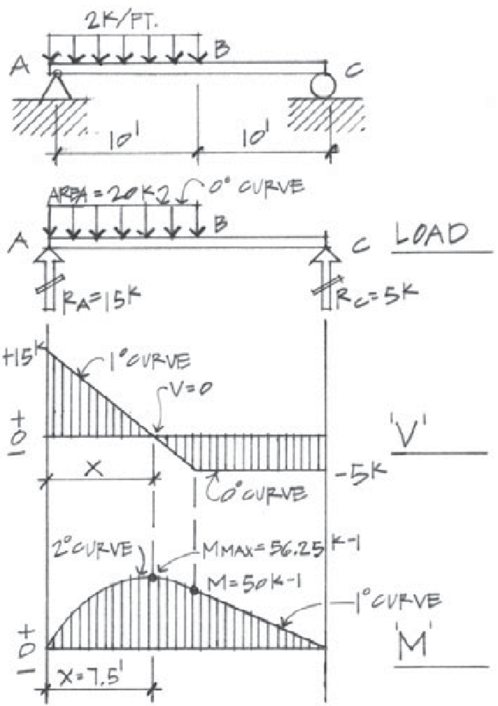
Neat trick: $M_2 - M_1 = (V_{1\to 2}^{\text{average}})(x_2 - x_1)$

Draw load, V, and M diagrams for this simply supported beam with a partial uniform load.

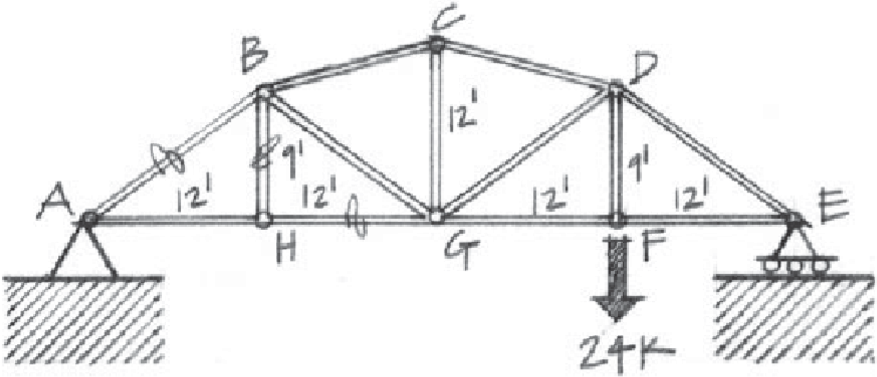




2K/FT.
A
B
C
10'
10'
AREA = 20K
0° CURVE
A
B
C
LOAD
RA=15K
RC=5K

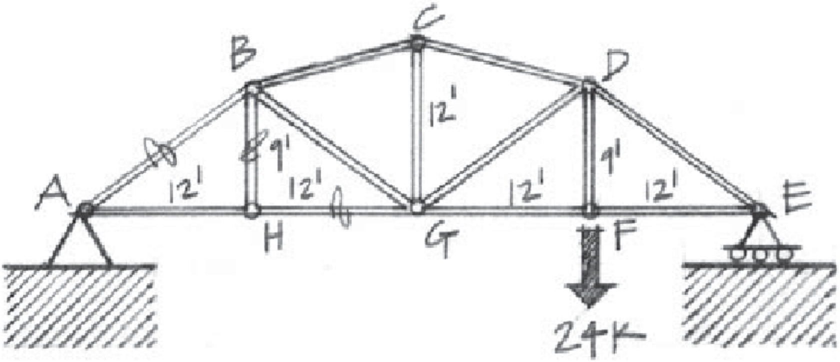

2K/FT.
A
B
C
10'
10'
AREA = 20K
0° CURVE
LOAD
R_A=15K
R_C=5K
+15K
1° CURVE
V=0
+
0
−
X
-5K
0° CURVE
'V'
2° CURVE
MMAX=56.25K-1
M=50K-1
1° CURVE
X=7.5'
'M'

Using the method of sections, solve for the forces in members AB, BH, and HG in the truss shown below. Indicate whether each of these members is in tension or in compression. Use only one section cut through the truss.



If we have time left, let's solve this truss problem together. It's actually pretty quick, using method of sections. First solve for vertical support force at A, then analyze left side of section.

Using the method of sections, solve for the forces in members AB, BH, and HG in the truss shown below. Indicate whether each of these members is in tension or in compression. Use only one section cut through the truss.



XC2. **(I haven't checked this with anyone else yet.)** For the truss as a whole $\sum F_x = 0$ gives $\boxed{R_{Ax} = 0}$. Then $\sum M_A = 0 = R_{Ey}(48') - 24\text{k}(36')$ gives $\boxed{R_{Ey} = 18\text{k}}$. Then $\sum F_y = 0 = R_{Ay} + R_{Ey} - 24\text{k}$ gives $\boxed{R_{Ay} = 6\text{k}}$. Now section the truss through members AB, BH, and GH and analyze the left-hand side. Then $\sum M_A = 0 = T_{BH}(12')$ gives $\boxed{T_{BH} = 0}$, which one can see by inspection of the vertical forces at joint H: bar BH is a "zero-force member." Then $\sum F_y = 0 = +6\text{k} + (3/5)T_{AB} + T_{BH}$ gives $\boxed{T_{AB} = -10\text{k}}$ (i.e. compression). Finally, $\sum F_x = 0 = (4/5)T_{AB} + T_{GH}$ gives $\boxed{T_{GH} = 8\text{k}}$.

Physics 8 — Monday, November 25, 2019

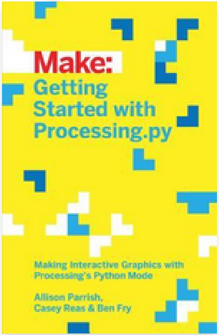
- Turn in HW11.
- Today is our last day on beams. Then we'll spend a week on oscillation / vibration / periodic motion.
- I'll put the PDF of the take-home practice exam online before Thanksgiving. I intend for it to be similar in coverage to the in-class final exam (Dec 12, noon, A1), though the in-class exam will be shorter than the take-home. If you turn it in on Friday, Dec 6, then I will email it back to you after class on Monday, Dec 9. If you turn it in on Monday, Dec 9, then I will give it back to you at the Wednesday (Dec 11) review session.
- I plan to do a review session on Dec 11, time/place TBD.
- Prof. Farley plans to join us today.
- Wednesday, you don't have to show up, but if you do, you'll get a bonus point. I will spend the hour introducing "Python Mode for Processing" (`py.processing.org`) — a visual-arts-oriented approach to writing code to do drawing and animation.

Physics 8 — Wednesday, November 27, 2019

- I'll put take-home practice exam (due 12/6 or 12/9) online this evening. I put 4 years' old exams online at

  http://positron.hep.upenn.edu/p8/files/oldexams/
- Today: a tutorial of the "Processing.py" computer programming language — whose purpose is to learn how to code within the context of the visual arts. It makes coding fun and visual. Processing.py is a Python-based version of the (Java-based) Processing programming environment.
- Extra-credit options (if you're interested):
  - Learn to use Mathematica (ask me how), which is a system for doing mathematics by computer. (It is the brains behind Wolfram Alpha.) Penn's site license makes Mathematica free-of-charge for SAS and Wharton students.
  - Use "Processing.py" (or ordinary "Processing") to write a program to draw or animate something that interests you. (Not necessarily physics-related.)
  - Knowing "how to code" is empowering & enlightening. So I offer you an excuse to give it a try, for extra credit, if you wish.
- Today's examples online at

  http://positron.hep.upenn.edu/p8/files/pyprocessing/



Want to Read

Rate this book

Getting Started with Processing.Py: Making Interactive Graphics with Processing's Python Mode

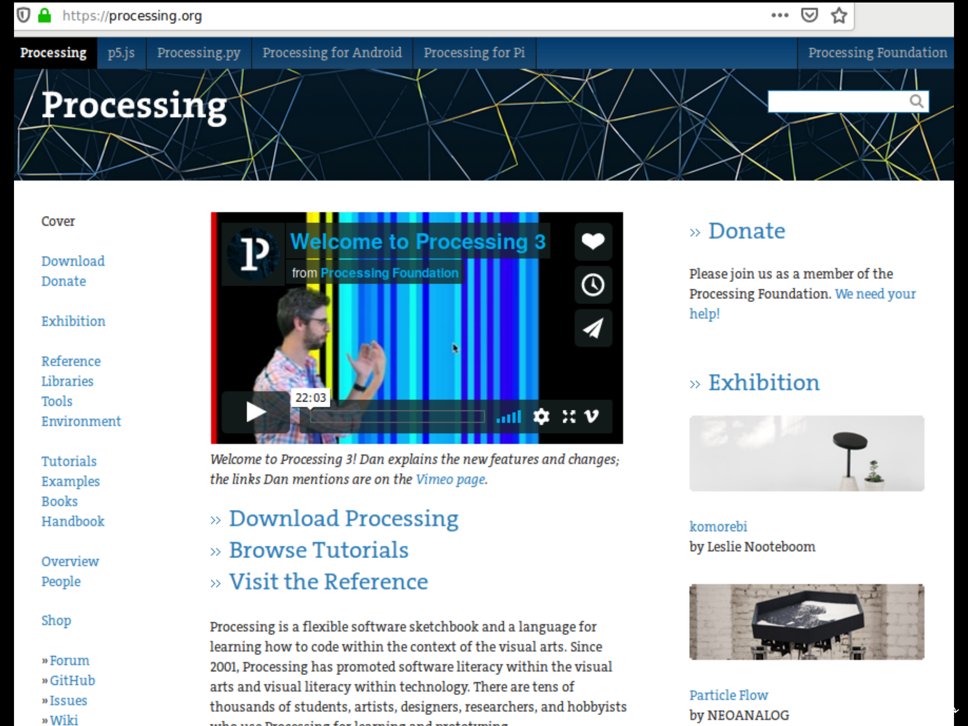
by Allison Parrish, Ben Fry, Casey Reas

4.88 · Rating details · 8 Ratings · 3 Reviews

Processing opened up the world of programming to artists, designers, educators, and beginners. The Processing.py Python implementation of Processing reinterprets it for today's web. This short book gently introduces the core concepts of computer programming and working with Processing. Written by the co-founders of the Processing project, Reas and Fry, along with co-author Allison Parrish, Getting Started with Processing.py is your fast track to using Python's Processing mode. (less)

The software is free & open-source. Runs on Mac, Windows, Linux. The “getting started” book will set you back about $15.

or start with the in-browser video tutorial (no download needed): `http://hello.processing.org` (Processing, **not** Processing.py)


https://processing.org
Processing
p5.js
Processing.py
Processing for Android
Processing for Pi
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Welcome to Processing 3
from Processing Foundation
22:03
Welcome to Processing 3! Dan explains the new features and changes; the links Dan mentions are on the Vimeo page.
» Download Processing
» Browse Tutorials
» Visit the Reference
Processing is a flexible software sketchbook and a language for learning how to code within the context of the visual arts. Since 2001, Processing has promoted software literacy within the visual arts and visual literacy within technology. There are tens of thousands of students, artists, designers, researchers, and hobbyists
» Donate
Please join us as a member of the Processing Foundation. We need your help!
» Exhibition
komorebi
by Leslie Nooteboom
Particle Flow
by NEOANALOG

Python Mode for Processing

You write Processing code. In Python.

Processing is a programming language, development environment, and online community. Since 2001, Processing has promoted software literacy within the visual arts and visual literacy within technology. Today, there are tens of thousands of students, artists, designers, researchers, and hobbyists who use Processing for learning, prototyping, and production.

Processing was initially released with a Java-based syntax, and with a lexicon of graphical primitives that took inspiration from OpenGL, Postscript, Design by Numbers, and other sources. With the gradual addition of alternative progamming interfaces — including JavaScript, Python, and Ruby — it has become increasingly clear that Processing is not a single language, but rather, an arts-oriented approach to learning, teaching, and making things with code.

https://processing.org/download/

Processing | p5.js | Processing.py | Processing for Android | Processing for Pi | Processing Founda

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Environment

Tutorials
Examples
Books
Handbook

Overview
People

Download Processing. Processing is available for Linux, Mac OS X, and Windows. Select your choice to download the software below.

3.4 (26 July 2018)

Windows 64-bit
Windows 32-bit

Linux 64-bit
Linux 32-bit
Linux ARM
(running on Pi?)

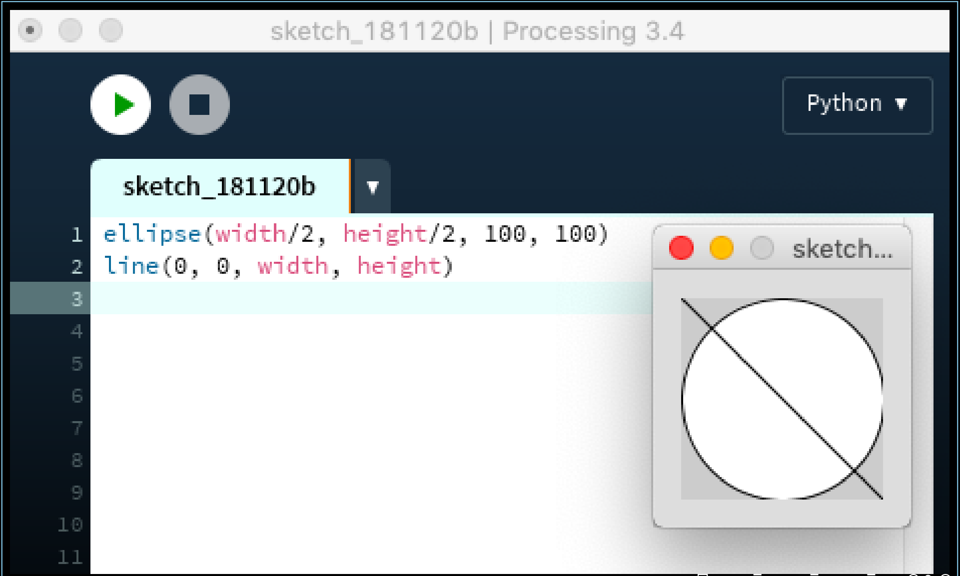
Mac OS X

» Github
» Report Bugs
» Wiki
» Supported Platforms

Read about the changes in 3.0. The list of revisions covers the differences between releases in detail.

“hello world” program

Let’s draw a circle and a line.



More commonly, a Processing program has a function called `setup()` that runs once when the program starts, and another function called `draw()` that runs once per frame.

```
def setup():
  # this function runs once when the program starts up
  size(900, 450)  # sets width & height of window (in pixels)

def draw():
  # this function runs once per frame of the animation
  line(0, frameCount, width, height-frameCount)
```

Let's make it do something repetitive

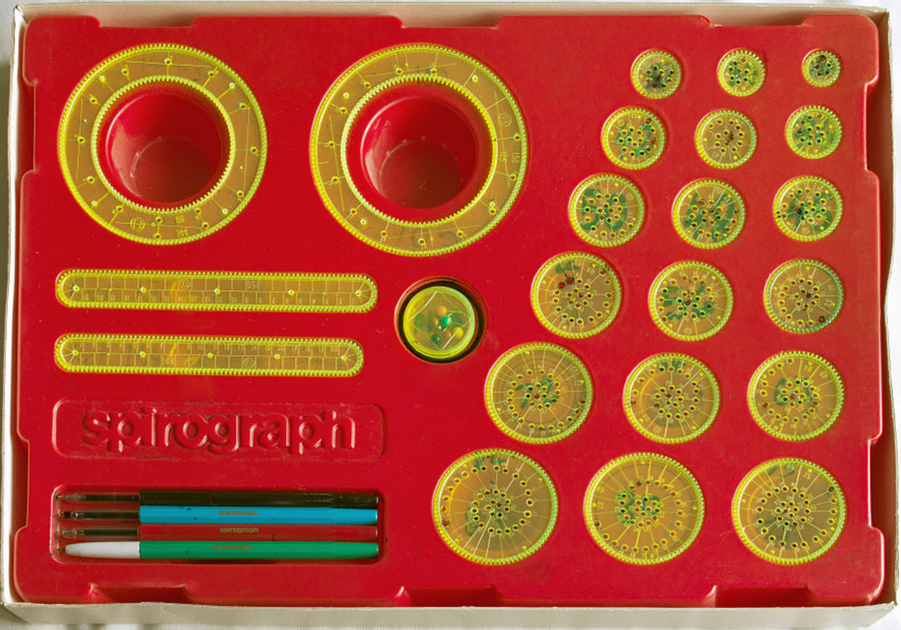
```
def setup():
  # this function runs once when the program starts up
  size(900, 450)  # sets width & height of window (in pixels)

def draw():
  # this function runs once per frame of the animation
  dy = 0.5*height + 0.5*height*sin(0.01*frameCount)
  line(0, dy, width, height-dy)
```

How about repeating something more exciting?

```
def setup():
  # this function runs once when the program starts up
  size(900, 450)  # sets width & height of window (in pixels)

def draw():
  # this function runs once per frame of the animation
  dy = 0.5*height + 0.5*height*sin(0.01*frameCount)
  line(0, dy, width, height-dy)
  t = 0.02*frameCount
  x = 0.5*width + 200*cos(t)
  y = 0.5*height + 200*sin(t)
  ellipse(x, y, 20, 20)
```

Did you ever have a Spirograph toy when you were a kid?

```
def setup():
  size(900, 450)

def draw():
  t = 0.02*frameCount
  x = 0.5*width + 200*cos(t) + 30*cos(11*t)
  y = 0.5*height + 200*sin(t) - 30*sin(11*t)
  ellipse(x, y, 5, 5)
```


spirograph

How about something that starts to resemble physics? A really, really low-tech animation of an planet orbiting a star.

```
def setup():
  size(900, 450)

def draw():
  t = 0.01*frameCount
  xsun = 0.5*width
  ysun = 0.5*height
  ellipse(xsun, ysun, 20, 20)
  rplanet = 200
  xplanet = xsun + rplanet*cos(t)
  yplanet = ysun + rplanet*sin(t)
  ellipse(xplanet, yplanet, 10, 10)
```

Let's add a moon in orbit around the planet.

```
def draw():
  t = 0.01*frameCount
  xsun = 0.5*width
  ysun = 0.5*height
  # clear screen before each new frame
  background(128)
  # draw sun
  ellipse(xsun, ysun, 20, 20)
  rplanet = 200
  xplanet = xsun + rplanet*cos(t)
  yplanet = ysun + rplanet*sin(t)
  # draw planet
  ellipse(xplanet, yplanet, 10, 10)
  rmoon = 30
  xmoon = xplanet + rmoon*cos(t*365/27.3)
  ymoon = yplanet + rmoon*sin(t*365/27.3)
  # draw moon
  ellipse(xmoon, ymoon, 5, 5)
```

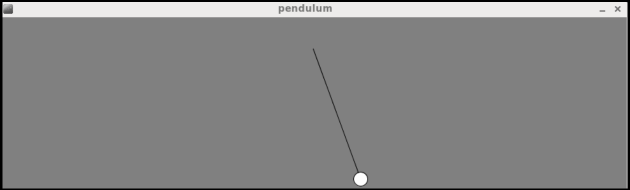
How about adding an inner planet?

```
def draw():
   ... other stuff suppressed ...
  # draw moon
  ellipse(xmoon, ymoon, 5, 5)
  # add second planet
  year_mercury_days = 115.88  # from Wikipedia
  T_ratio = year_mercury_days/365.25
  R_ratio = T_ratio**(2.0/3)
  xplanet = xsun + R_ratio*rplanet*cos(t/T_ratio)
  yplanet = ysun + R_ratio*rplanet*sin(t/T_ratio)
  ellipse(xplanet, yplanet, 7, 7)
```

Animate a pendulum (skip?)

```
def setup():
  size(900, 450)

def draw():
  t = 0.01*frameCount
  g = 9.8
  L = 2.0
  degree = PI/180
  amplitude = 20*degree
  omega = sqrt(g/L)
  theta = amplitude * sin(omega*t)
  xbob = L * sin(theta)
  ybob = L * cos(theta)
  # convert coordinates into pixel coordinates
  ... continued on next slide ...
```

```
def draw():
  ... continued from previous slide ...
  # convert coordinates into pixel coordinates
  xpixel_pivot = 0.5*width
  ypixel_pivot = 0.1*height
  scale = 100.0  # pixels per meter
  xpixel_bob = xpixel_pivot + scale*xbob
  ypixel_bob = ypixel_pivot + scale*ybob
  # clear the screen for each new frame of animation
  background(128)
  # draw the string
  line(xpixel_pivot, ypixel_pivot, xpixel_bob, ypixel_bob)
  # draw the bob
  ellipse(xpixel_bob, ypixel_bob, 20, 20)
```

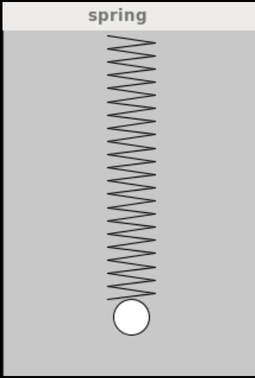


Animate a mass bobbing on a spring

```
def draw():
  t = 0.01*frameCount
  omega = 1.0
  amplitude = 0.5
  Lequilibrium = 2.0
  xbob = 0
  ybob = Lequilibrium + amplitude * cos(omega*t)
  xpixel_anchor = 0.5*width
  ypixel_anchor = 0.01*height
  scale = 100.0
  xpixel_bob = xpixel_anchor + scale*xbob
  ypixel_bob = ypixel_anchor + scale*ybob
  // draw the bob
  rbob = 15
  ellipse(xpixel_bob, ypixel_bob, 2*rbob, 2*rbob)
```

Clear screen between frames; draw the spring

```
def draw():
  ... other stuff suppressed ...
  # clear the screen for each new frame
  background(200)
  # draw the bob
  rbob = 15
  ellipse(xpixel_bob, ypixel_bob, 2*rbob, 2*rbob)
  # draw the spring as a series of zig-zag lines
  nzigzag = 20
  for i in range(nzigzag):
    spring_top = ypixel_anchor
    spring_bottom = ypixel_bob - rbob
    dy = (spring_bottom-spring_top)/nzigzag
    xzig = xpixel_anchor - 20
    yzig = ypixel_anchor + i*dy
    xzag = xpixel_anchor + 20
    ymid = yzig + 0.5*dy
    yzag = yzig + dy
    line(xzig, yzig, xzag, ymid)
    line(xzag, ymid, xzig, yzag)
```

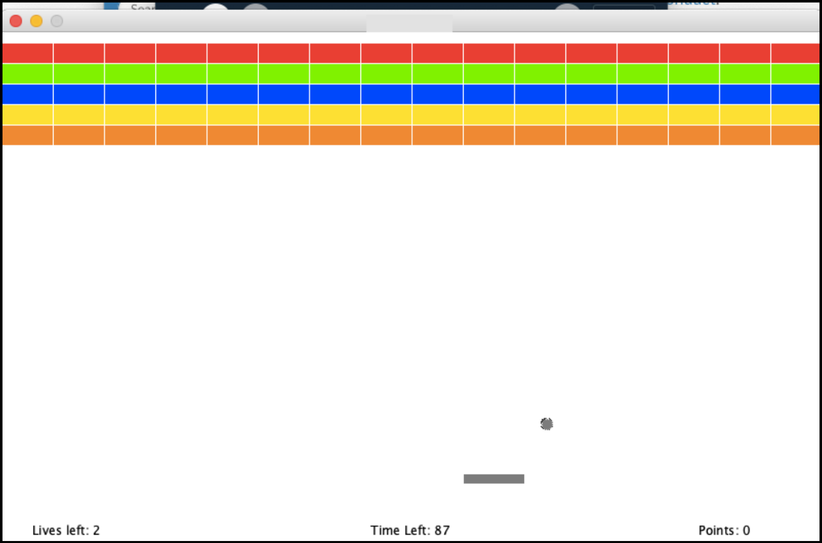


Let's add some "physics" to the spring.

```
# we will update position & velocity frame-by-frame,
# so we store them in these "global" variables
y = 1.49  # need to change this to make anything happen!
vy = 0.0

def draw():
  dt = 0.01
  k = 20.0
  m = 1.0
  g = 9.8
  Lrelaxed = 1.0
  y = y + vy*dt
  Fy = m*g - k*(y-Lrelaxed)
  vy = vy + (Fy/m)*dt
  xbob = 0
  ybob = Lrelaxed + y
  ... the rest is unchanged ...
```

https://en.wikipedia.org/wiki/Leapfrog_integration

A "breakout" game coded by a Fall 2017 (and Fall 2018) student.



This was done in Java Processing. Let's try to imitate it in Python!

```
def setup():
    size(900, 450)
    global b
    # Make rectangle location be its center position
    rectMode(CENTER)
    # Instantiate the state of the game board
    b = Breakout()

def draw():
    global b
    b.update()
    b.draw()

class Breakout:

    # "Constructor" for new Breakout object
    def __init__(self):
    ... continued on next slide ...
```

```
class Breakout:

    def __init__(self):
        # various screen boundaries
        self.ytop = 0.0
        self.ybot = height
        self.xleft = 0.0
        self.xright = width
        # ball's size, position, velocity
        self.rball = 7.0
        self.xball = 0.5*width
        self.yball = 0.5*height
        self.speed = 3.0
        self.vxball = self.speed/sqrt(2)
        self.vyball = self.speed/sqrt(2)

    def update(self):
        ... see next slide ...

    def draw(self):
        ... see next slide ...
```

```
class Breakout:

    def __init__(self):
        ... see previous slide ...

    def update(self):
        dt = 1.0
        # use ball velocity to update ball position
        self.xball += self.vxball*dt
        self.yball += self.vyball*dt
        # update ball velocity if it hits the game boundary
        if ((self.xball >= self.xright) or
            (self.xball <= self.xleft)):
            self.vxball *= -1.0
        if ((self.yball >= self.ybot) or
            (self.yball <= self.ytop)):
            self.vyball *= -1.0

    def draw(self):
        ... see next slide ...
```

```
class Breakout:

    def __init__(self):
        ... see earlier slide ...

    def update(self):
        ... see previous slide ...

    def draw(self):
        # clear the screen
        background(200)
        # draw the ball (black)
        fill(color(0, 0, 0))
        ellipse(self.xball, self.yball,
                2*self.rball, 2*self.rball)
```

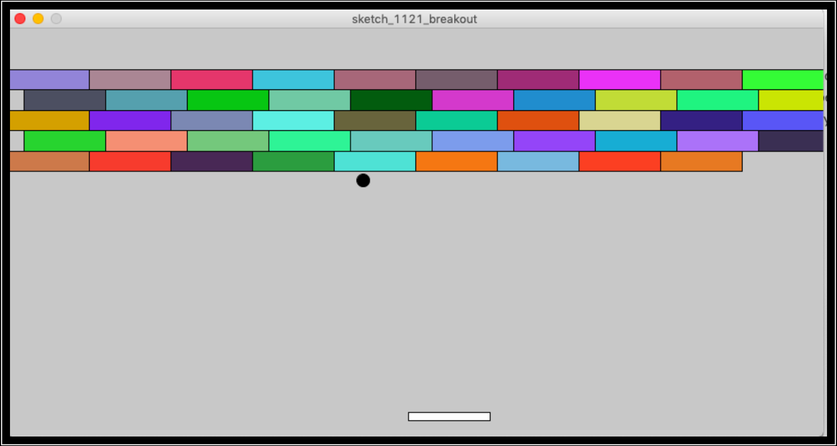
```
... insert this into Breakout :: __init__
# paddle's location and x,y thickness
self.xpaddle = 0.5*width
self.ypaddle = 0.95*height
self.dxpaddle = 0.1*width
self.dypaddle = 0.02*height

... insert this into Breakout :: update
# make the paddle follow the horizontal mouse position
self.xpaddle = mouseX
# check for ball bouncing off of the paddle
if (abs(self.yball - self.ypaddle) < self.dypaddle/2 and
    abs(self.xball - self.xpaddle) < self.dxpaddle/2 and
        self.vyball > 0):
    self.vyball *= -1.0

... insert this into Breakout :: draw
# draw the paddle (white)
fill(color(255, 255, 255))
rect(self.xpaddle, self.ypaddle,
     self.dxpaddle, self.dypaddle)
```

```python
class Brick:

    def __init__(self, x, y, dx, dy):
        self.x = x
        self.y = y
        self.dx = dx
        self.dy = dy
        self.rcolor = random(0, 255)
        self.gcolor = random(0, 255)
        self.bcolor = random(0, 255)

    def checkCollision(self, x, y):
        if abs(x-self.x) > 0.5*self.dx:
            return False
        if abs(y-self.y) > 0.5*self.dy:
            return False
        return True

    def draw(self):
        fill(color(self.rcolor, self.gcolor, self.bcolor))
        rect(self.x, self.y, self.dx, self.dy)
```

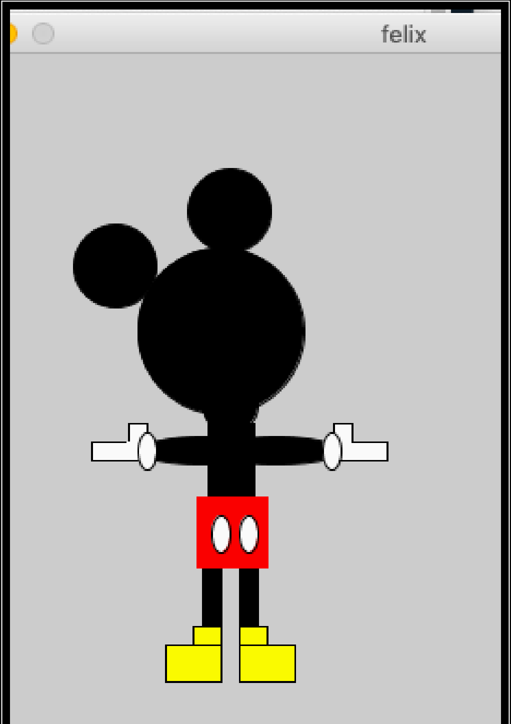
```
... insert into Breakout :: __init__
# make list of bricks
self.bricks = []
ncol = 10
for irow in range(5):
    for jcol in range(ncol):
        dxbrick = 1.0*width/ncol
        dybrick = 0.05*height
        xbrick = (jcol+0.5)*dxbrick
        if (irow % 2) != 0:
            xbrick += 0.5*dxbrick
        ybrick = 0.1*height + (irow+0.5)*dybrick
        self.bricks.append(Brick(x=xbrick, y=ybrick,
                                 dx=dxbrick, dy=dybrick))

... insert into Breakout :: draw
# draw the bricks
for b in self.bricks:
    b.draw()
```

```
... insert into Breakout :: update
# check for collisions with bricks
for i in range(len(self.bricks)):
    b = self.bricks[i]
    if b.checkCollision(self.xball, self.yball):
        # collision!  reverse the ball's velocity
        self.vxball *= -1.0
        self.vyball *= -1.0
        # delete the struck brick from the list!
        self.bricks.pop(i)
        # don't check any more bricks this frame,
        # as we modified the list of bricks
        break
```

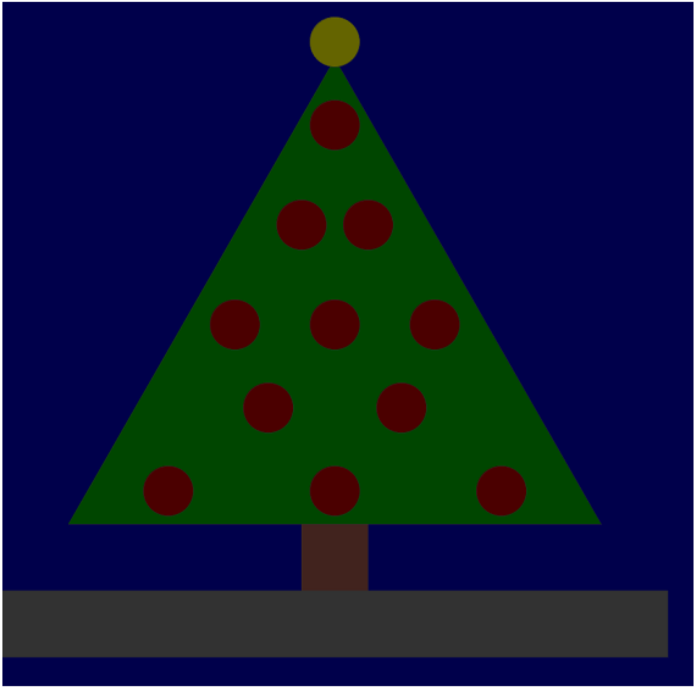
```
        ... in Breakout :: __init__
        self.previousMouseX = mouseX
...
        ... in Breakout :: update
        # estimate the horizontal velocity of the paddle
        vxpaddle = (mouseX - self.previousMouseX)/dt
        self.previousMouseX = mouseX
...
            ... upon detecting collision with paddle
            # allow paddle velocity to affect horizontal
            # ball velocity, since otherwise we can get
            # stuck with bricks that cannot be reached
            self.vxball += vxpaddle
            # don't let ball velocity become too horizontal
            minvh = 0.5*self.speed
            if abs(self.vyball) < minvh:
                self.vyball = -minvh
            # but keep the overall ball speed constant
            temp_speed = sqrt(self.vxball**2 + self.vyball**2)
            self.vxball *= self.speed/temp_speed
            self.vyball *= self.speed/temp_speed
```

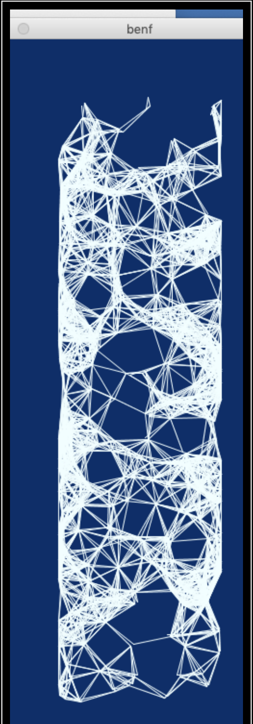

sketch_1121_breakout

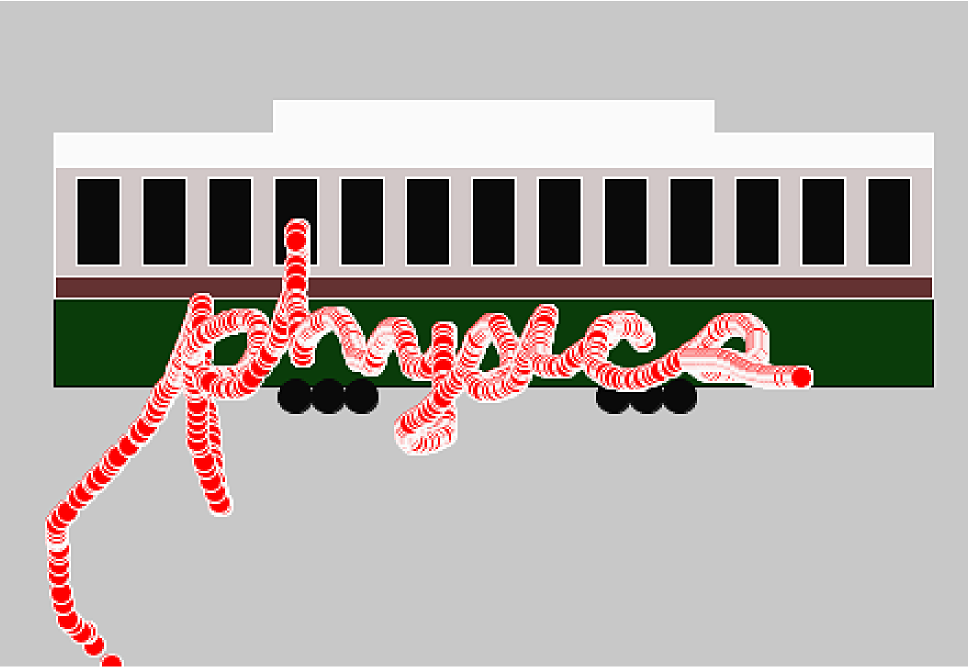
- The easiest way to get started with the original Java-based version of Processing is to start with this easy online video tutorial that will get you coding in Processing in about an hour! No download or software install is needed for this tutorial — you type your first programs directly into your web browser as you follow along with the video. `http://hello.processing.org`
- For the Python version, work through the first few tutorials at `http://py.processing.org/tutorials`
- If you're in Addams Hall often, you might ask Orkan Telhan if he has ideas — I believe he still teaches Processing in FNAR 264 / VLST 264, "Art, Design, and Digital Culture."
- There are also tons of examples at `http://processing.org` that you could use as starting points or for inspiration, though again these examples use the Java version of Processing.
- In Fall 2017, ten students sent me Processing sketches! I include a few screen captures on the next few slides.


felix

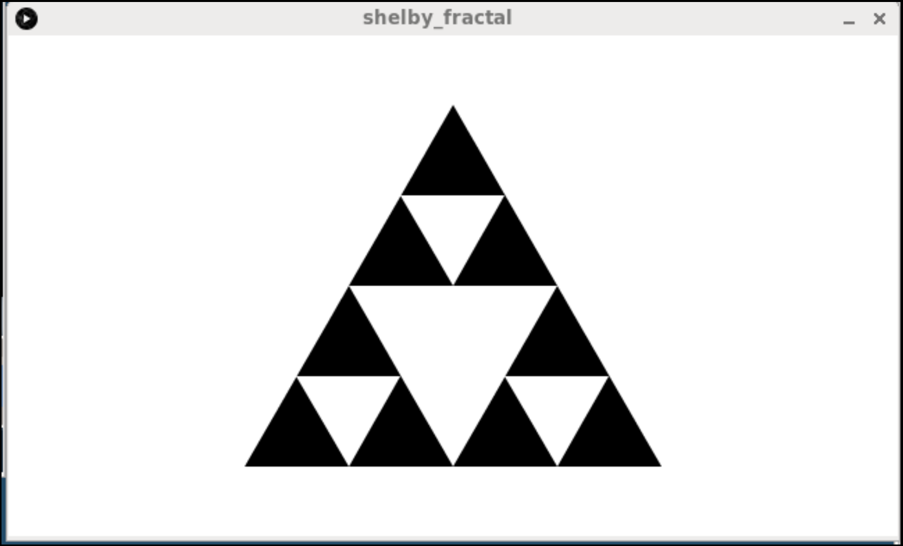

galena




benf

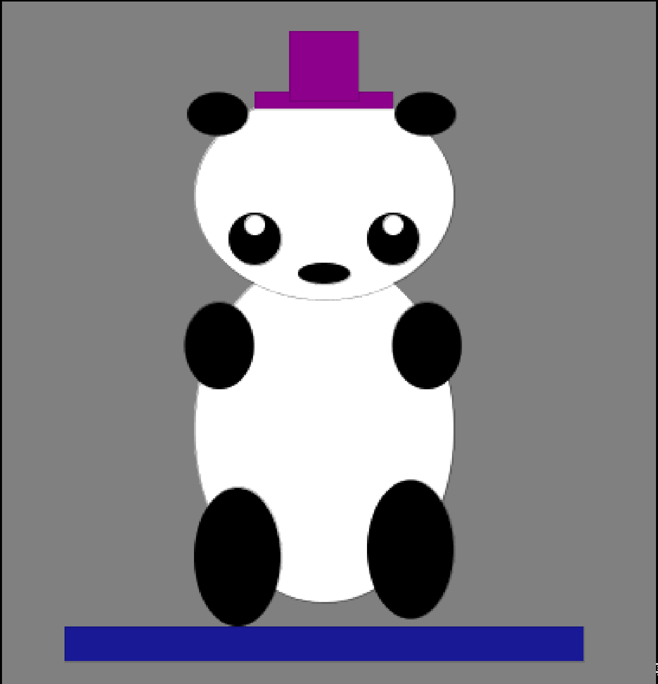


An example from a Fall 2013 student: drawing a fractal.



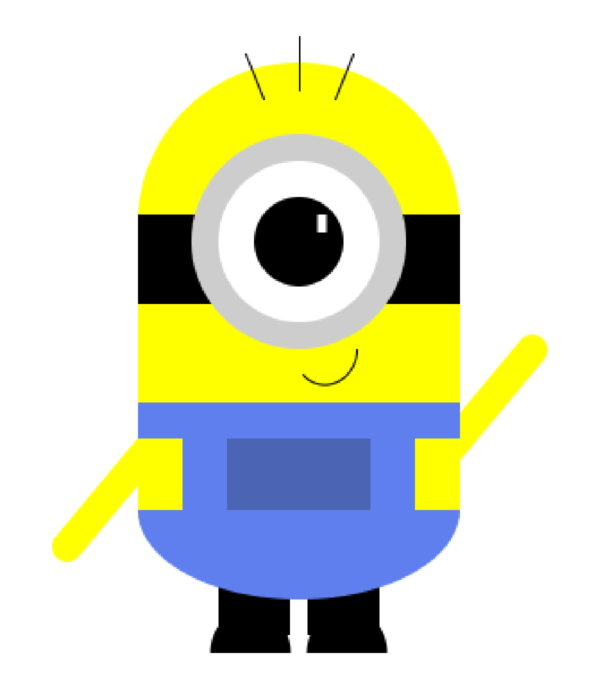
Another Fall 2013 student: ball bouncing between two springs

An example from a Fall 2015 student: an animated panda.

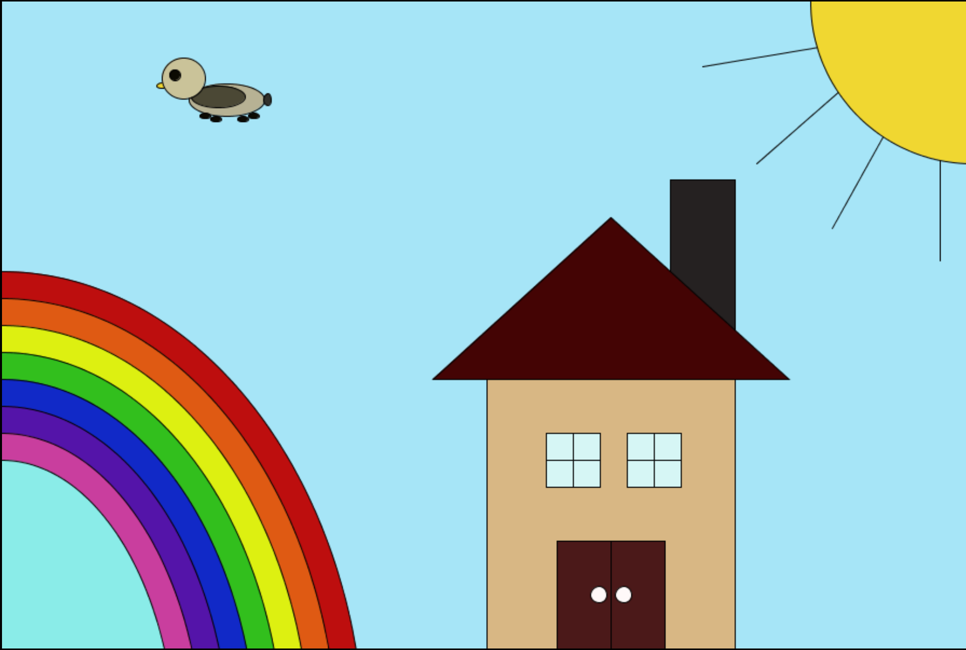


An example from a Fall 2015 student: a rotating fractal.

An example from a Fall 2015 student: a minion.



Fall 2015 student: bird moves where you move the mouse pointer.



Physics 8 — Wednesday, November 27, 2019

- I'll put take-home practice exam (due 12/6 or 12/9) online this evening. I put 4 years' old exams online at

  `http://positron.hep.upenn.edu/p8/files/oldexams/`
- Extra-credit options (if you're interested):
  - Learn to use Mathematica (ask me how), which is a system for doing mathematics by computer. (It is the brains behind Wolfram Alpha.) Penn's site license makes Mathematica free-of-charge for SAS and Wharton students.
  - Use "Processing.py" (or ordinary "Processing") to write a program to draw or animate whatever interests you. (Not necessarily physics-related.)
  - Read O/K ch9 on columns & summarize what you learned.
  - Read Mazur ch13 on gravity & summarize what you learned.
  - Go through Prof. Phil Nelson's book on using Python for data modeling. Several Huntsman students seem keen to do this.
  - Respond to podcast about near-fatal flaw in Citigroup Center, 601 Lexington Ave, NYC

`http://positron.hep.upenn.edu/p8/files/citicorp_tower.mp3`

  - Pursue your own XC idea: phys/math/coding/structures/etc
- Today's examples online at

`http://positron.hep.upenn.edu/p8/2018/files/pyprocessing/`

Physics 8 — Monday, December 2, 2019

- Final exam (25%) is Thu, Dec 12, noon–2pm, DRL A1.
- I'll try to book a room for a review session on Wed, Dec 11, preferably mid-afternoon.
- Pick up take-home practice exam (10%) in back of room.
- If you turn it in on Friday (in class, or in my office, DRL 1W15, by 5pm), I'll grade it and return it to you (email PDF) on Monday evening, Dec 9.
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- 4 previous years' exams & practice exams are at `http://positron.hep.upenn.edu/p8/files/oldexams`
- Periodic motion (oscillation, vibration) is our last topic this term. Alas, this year's exam schedule doesn't allow us to include it in the homework or the exam.

Vibrations/oscillations

- Are ubiquitous (look around — or listen — for examples!) because anything in stable equilibrium can oscillate about the equilibrium point. (Illustration.)

Picture a ball at the bottom of a round container. Is it in stable equilibrium at the bottom? What happens if I slide it over a bit and then let go of it?

- The restoring force pushes it back toward the equilibrium position. Once it reaches the equilibrium position, the net force is zero, but by that point the ball is in motion, so it continues past the equilibrium point until the restoring force eventually reverses its direction. It keeps moving back and forth until eventually the energy is dissipated by friction, and the ball comes to rest in the equilibrium position.
- Contrast with *neutral* or *unstable* equilibrium: no restoring force in these cases.

Oscillations / vibrations

- The restoring force that keeps an object stable is the same restoring force that causes the object to vibrate when displaced.
- The simplest form for a restoring force is Hooke's law:

$$F_x = -k\ (x - x_0)$$

- A linear restoring force is the most common case, for small displacements. We study it because it is ubiquitous and because it is relatively easy to analyze.
- If there is a linear restoring force (i.e. if the force is proportional to the displacement) and negligible friction, then the math works out cleanly with sines and cosines, and we call the motion *Simple Harmonic Motion*.

skip math — here in case you're curious

Hooke's law for a block on a horizontal spring is

$$F_x = -k\ (x - x_0)$$

(Note: for vertical orientation, the equilibrium position is offset downward by mg/k, after which the math is identical.)

Newton's 2nd law for the block of mass m then reads

$$ma_x = -k\ (x - x_0)$$

To simplify the math, let $x_0 = 0$ for the moment. Then

$$ma_x = -kx$$

$$m\ \frac{\mathrm{d}^2 x}{\mathrm{d}t^2} = -kx$$

A function whose second derivative is proportional to the original function (with a negative coefficient) is a sine or a cosine.

$$x(t) = A\ \sin(\omega t + \phi_i)$$

Plugging **[skip math — it's here for your curiosity]**

$$x(t) = A\ \sin(\omega t + \phi_i)$$

into

$$ma_x = -kx$$

works, using "angular frequency" ω (radians/second)

$$\omega = \sqrt{\frac{k}{m}}$$

Or (more familiar) "frequency" (cycles/second, or Hz)

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

---

check:

$$v_x(t) = \frac{\mathrm{d}x}{\mathrm{d}t} = \omega A\ \cos(\omega t + \phi_i)$$

$$ma_x(t) = m\ \frac{\mathrm{d}v_x}{\mathrm{d}t} = -m\omega^2 A\ \sin(\omega t + \phi_i) = -kA\ \sin(\omega t + \phi_i) = -kx$$

For a mass oscillating on a spring at its “natural frequency,” i.e. the frequency at which it oscillates if I pluck it or whack it and then leave it alone

$$\omega_0 = 2\pi f_0 = \sqrt{\frac{k}{m}}$$

and the motion is sinusoidal in time:

$$x(t) = x_{\rm eq} + A\ \sin(\omega_0 t + \phi_i)$$

- x_{eq} is equilibrium position (usually we choose $x_{\rm eq} = 0$)
- A is called the amplitude
- The “initial phase” ϕ_i tells you what’s happening at $t = 0$
- $\phi_i = 0$ or π means displacement w.r.t. $x_{\rm eq}$ is zero at $t = 0$
- $\phi_i = \pm\pi/2$ means displacement is max(min)imum at $t = 0$
- notice $\sin(\omega t \pm \pi/2) = \pm\cos(\omega t)$

$$x(t) = x_{\text{eq}} + A\ \sin(\omega_0 t + \phi_i)$$

Writing $x(t)$ this way is usually more complicated than necessary. The most common cases for ϕ_i are:

- $\phi_i = 0$: at $t = 0$, $(x - x_{\text{eq}}) = 0$ and $v_x > 0$ (maximum)

$$x(t) = x_{\text{eq}} + A\ \sin(\omega_0 t)$$

- $\phi_i = \pi/2$: at $t = 0$, $(x - x_{\text{eq}}) > 0$ (maximum) and $v_x = 0$

$$x(t) = x_{\text{eq}} + A\ \cos(\omega_0 t)$$

- $\phi_i = \pi$: at $t = 0$, $(x - x_{\text{eq}}) = 0$ and $v_x < 0$ (minimum)

$$x(t) = x_{\text{eq}} - A\ \sin(\omega_0 t)$$

- $\phi_i = -\pi/2$: at $t = 0$, $(x - x_{\text{eq}}) < 0$ (minimum) and $v_x = 0$

$$x(t) = x_{\text{eq}} - A\ \cos(\omega_0 t)$$

As another simplification, usually we define the x axis so that $x_{\mathrm{eq}} = 0$. Then for the two most common cases:

- at $t = 0, \quad x = 0$ and $v_x > 0$

$$x(t) = A \, \sin(\omega_0 t)$$

$$v_x(t) = \omega_0 A \, \cos(\omega_0 t)$$

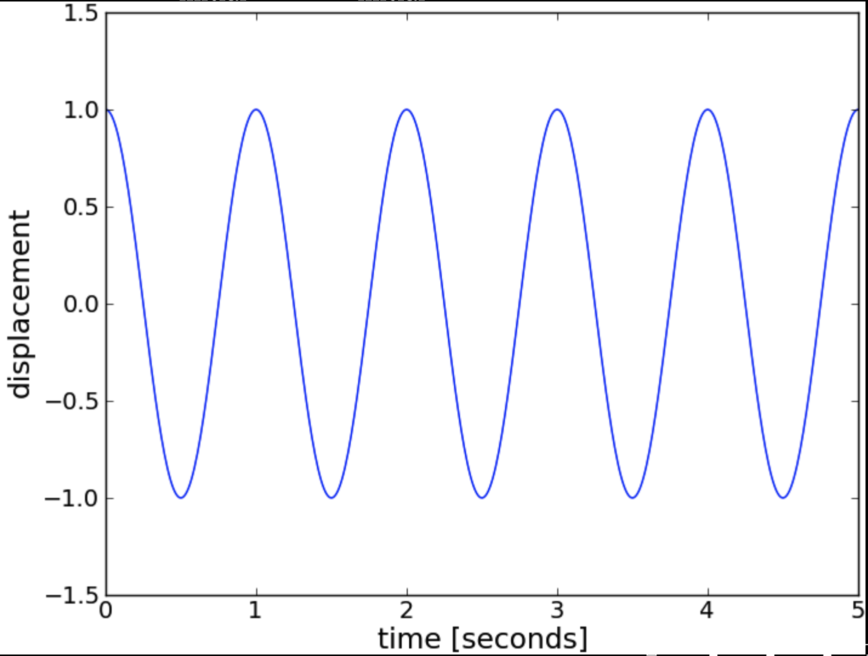
- at $t = 0, \quad x > 0$ and $v_x = 0$

$$x(t) = A \, \cos(\omega_0 t)$$

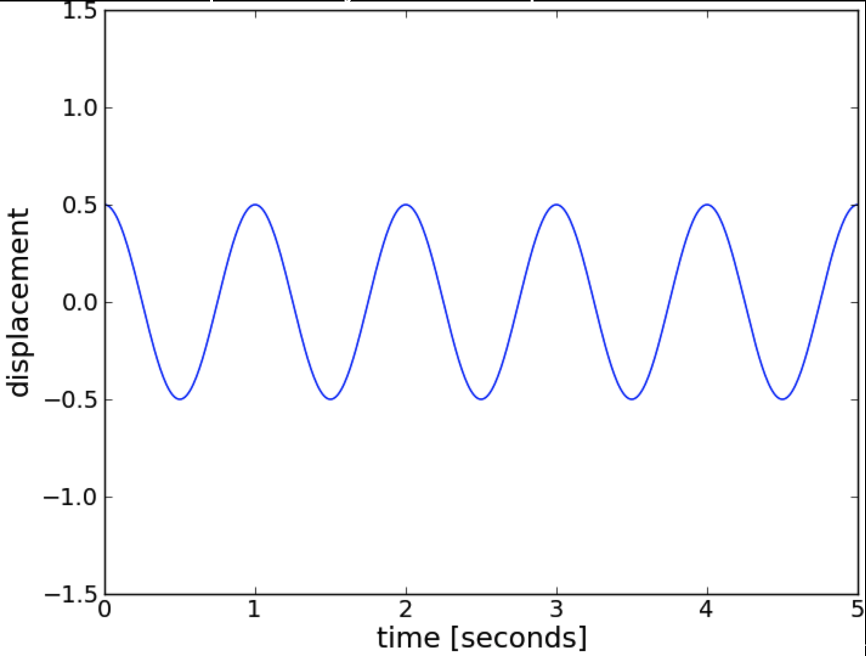
$$v_x(t) = -\omega_0 A \, \sin(\omega_0 t)$$

Let's try this with graphs instead of equations. The next few graphs will assume that we choose $x_{\mathrm{eq}} = 0$.

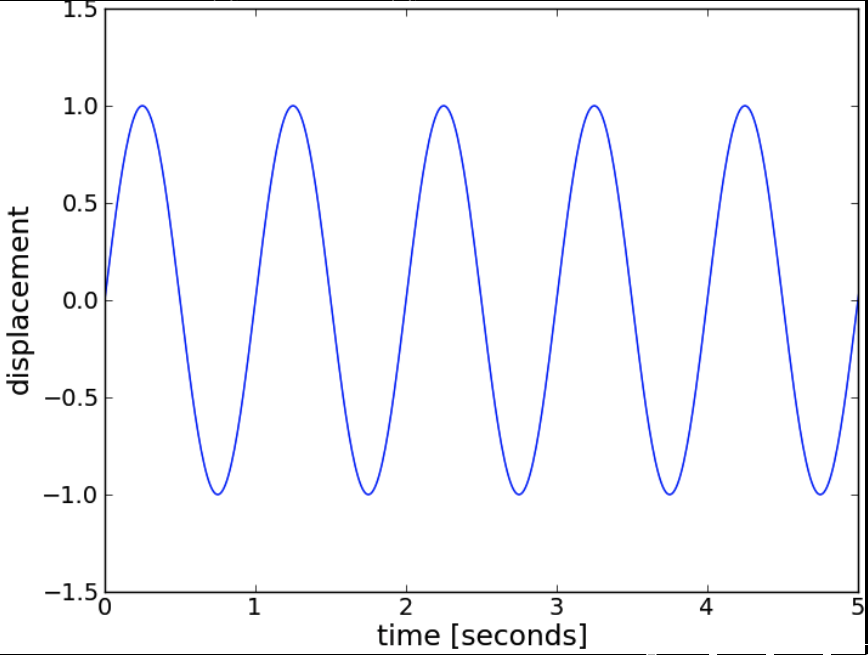
Oscillation: $v_{\mathrm{initial}} = 0$, $x_{\mathrm{initial}} > 0$: looks like a **cosine**



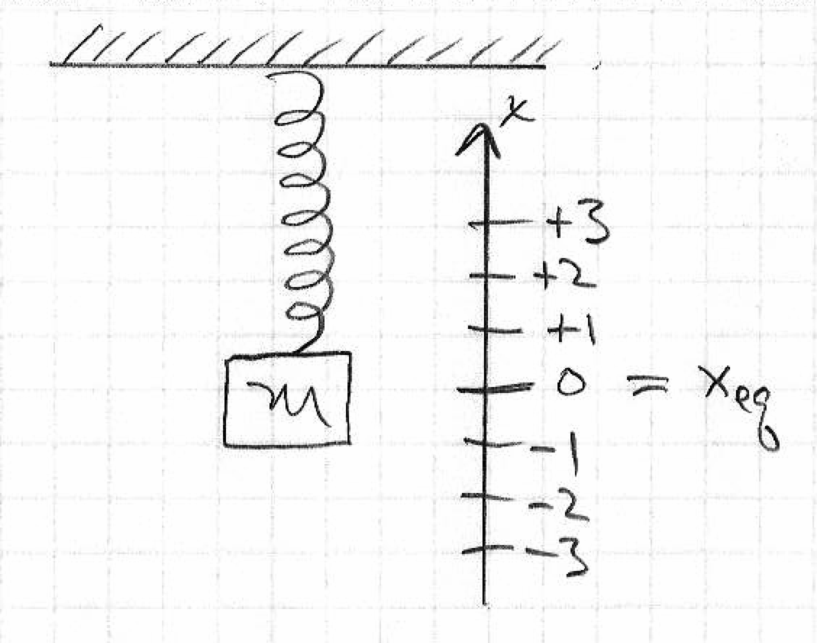
now reduce amplitude by half from previous slide

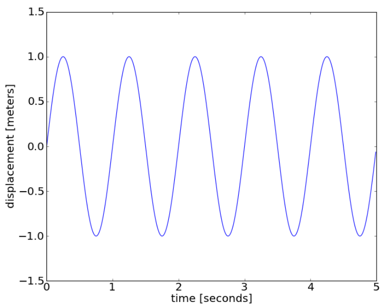


Oscillation: $v_{\text{initial}} > 0$, $x_{\text{initial}} = 0$: looks like a **sine**

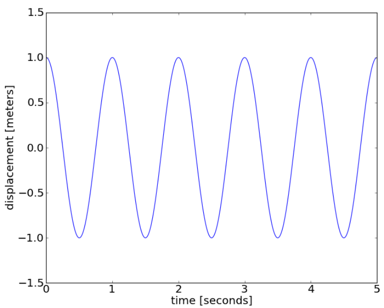


Let's try some examples using a coordinate system that looks like this. So $x_{\text{eq}} = 0$ and the x axis points upward.




1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
displacement [meters]
0
1
2
3
4
5
time [seconds]

A


1.5
1.0
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displacement [meters]
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1
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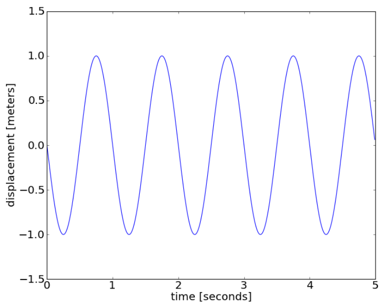
4
5
time [seconds]

B


1.5
1.0
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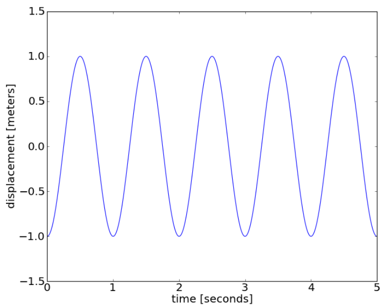
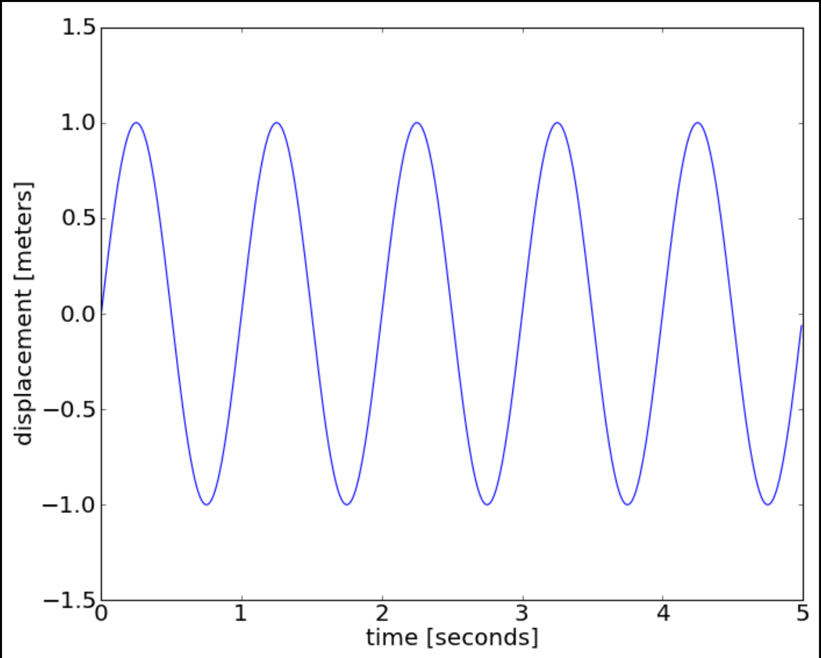
displacement [meters]
0
1
2
3
4
5
time [seconds]

C

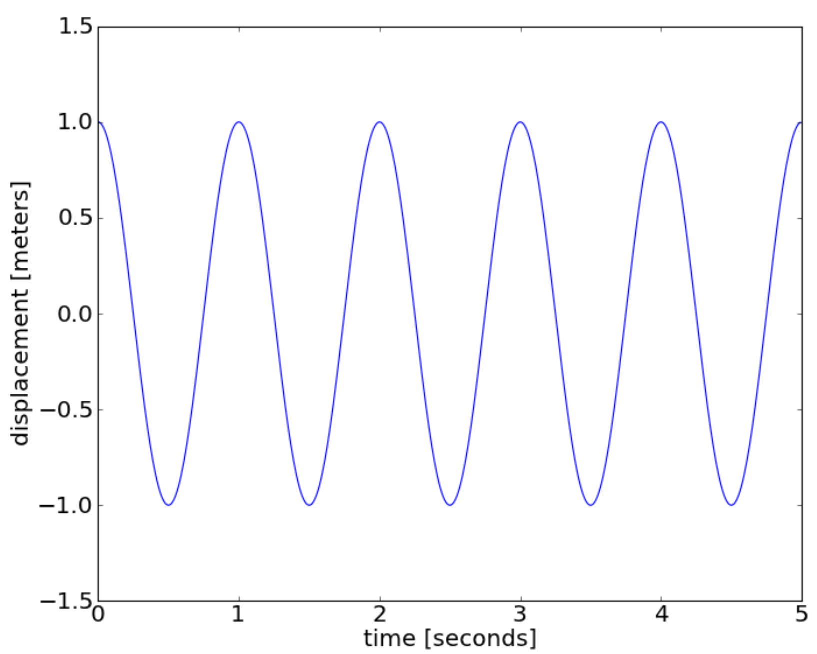

1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
displacement [meters]
0
1
2
3
4
5
time [seconds]

D

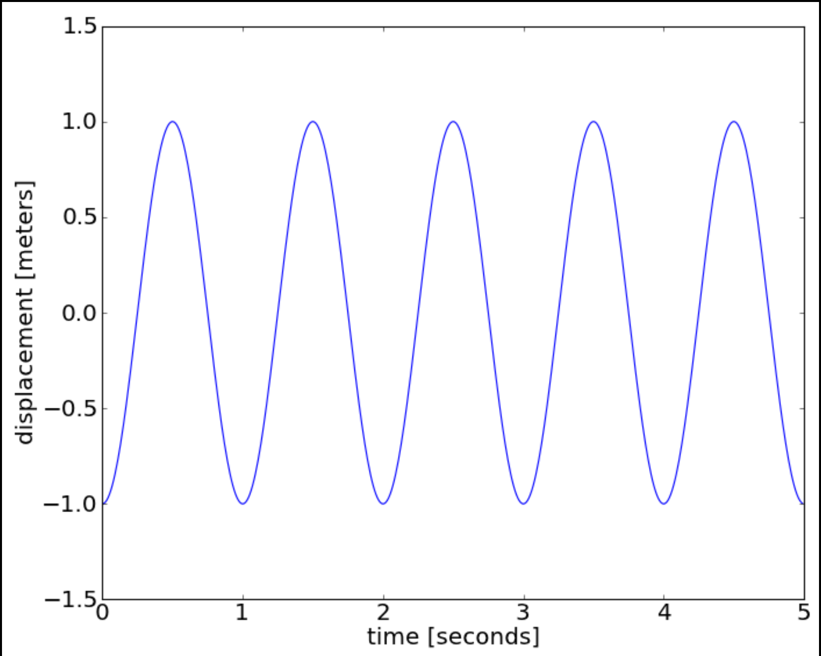
What's the amplitude of this motion? Period? Frequency?



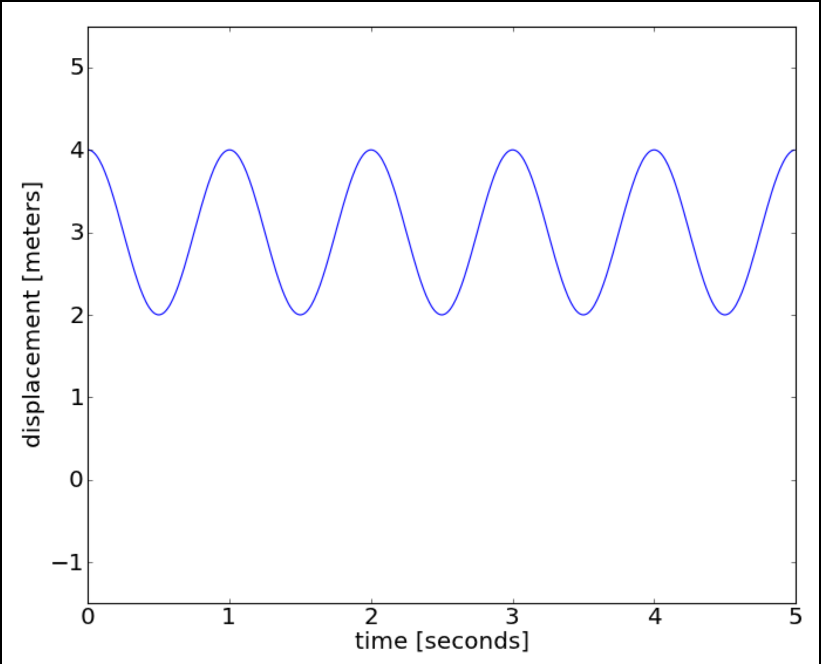
What's the amplitude of this motion?



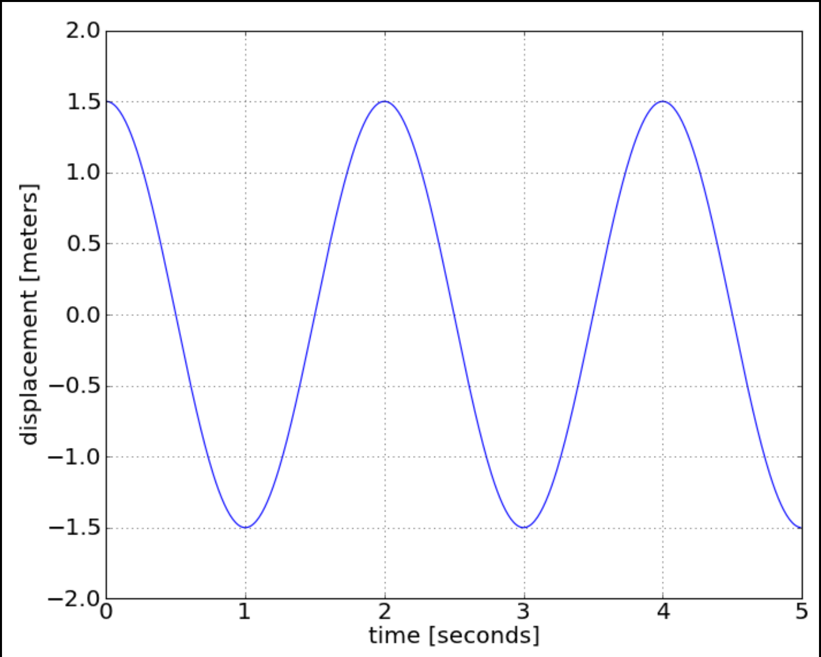
What's the amplitude of this motion?



What's the amplitude of this motion? What is x_{eq}?



Amplitude? x_{eq}? Period? Frequency? Angular frequency?

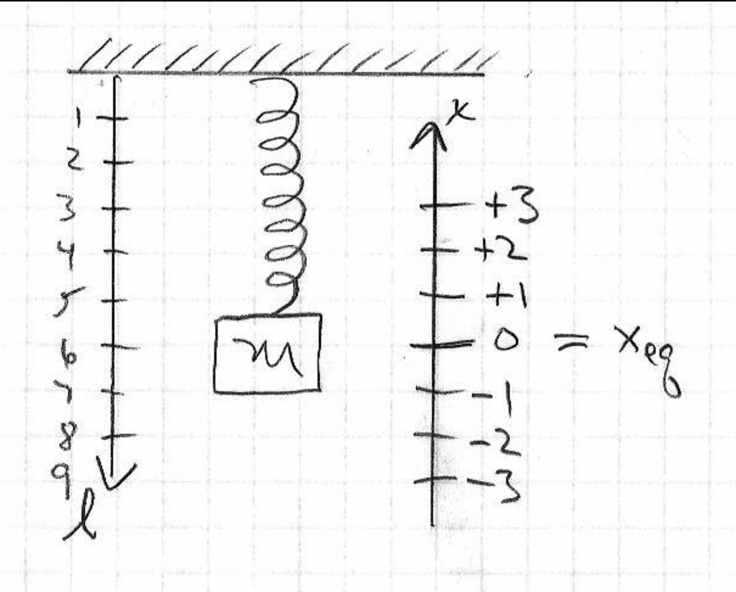


- Worth remembering: natural frequency for a mass on a spring

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

- double $k \quad \rightarrow \quad$ multiply f by $\sqrt{2}$
- double $m \quad \rightarrow \quad$ divide f by $\sqrt{2}$

- For a wide range of equilibrium situations in which the restoring force is provided by some form of elasticity,
  - more stiffness $\rightarrow$ higher f
  - more mass $\rightarrow$ lower f
- See same $\sqrt{\frac{\text{stiffness}}{\text{inertia}}}$ trend in beams, skyscrapers, etc.
- But pendulum is an exception, because restoring force $\propto \; m$. We'll see in a moment.
- Another surprising result: frequency of oscillation is independent of amplitude
- Let's use a much stiffer spring and a much larger mass to illustrate this last result! How can we measure k?
- **fall 2019: measured period 15% larger than predicted; next time try measuring k by adding 14 pound bowling**



Caution: for this situation, if you want to graph the length of the spring vs. time, the "length" coordinate increases in the **downward** direction, and "$\ell = 0$" is at the ceiling.

We wrote $x(t)$ in terms of $\omega =$ "natural angular frequency:"

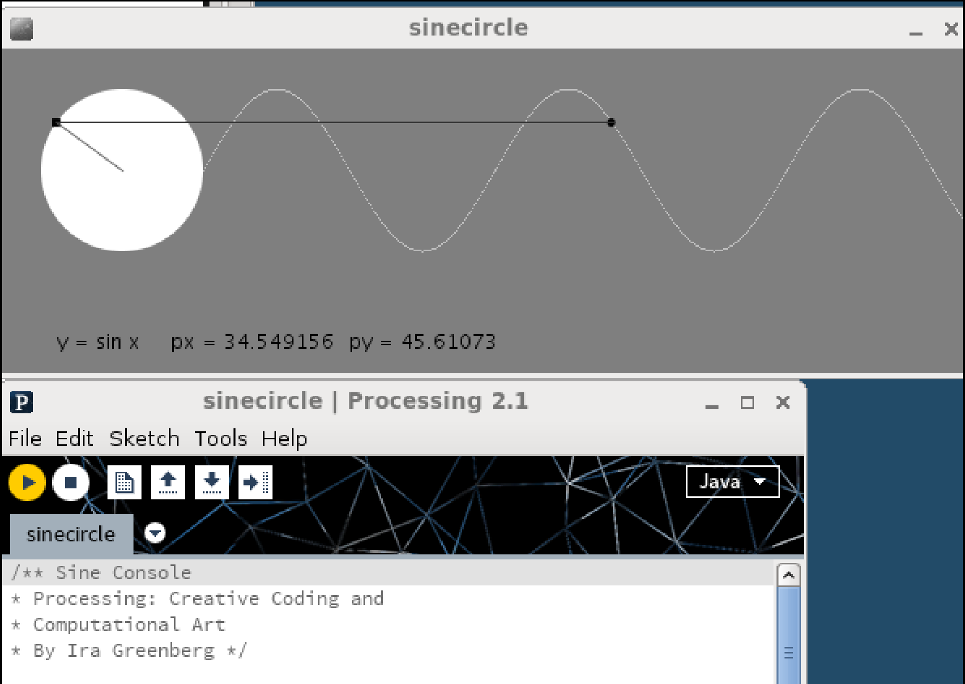
$$x = A \sin(\omega t + \phi_i)$$

but we could have equivalently used $f =$ "natural frequency:"

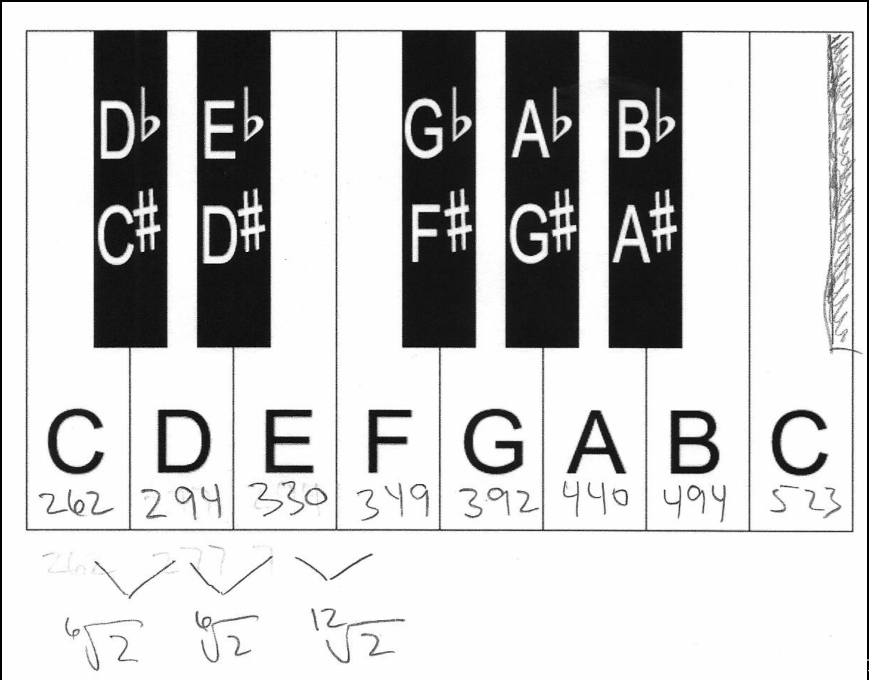
$$x = A \sin(2\pi f \; t + \phi_i)$$

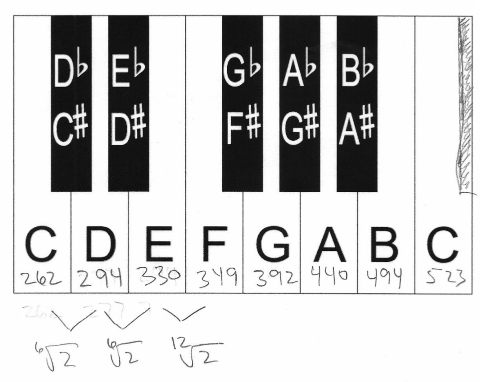
- $f =$ *frequency*, measured in cycles/sec, or Hz (hertz)
- $\omega = \frac{f}{2\pi}$ is *angular frequency*, measured in radians/sec, or s^{-1}
- The frequency $f = 2\pi\omega$ is much more intuitive than ω
- Using ω keeps the equations cleaner — can be helpful for derivations, etc., so that you don't have to keep writing 2π

"angular velocity" ω is our old friend from studying circular motion:



“frequency” $f = \frac{\omega}{2\pi}$ is more familiar from music, etc.





- A above middle C: 440 Hz
- Middle C: 261.63 Hz
- $440 \times (\frac{1}{2})^{\frac{3}{4}} = 261.63$
- Octave = factor of 2 in frequency f
- Half step = factor of $\sqrt[12]{2}$ in frequency
- Whole step = factor of $\sqrt[6]{2}$ in frequency
- Major scale (white keys, starting from C) = (root) W W H W W W H
- Minor scale (white keys, starting from A) = (root) W H W W H W W

If the amplitude of simple harmonic motion doubles, what happens to the frequency (i.e. the natural frequency) of the system?

(A) The frequency is $1/2$ as large.
(B) The frequency is $1/\sqrt{2}$ as large.
(C) The frequency is unchanged.
(D) The frequency is $\sqrt{2}$ times as large.
(E) The frequency is 2 times as large.

If the amplitude of simple harmonic motion doubles, what happens to the energy of the system?

(A) The energy is unchanged.
(B) The energy is $\sqrt{2}$ times as large.
(C) The energy is 2 times as large.
(D) The energy is 4 times as large.

One way to see that **(D)** is correct is to write

$$x = A\sin(\omega t) \qquad v_x = \omega A\cos(\omega t)$$

and then write out the energy

$$\frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

and see that energy is proportional to A^2.

Physics 8 — Monday, December 2, 2019

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- Pick up take-home practice exam (10%) in back of room.
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- Periodic motion (oscillation, vibration) is our last topic this term. Alas, this year's exam schedule doesn't allow us to include it in the homework or the exam.

Physics 8 — Wednesday, December 4, 2019

- Practice exam: If you turn it in Monday (in class, or in my office, by 5pm), I'll return it to you on Wed, Dec 11. If I don't have your exam by 5pm on Monday, Dec 9, your score is zero, no exceptions, so that I can return graded exams promptly.
- 4 previous years' exams & practice exams are at `http://positron.hep.upenn.edu/p8/files/oldexams`
- Extra credit options (until Thu, Dec 19):
  - O/K ch9 (columns)
  - Citigroup Center "structural integrity" podcast
  - Mazur ch13 (gravity), ch14 (Einstein relativity)
  - Code something in Processing or Py.Processing
  - Go through tutorials to learn Wolfram Mathematica
  - Go through Prof. Nelson's python data modeling book
  - You can suggest something else!

- Worth remembering: natural frequency for a mass on a spring

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

- double $k \rightarrow$ multiply f by $\sqrt{2}$
- double $m \rightarrow$ divide f by $\sqrt{2}$

- For a wide range of equilibrium situations in which the restoring force is provided by some form of elasticity,
  - more stiffness $\rightarrow$ higher f
  - more mass $\rightarrow$ lower f
- See same $\sqrt{\frac{\text{stiffness}}{\text{inertia}}}$ trend in beams, skyscrapers, etc.
- But pendulum is an exception, because restoring force $\propto m$. We'll see in a moment.
- Another surprising result: frequency of oscillation is independent of amplitude
- Unfortunately, with A6 booked for back-to-back classes, I didn't have occasion to figure out why Monday's measured period for me-on-spring was 15% larger than our prediction.
- Let's make sure we understand how we got the prediction.

(From 2017 practice exam): Your physics teacher Bill gets the crazy idea that he himself will be the "mass" bobbing up and down on the end of a stiff spring that is attached to the ceiling of DRL room A2. Bill first hangs the spring from the ceiling and measures its relaxed length to be 0.85 meters. Then he climbs the ladder, gradually applies his full weight to the lower end of the spring (by sitting on a little attached bar), and measures the spring's new equilibrium length (the length of the spring when Bill is in static equilibrium) to be 1.55 meters.

[a] If Bill's mass is 70 kg, what is the spring constant k of the spring?

(A) $(70)/(1.55-0.85) = 100\,\mathrm{N/m}$

(B) $(1.55-0.85)/(70\times 9.8) = 1.02\times 10^{-3}\,\mathrm{N/m}$

(C) $(70\times 9.8)/(0.85) = 807\,\mathrm{N/m}$

(D) $(70\times 9.8)/(1.55-0.85) = 980\,\mathrm{N/m}$

[b] If someone pulls down on Bill's feet until the spring's length is 1.85 meters, holds them there for a moment, then lets go (without giving any sort of push), will Bill's motion repeat itself periodically? If so, how often? If not, why not?

(A) No: the spring will return to $\ell = 1.55\,\mathrm{m}$ and stay there.

(B) Yes, with period $T = 2\pi\sqrt{70/980} = 1.7\,\mathrm{s}$

(C) Yes, with period $T = 2\pi\sqrt{70 \times 9.8/980} = 0.83\,\mathrm{s}$

(D) Yes, with period $T = 2\pi\sqrt{980/70} = 23.5\,\mathrm{s}$

(E) Yes, with period $T = \sqrt{70/980} = 0.26\,\mathrm{s}$

(F) Yes, with period $T = \sqrt{70/980}/(2\pi) = 0.043\,\mathrm{s}$

(G) Yes, with period $T = \sqrt{980/70} = 3.7\,\mathrm{s}$

(H) Yes, with period $T = \sqrt{980/70}/(2\pi) = 0.60\,\mathrm{s}$

[c] Sketch a graph of the length of the spring as a function of time, where $t = 0$ is where the person lets go of Bill's feet. Be sure to label the important features of the graph, e.g. period and amplitude.

[d] If the person instead pulls down on Bill's feet until the spring's length is 1.70 meters, then lets go, how will the period of the motion be affected? (State what the period will be.)

[e] How will the amplitude of the motion be affected? (State what the amplitude will be.)

[f] If Bill somehow managed to hold a 70 kg medicine ball while sitting on this same spring, thus effectively doubling his mass, would the natural period of the motion be affected? (State what the period would be.)

If the amplitude of simple harmonic motion doubles, what happens to the frequency (i.e. the natural frequency) of the system?

(A) The frequency is $1/2$ as large.
(B) The frequency is $1/\sqrt{2}$ as large.
(C) The frequency is unchanged.
(D) The frequency is $\sqrt{2}$ times as large.
(E) The frequency is 2 times as large.

If the amplitude of simple harmonic motion doubles, what happens to the energy of the system?

(A) The energy is unchanged.
(B) The energy is $\sqrt{2}$ times as large.
(C) The energy is 2 times as large.
(D) The energy is 4 times as large.

One way to see that **(D)** is correct is to write

$$x = A\sin(\omega t) \qquad v_x = \omega A\cos(\omega t)$$

and then write out the energy

$$\frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

and see that energy is proportional to A^2.

Pendulum: gravity provides restoring torque

$$\tau = I\alpha \quad \Rightarrow \quad \alpha = \frac{\tau}{I}$$

$$\tau = -mg\ell \sin\theta$$

$$I = m\ell^2$$

Using $\sin\theta = \theta - \theta^3/6 + \theta^5/120 + - \cdots$,

$$\alpha = -\frac{g\sin\theta}{\ell} \approx -\frac{g\theta}{\ell}$$

for small θ, so

$$\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} = -\frac{g}{\ell}\,\theta$$

So for a pendulum (a point mass on a string, small amplitude),

$$\omega_0 = \sqrt{\frac{g}{\ell}} \qquad \boxed{f_0 = \frac{1}{2\pi}\sqrt{\frac{g}{\ell}}}$$

(Mazur also generalizes this to objects with more complicated rotational inertias. That's only relevant for XC problems.)

Remember: oscillator period is **independent** of the amplitude

Mass on spring (use "0" to mean "natural"):

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \qquad T_0 = 2\pi\sqrt{\frac{m}{k}}$$

Simple pendulum (small heavy object at end of "massless" cable):

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{g}{\ell}} \qquad T_0 = 2\pi\sqrt{\frac{\ell}{g}}$$

For a pendulum, the period is also independent of the mass, because the restoring force (due to gravity) is proportional to mass, so the mass cancels out.

Let's measure the oscillation period T_0 for this ball on a string, for a few different values of ℓ.

Remember,

$$T_0 = \frac{1}{f_0} = 2\pi\sqrt{\frac{\ell}{g}}$$

To speed us up, I've pre-calculated everything but ℓ:

$$T_0 \;=\; 2\pi\sqrt{\frac{\ell}{g}} \;=\; \left(2\pi\,\sqrt{\frac{1\text{ meter}}{g}}\right)\sqrt{\frac{\ell}{1\text{ meter}}}$$

$$T_0 \;=\; (2.01\text{ seconds}) \;\times\; \sqrt{\frac{\ell}{1\text{ meter}}}$$

Does it depend on amplitude? Does it depend on mass?

Imagine your old playground swing set. I'll bet you remember everybody going back and forth at about the same time interval, even if some kids had different amplitudes, different phases, or even different masses!

You probably also remember the time between swings to be something like a few seconds:

$$T_0 = 2\pi\sqrt{\frac{\ell}{g}} \approx 2\pi\sqrt{\frac{3.1\ \mathrm{m}}{9.8\ \mathrm{m/s^2}}} \approx 3.5\ \mathrm{s}$$

Notice that this is independent of the mass of the kid.

Let's try this for a very lightweight and a much heavier "kid!"

By the way, how often should I "kick" if I want to make the swing go as high as possible? Time kicks to swing's natural motion!

Most important points about periodic motion

- Meaning of amplitude, period, frequency
- Drawing or interpreting a graph of periodic motion
- Don't confuse angular frequency vs. frequency ($\omega = 2\pi f$)
- Any system that is in stable equilibrium can undergo vibrations w.r.t. that stable position.
- Mass on spring: (natural) frequency is

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

- Pendulum: (natural) frequency is

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{g}{\ell}}$$

- For a given mass, a larger restoring force (more stiffness) increases f_0.
- If the restoring force is elastic (not gravitational), then a bigger mass decreases f_0. For pendulum, f_0 doesn't depend on mass, because restoring force is gravitational.

We often write $x(t)$ in terms of $\omega =$ "natural angular frequency:"

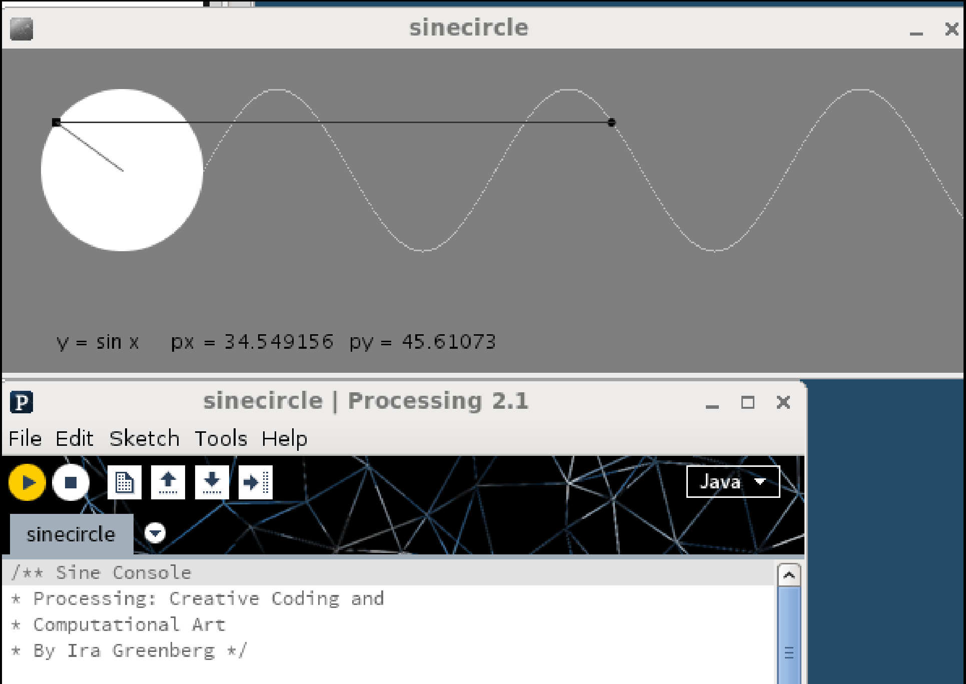
$$x = A\cos(\omega t + \phi_i)$$

but we can equivalently used $f =$ "natural frequency:"

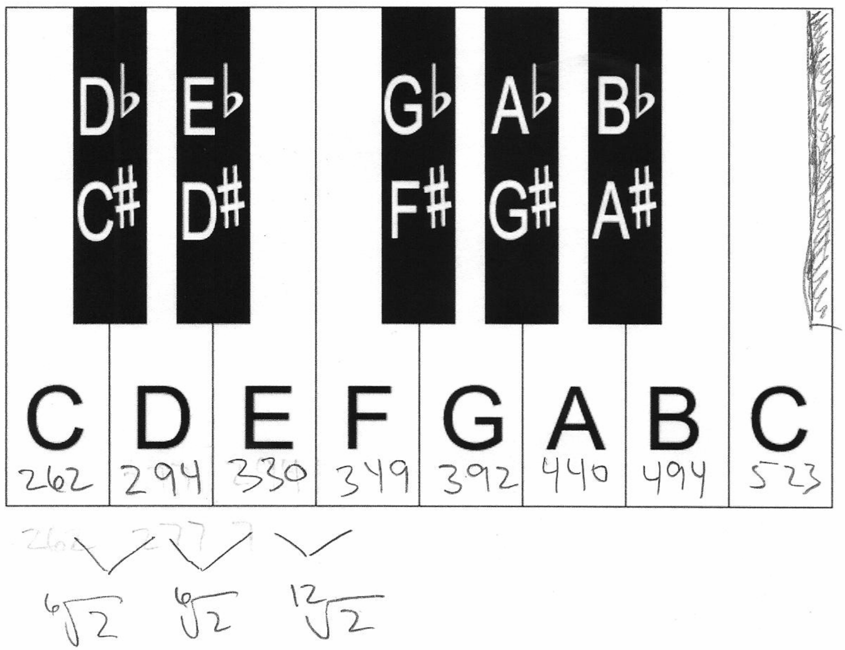
$$x = A\cos(2\pi f\ t + \phi_i)$$

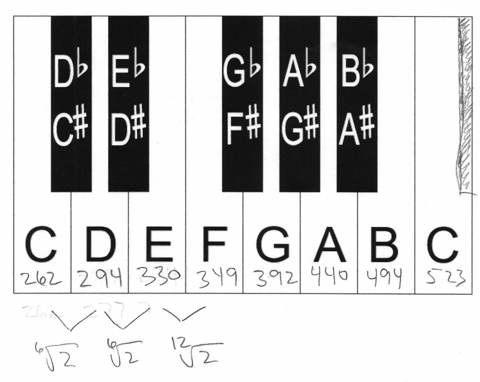
- $f =$ *frequency*, measured in cycles/sec, or Hz (hertz)
- $\omega = \frac{f}{2\pi}$ is *angular frequency*, measured in radians/sec, or s^{-1}
- The frequency $f = 2\pi\omega$ is much more intuitive than ω
- Using ω keeps the equations cleaner — can be helpful for derivations, etc., so that you don't have to keep writing 2π

"angular velocity" ω is our old friend from studying circular motion:



“frequency” $f = \frac{\omega}{2\pi}$ is more familiar from music, etc.





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resonance: Tacoma Narrows Bridge collapse

http://www.youtube.com/watch?v=j-zczJXSxnw

- We may not have time for this 6-minute video in class. If you've never seen it, I highly recommend watching it!
- This one has no audio. There's another version of this video out there that has newsreel-style audio.

Let's return to our two favorite examples of oscillating systems.

Natural frequency & period for mass on spring:

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \qquad T_0 = 2\pi\sqrt{\frac{m}{k}}$$

Natural frequency & period for simple pendulum (small heavy object at end of "massless" cable):

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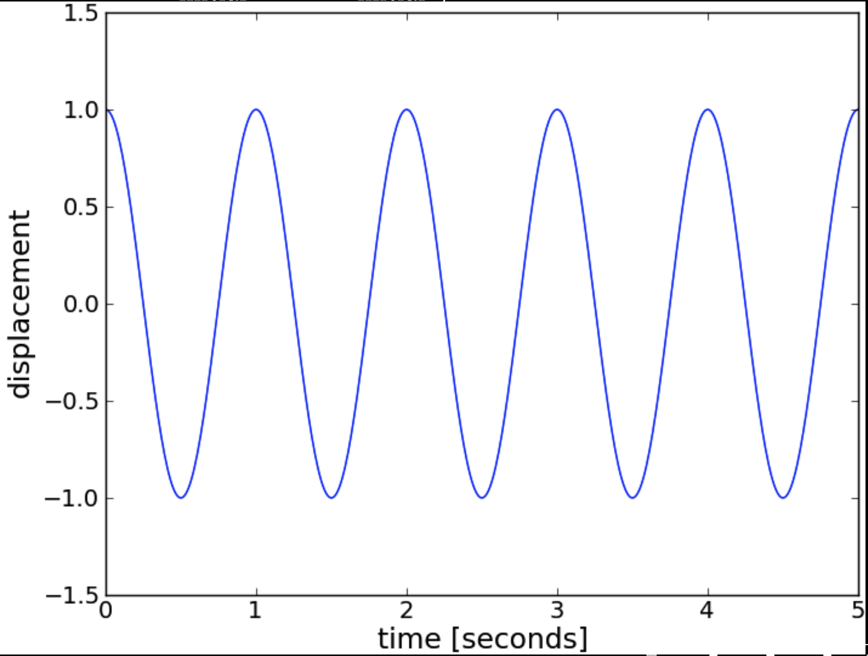
Oscillation: $v_{\text{initial}} = 0$, $x_{\text{initial}} \neq 0$



Missing from previous picture: **damping**

- Without some kind of external push, a swingset eventually slows to a stop, right? Eventually the mechanical energy is dissipated by friction, air resistance, etc.
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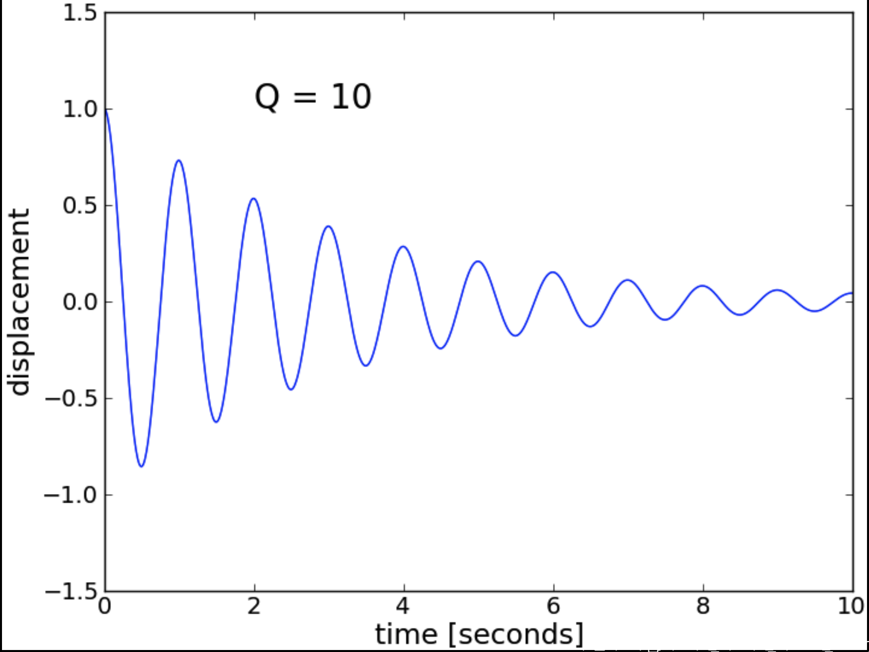
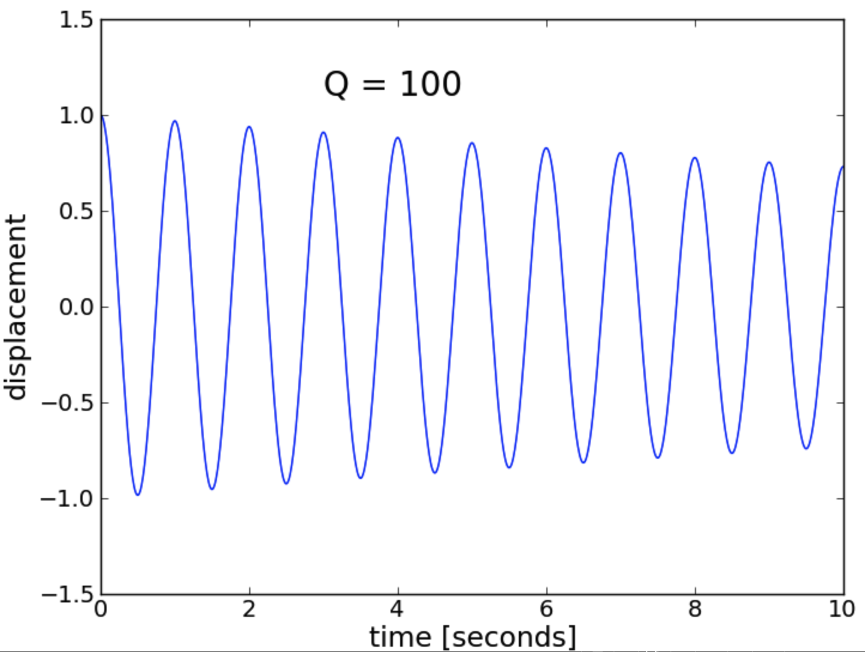
Oscillation (damped, Q=10, f=1 Hz)



Oscillation (damped, Q=100, f=1 Hz)



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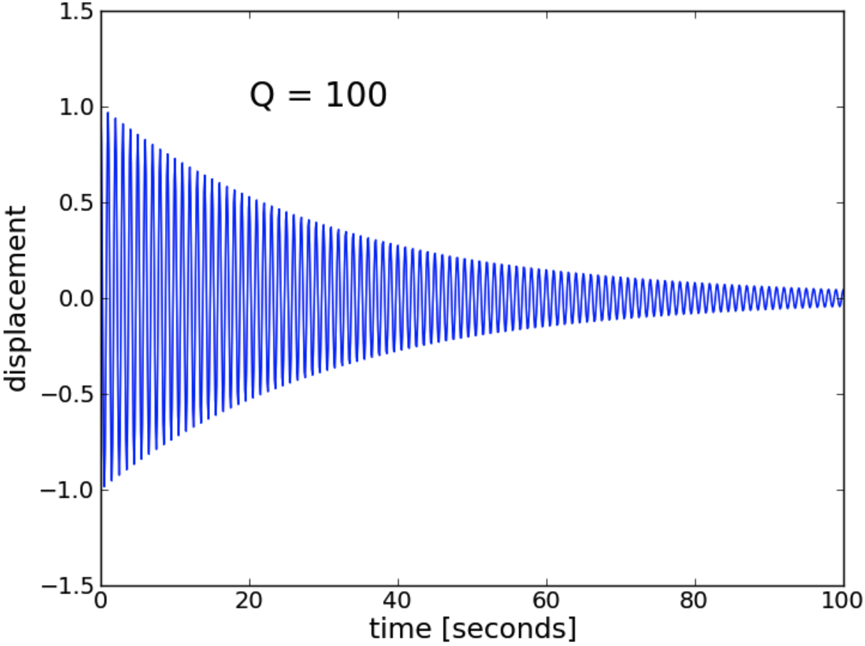
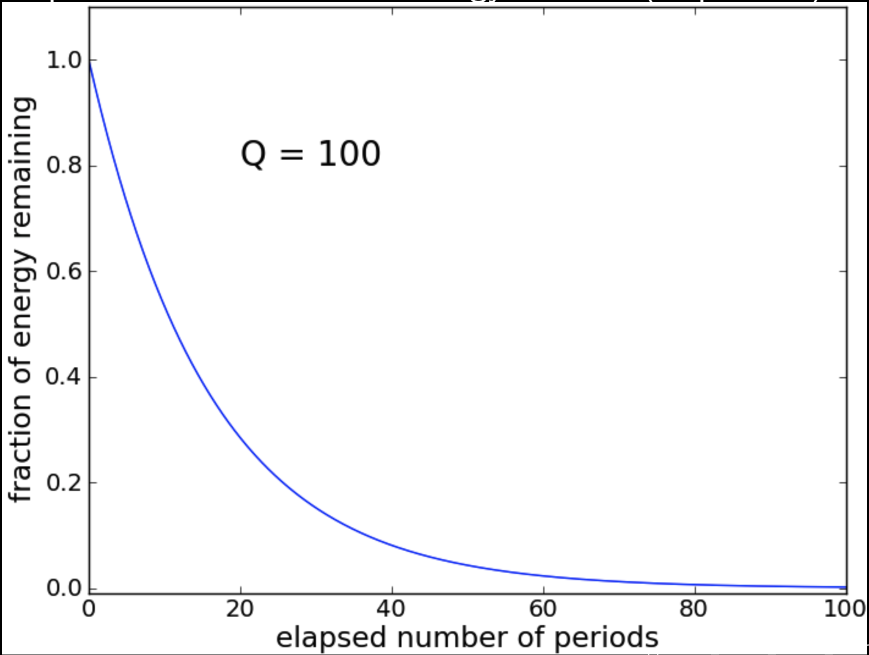
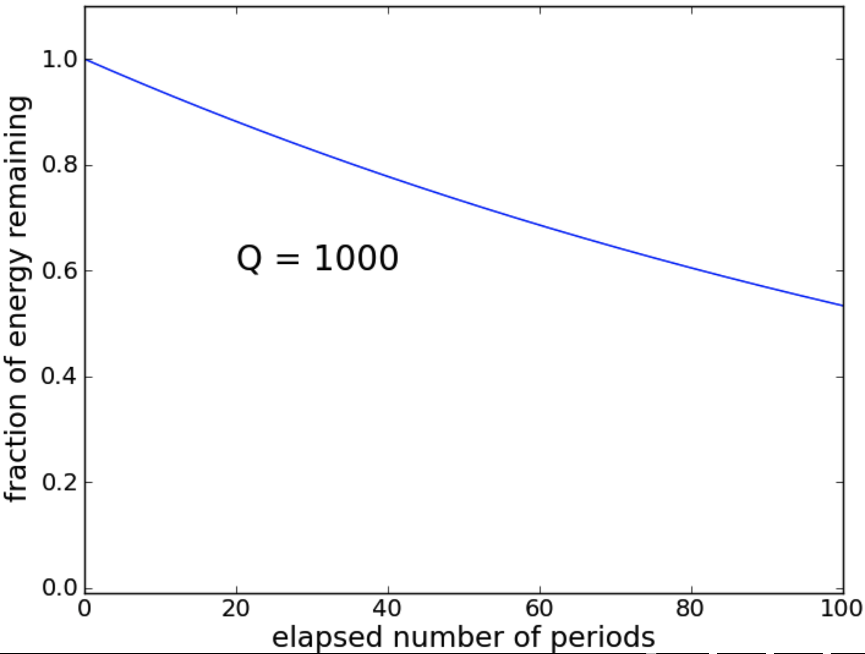


Damped, Q=100, f=1 Hz: energy vs time (in periods)



Damped, Q=1000, f=1 Hz: energy vs time (in periods)



For a given frequency f,

- Less damping $\leftrightarrow$ higher Q
- More damping $\leftrightarrow$ lower Q

- $Q = \omega\tau$ is number of radians after which energy has decreased by a factor $e^{-1} \approx 0.37$
- Equivalently, $Q = 2\pi f \tau$ is number of cycles after which energy has decreased by a factor $e^{-2\pi} \approx 0.002$

- More simply, Q is roughly the number of periods after which nearly all of the energy has been dissipated.

- "Tinny" sound of frying pan $\leftrightarrow$ low Q (fast dissipation)
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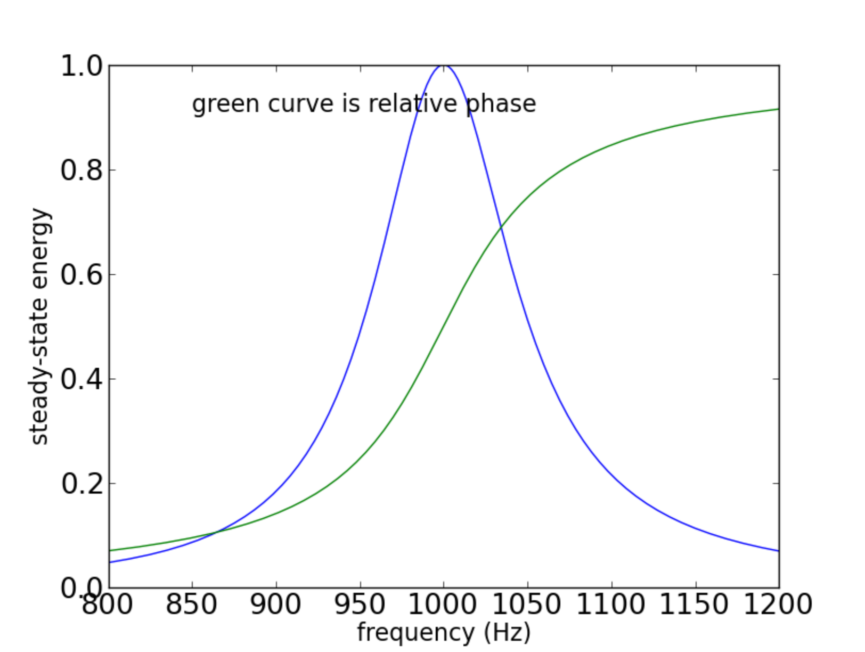
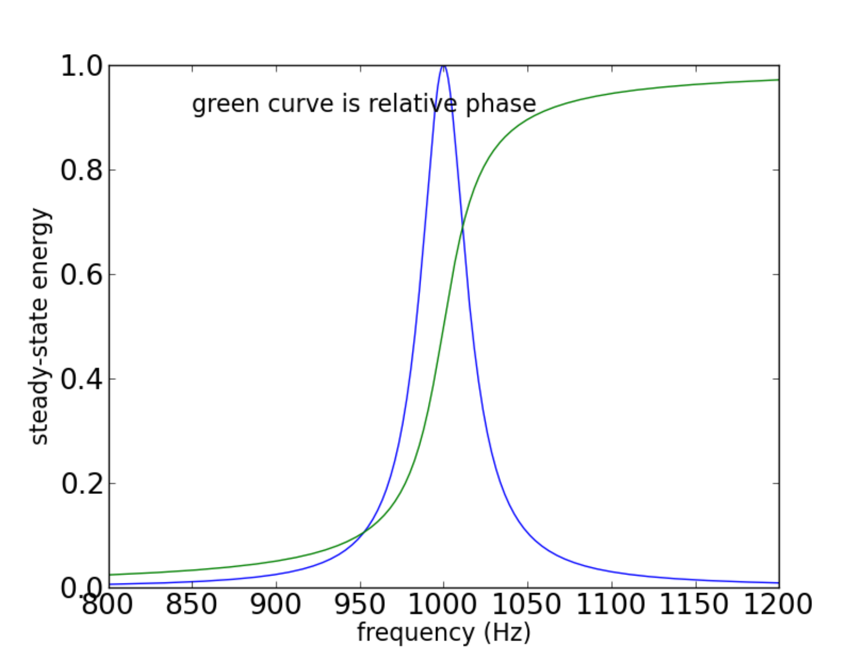
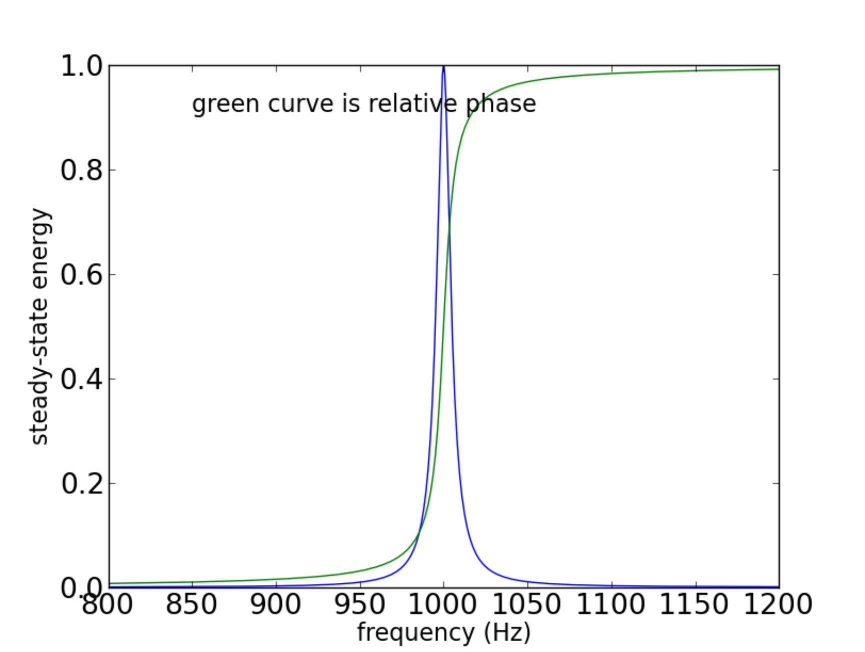
$f_0 = 1000$ Hz, $Q = 10$: energy and phase vs. f_{push}



$f_0 = 1000$ Hz, $Q = 30$: energy and phase vs. f_{push}



$f_0 = 1000$ Hz, $Q = 100$: energy and phase vs. $f_{\rm push}$



Physics 8 — Wednesday, December 4, 2019

- Practice exam: If you turn it in Monday (in class, or in my office, by 5pm), I'll return it to you on Wed, Dec 11. If I don't have your exam by 5pm on Monday, Dec 9, your score is zero, no exceptions, so that I can return graded exams promptly.
- 4 previous years' exams & practice exams are at `http://positron.hep.upenn.edu/p8/files/oldexams`
- Extra credit options (until Thu, Dec 19):
  - O/K ch9 (columns)
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  - Code something in Processing or Py.Processing
  - Go through tutorials to learn Wolfram Mathematica
  - Go through Prof. Nelson's python data modeling book
  - You can suggest something else!

Physics 8 — Friday, December 6, 2019

- Practice exam: If you turn it in today by 5pm, I'll email it back to you, graded, on Monday evening. If you turn it in Monday (in class, or in my office, by 5pm), I'll return it to you on Wed. If I don't have your exam by 5pm on Monday, your score is zero, no exceptions, so that I can return graded exams promptly. I'll post solutions online Monday evening.
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We often write $x(t)$ in terms of $\omega =$ "natural angular frequency:"

$$x = A\cos(\omega t + \phi_i)$$

but we can equivalently used $f =$ "natural frequency:"

$$x = A\cos(2\pi f\ t + \phi_i)$$

- f = *frequency*, measured in cycles/sec, or Hz (hertz)
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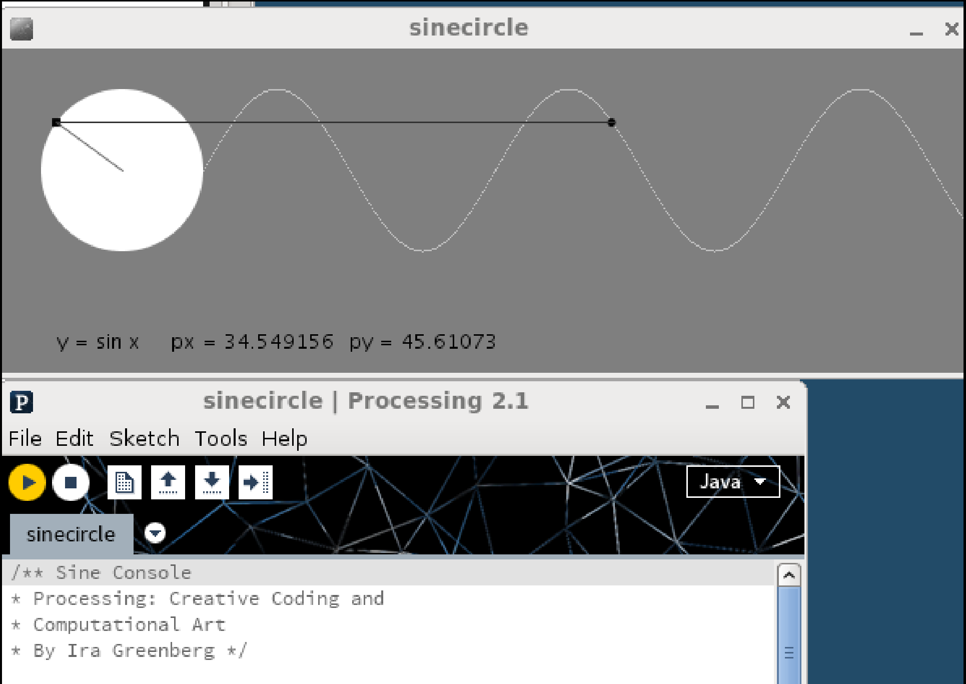
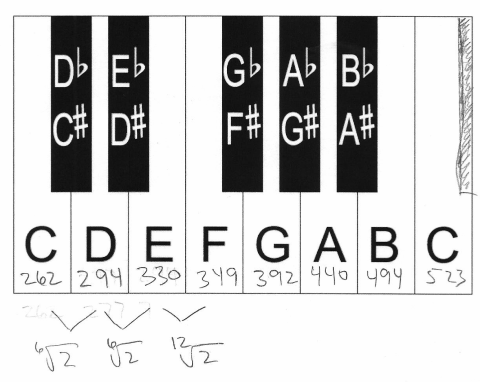


“frequency” $f = \frac{\omega}{2\pi}$ is more familiar from music, etc.



$(\sqrt[12]{2})^4 = 1.2599 \approx \frac{5}{4}$ (major 3rd)

$(\sqrt[12]{2})^5 = 1.3348 \approx \frac{4}{3}$ (perfect 4th)

$(\sqrt[12]{2})^7 = 1.4984 \approx \frac{3}{2}$ (perfect 5th)

$(\sqrt[12]{2})^{12} = 2$ (an octave!)

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- Middle C: 261.63 Hz
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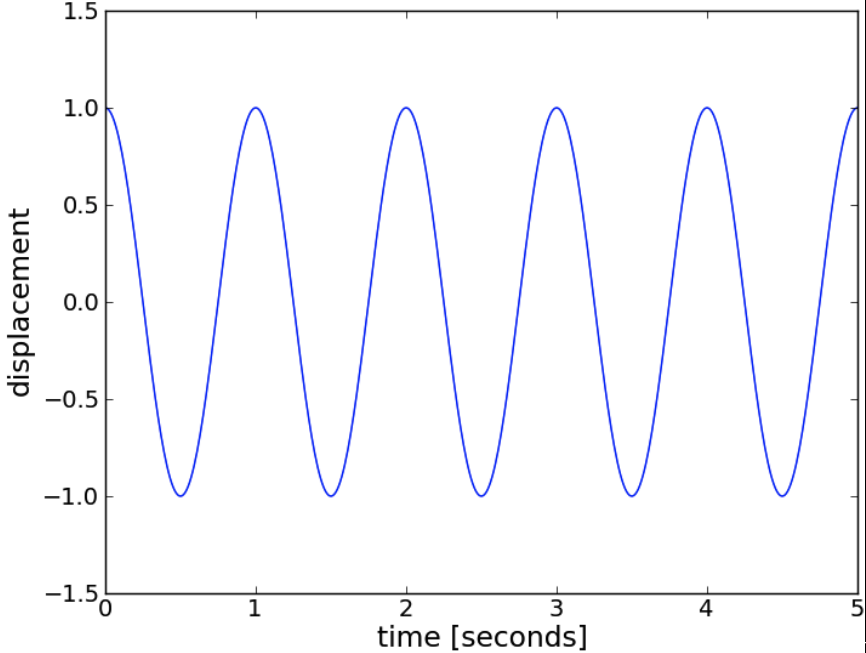

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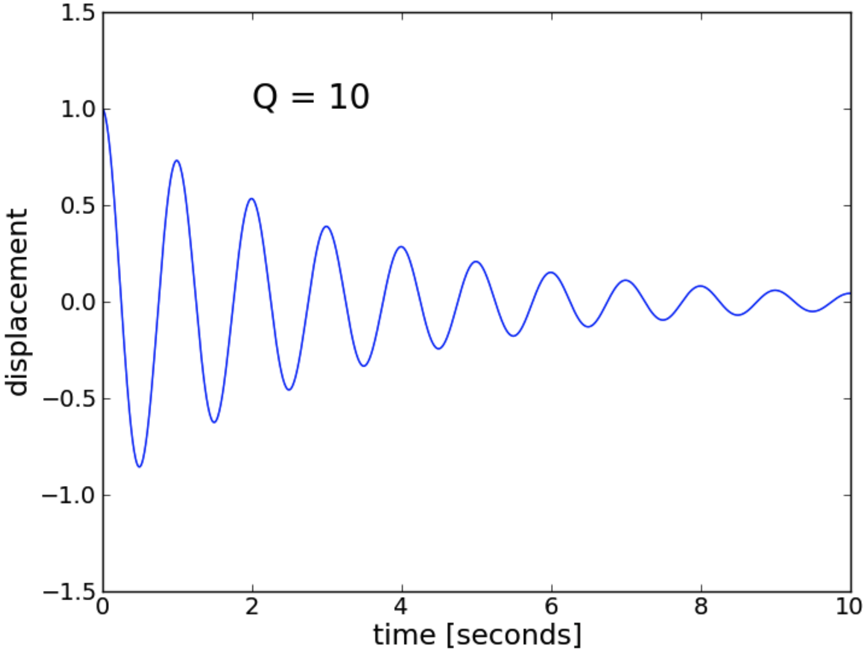

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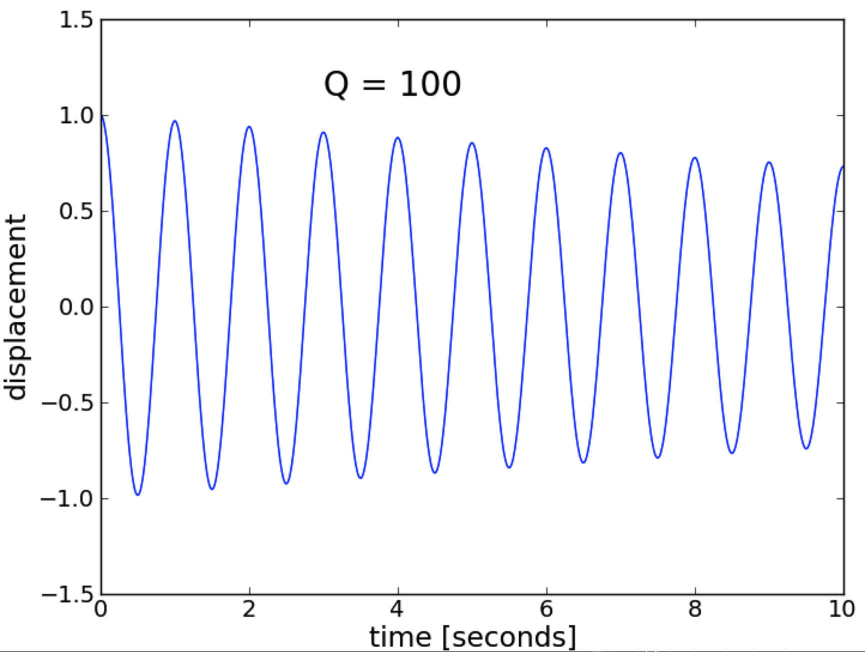

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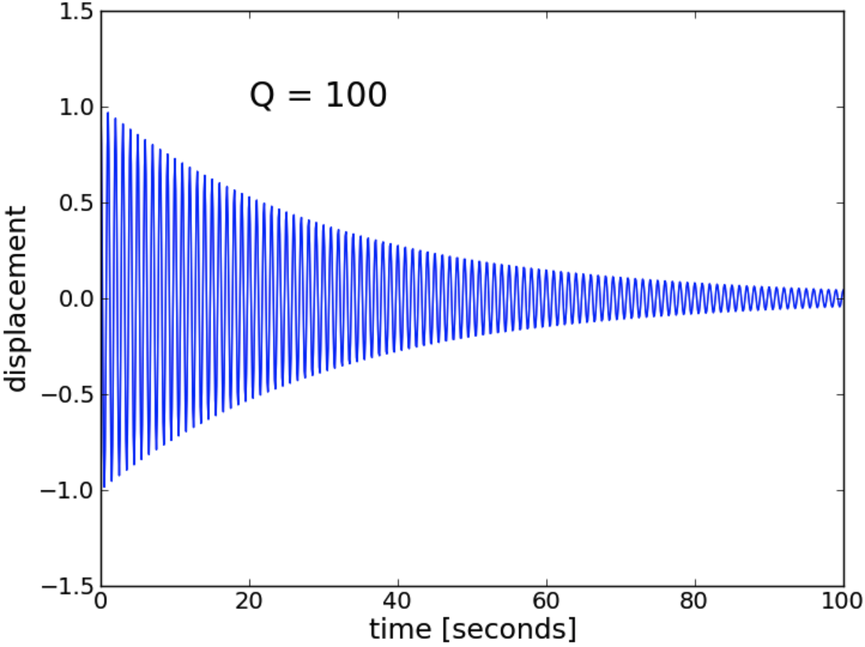
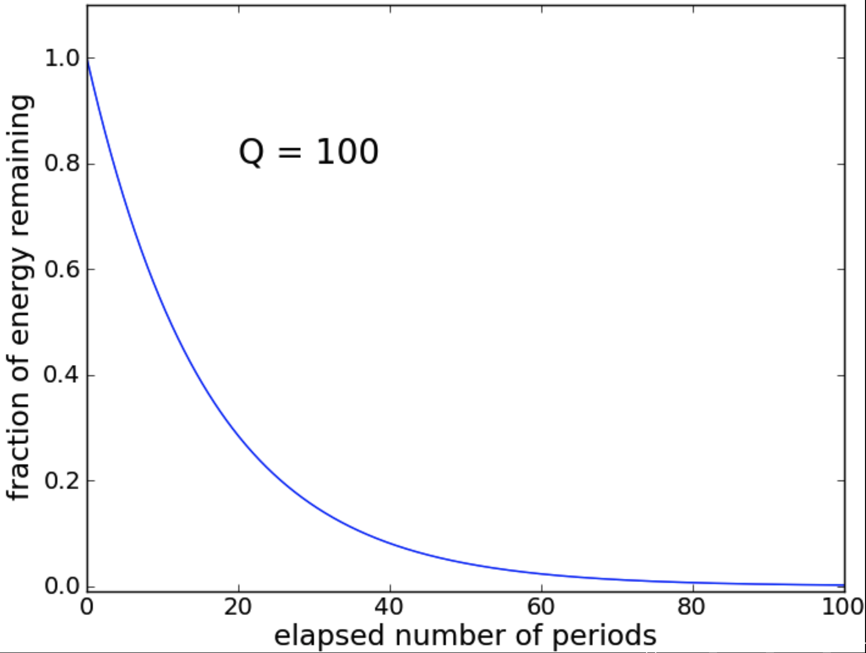
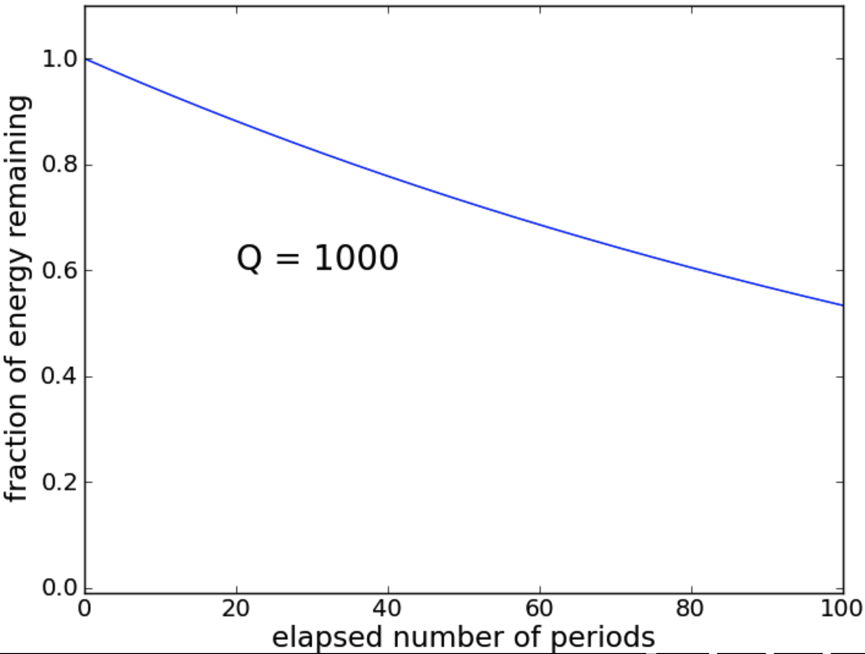


Damped, Q=100, f=1 Hz: energy vs time (in periods)



Damped, Q=1000, f=1 Hz: energy vs time (in periods)



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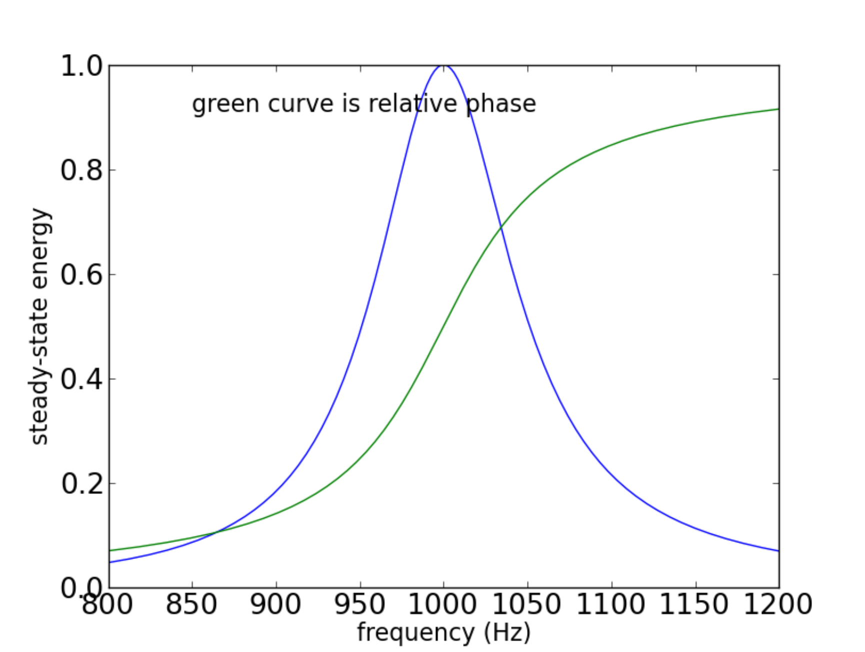

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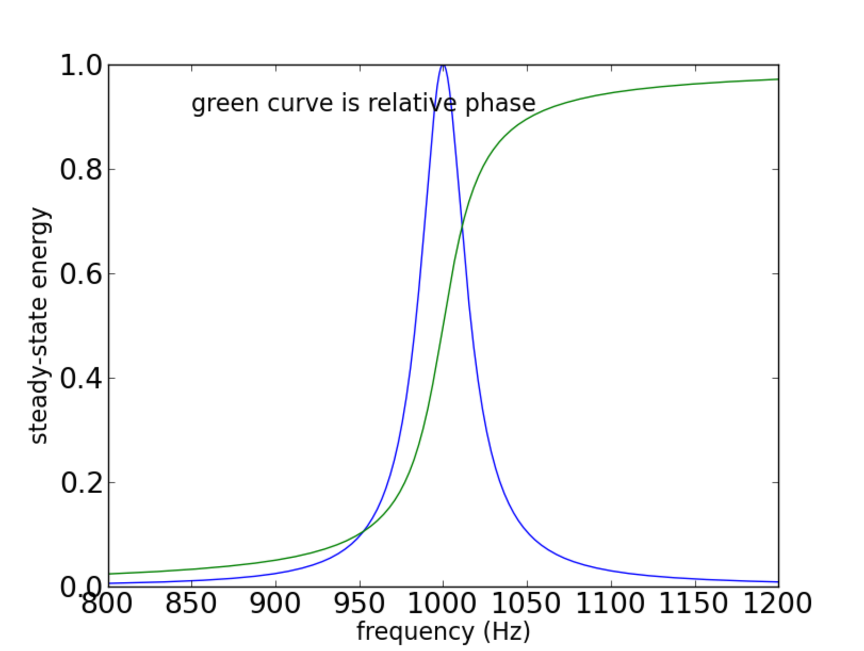

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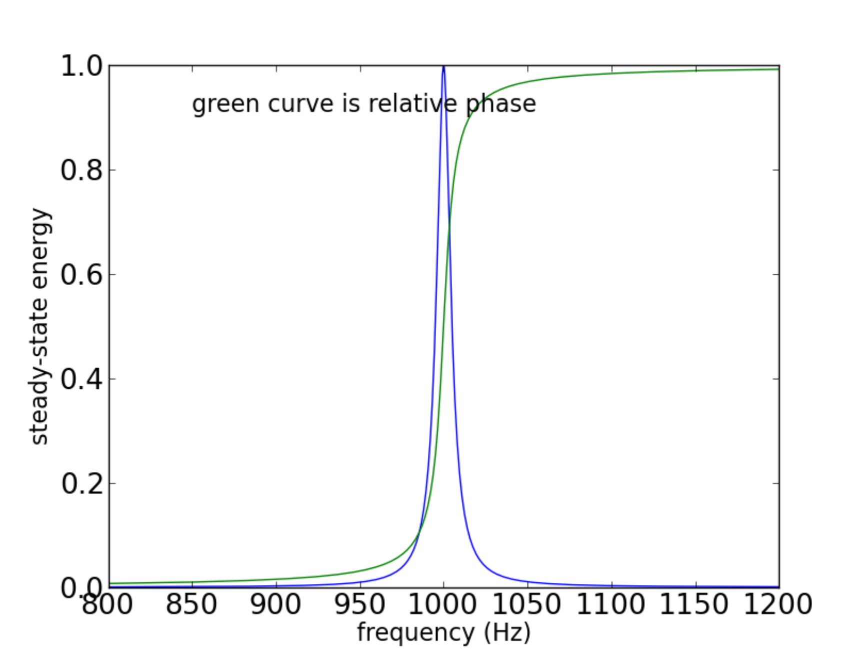

(avoiding) resonance in structures

https://99percentinvisible.org/episode/supertall-101/

(avoiding) resonance in structures

Lateral Loads and Stability

Natural period of oscillation

- harmonic motion
- period of a structure is proportional to weight and inversely proportional to stiffness

Lateral stability of structures

- braced frame
- rigid frame
- shear wall

Code and Safety in Design

- factor of safety
- resilience

Design strategies

- absorb energy in the structure (flexible joints, shock absorbers) (example Bell Atlantic, west coast buildings)
- tuned mass dampers (CitiGorp)
- building shape (Burj Khalifa)

Lateral Stability in Structures

Physics 8 — Friday, December 6, 2019

- Practice exam: If you turn it in today by 5pm, I'll email it back to you, graded, on Monday evening. If you turn it in Monday (in class, or in my office, by 5pm), I'll return it to you on Wed. If I don't have your exam by 5pm on Monday, your score is zero, no exceptions, so that I can return graded exams promptly. I'll post solutions online Monday evening.
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  - Citigroup Center "structural integrity" podcast
  - Mazur ch13 (gravity), ch14 (Einstein relativity)
  - Code something in Processing or Py.Processing
  - Go through tutorials to learn Wolfram Mathematica
  - Go through Prof. Nelson's python data modeling book
  - You can suggest something else!

Physics 8 — Monday, December 9, 2019

- Practice exam: If you turned it in Friday, I'll email it back to you, graded, this evening. Otherwise, I'll return it to you on Wed. If I don't have your exam by 5pm today, your score is zero, with no exceptions, so that I can put solutions online tonight.
  `http://www.hep.upenn.edu/~ashmansk/drl_1w15.jpg`
- 3×5 card + "dumb" calculator on final exam (no internet, no reference materials except 3×5 card)
- Review session (optional) Wed 2–4pm DRL A6
- 4 previous years' exams & practice exams are at
  `http://positron.hep.upenn.edu/p8/files/oldexams`
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  - Citigroup Center "structural integrity" podcast
  - Learn about Taipei 101 "Tuned Mass Damper"
  - Mazur ch13 (gravity), ch14 (Einstein relativity)
  - Code something in Processing or Py.Processing
  - Go through tutorials to learn Wolfram Mathematica
  - Go through Prof. Nelson's python data modeling book
  - Suggest something else: `https://youtu.be/Wiln4BUOzDg`
- If you have your clicker here, please turn it in after class.

(avoiding) resonance in structures

https://99percentinvisible.org/episode/supertall-101/

Taipei 101 “Damper babies” speaking in their made-up nonsense language about the Tuned Mass Damper
https://youtu.be/1kwMnBOPAVQ

Short documentary by actor/presenter includes “eating soup on moving bus” demo of non-inertial reference frames, and interview with the Taipei 101 architect including architect’s demo that resembles our in-class meter-stick demos:
https://youtu.be/OSEYOavsKxA

Physics can give us new insights into the everyday world. We should go through this video a second time at end of semester.

https://www.youtube.com/watch?v=XfZFuw7a13E

https://www.youtube.com/watch?v=XfZFuw7a13E&t=35

- 0:35 — impulse
- 0:43 — rotational inertia, torque
- 0:51 — torque, periodic motion, velocity, projectile motion
- 2:53 — friction, circular motion, projectile motion (173 s)
- 3:30 — center of mass (210 s)
- 6:18 — friction, “normal force” (378 s)

https://www.youtube.com/watch?v=XfZFuw7a13E&t=378

(avoiding) resonance in structures [ARCH 535]

Lateral Loads and Stability

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- period of a structure is proportional to weight and inversely proportional to stiffness

Lateral stability of structures
- braced frame
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- tuned mass dampers (CitiGorp)
- building shape (Burj Khalifa)

Lateral Stability in Structures

I list at the link below some (totally optional) questions about the format of the course. Please take a look. If you're willing to think about them, you can answer either via the usual response form or else include your thoughts anonymously with your course review. E.g. type anonymous comments now and copy/paste them later.

`http://positron.hep.upenn.edu/q008/?date=2019-12-06`

- About how many hours did this practice exam take you?
- Are there particular problems from [the practice exam] or topics from the course that you'd like me to go over in the review session before the final exam?
- What topics did you enjoy most and least from this course? Will anything that you have learned in Physics 8 be useful in your future career? Would you adjust the balance between covering traditional physics topics and covering applications to architectural structures? As a result of feedback on this question from past years, I have dropped Mazur's chapter 13 (gravity) and substantially expanded the Onouye/Kane segment of the course.

- Do you have any comments on the format of Physics 8 (how to use classroom time, whether to have midterms or quizzes, how much emphasis to place on homework and reading)? Do you have suggestions for formatting Physics 9 for next fall in a way that will help you to learn the material more readily or will make the course more engaging for you?
- Here is what I am thinking for reading assignments next time. I welcome your comments and suggestions! I like Mazur's non-traditional ordering of topics (momentum and energy before forces, focus initially on one-dimensional problems), and I like the conceptual half (the first half) of each of his chapters. But I think that the second half of each Mazur chapter, many students in Physics 8 aren't sure where to focus their attention. So I'm tempted to keep the first half of each Mazur chapter but to replace the second half with notes that I would write up to summarize the key results, with pointers to the textbook for longer discussions.

- For the Onouye/Kane book, I love the illustrations and the many architecture-related worked examples. But I think in many cases it would be nice to have a clearer explanation of the key results and how to use them. So I'm tempted to supplement each O/K chapter with my own notes summarizing the most important results. If I supply notes, you would read my notes carefully and then just quickly skim through the corresponding textbook material. Do you think that would be an improvement? Other suggestions?
- I'd like to make the reading go more efficiently for you, and better synchronize each week's reading to that week's classroom time. I'd like to distill the O/K material to the key ideas, simplify the discussion of beams, and add a short discussion of columns and of stone arches. I also want to expunge the non-metric units from most of the examples!

- If you have your clicker, please turn it in after class.
- Would 2 lectures @ 90 minutes work better? e.g. MW 3-4:30?
- Future "quiz" idea, to cover a topic from homework already done, graded, & handed back with solutions:
- pass 1: spend 5 minutes solving the problem on your own, and hand that in.
- pass 2: spend 5 minutes discussing & solving the problem with your neighbors, and hand in your copy of that group result.
- If you fix on pass 2 any mistakes from pass 1, you earn back 50% of the corresponding points you lost on pass 1.
- You would do this at the start of class, about once a week, while I'm setting up demos or handing back graded work.
- Another idea: try to make one question per reading assignment be some kind of simple calculation (similar in difficulty to Mazur's "self-quiz" questions) to help you test your understanding. Maybe Canvas would grade this automatically and would let you redo it until you get it right.

- what if we moved some of the HW problem-solving into class time, and moved some of the demonstrations etc into videos or animations?
- what if we blocked out an hour on fridays, potentially for some hands-on lab-like activity in class? (probably more relevant for physics 009.)
- how helpful would it be for me to replace a large fraction of the reading with my own typed-up notes, which would be more focused in content on what we emphasized during classroom time?
- for the current course content, assuming that we spend 3 hours a week together in some sort of classroom, what would be the best use of those hours?

- I really like the homework problems from this course. To me, the homework problems are the most valuable thing you do here, and I think that most of the problems we solve are a good fit. IMO, everything else I do is mainly to motivate you to solve & think carefully about the HW problems.
- It would help me if you could describe what niche this course fills for you, or what niche you'd like it to fill. Sometimes people suggest eliminating this course and having ARCH students take PHYS 150 instead. Sometimes people suggest having PHYS 8/9 be only one semester, not two. Sometimes people suggest adding "labs" or (preferably) hands-on versions of the demonstrations.
- I think that as long as enrollment stays roughly where it is now, it's advantageous to offer introductory physics courses tailored to the interests/needs of various groups of students.
- Anybody thinking of taking Physics 9? It was suggested to offer Physics 8 every year and eliminate Physics 9, but Prof. Farley and I both think the Physics 9 topics are quite valuable, even if they are less tangible than mechanics.

- By the way, the topics for Physics 9 can be summarized as:
- waves, sound, light, fluids, heat, electricity & circuits.
- Understanding these topics is relevant for environmental systems, energy efficiency, acoustics & soundproofing, mechanical & plumbing systems, etc.
  - If you've sat in an old DRL classroom while the medevac helicopter passed overhead, you know why a designer needs to understand sound propagation.
  - Since so much of what you do is visual, you're probably already curious about what light is and how it is emitted, reflected, absorbed, magnified, affected by passing through glass & water, observed by human eyes, etc.
  - If you've felt your ears pop when deep underwater or at high altitude, if you've marveled at the size of the Hoover Dam, or if you've seen the effect of a strong wind on the roof of a house of the wall of a skyscraper, then you may be curious how fluids (liquids and gases) work.
  - Since a key function of many buildings is to shelter occupants from variations in outdoor temperature, you may want to learn some of the physics behind temperature, heat, and energy.

- By the way, the topics for Physics 9 can be summarized as:
- waves, sound, light, fluids, heat, electricity & circuits.
  - Since it's difficult to imagine life in the modern world without electricity, you might be curious what volts and amps really are, or why so-called "high-tension" (really "high voltage") lines are used to transport electricity over long distances, or why an electrical outlet provides "alternating current," while a flashlight battery provides "direct current."
  - You might also be generally curious how electrical forces hold atoms together and are responsible for the chemical energy stored in food and fuel. Or how a rooftop solar panel converts sunlight into electrical energy.
- We are also tempted to make Physics 9 a more hands-on course than Physics 8. Sometimes Physics 9 is a smaller group, which makes it easier to be less formal.
- In 2016, we did some hands-on learning in class, e.g. by building and measuring some battery-powered circuits. We also spent a few classes learning to program tiny "Arduino" computers to let you create gadgets that can interact with the environment: blinking, sensing, producing sounds, etc.

- For the Physics 9 topics, there is much less need to spend a lot of time solving intricate homework problems. There are homework problems, but they are much less rigorous than in Physics 8.
- In Physics 8, we really want you to become highly skilled, through lots of practice, at working with forces, vectors, and torques/moments.
- In Physics 9, we want you to be aware of the many physical phenomena that affect a design: acoustics, light, the movement of air and water, heating/cooling, electrical power, electronic automation.
- So much of the reading takes the form of anecdotes that illustrate the relevance of the physics. In 2016 & 2018, we did this using "Physics and Technology for Future Presidents" by Richard Muller, which most students enjoyed. Next year we will probably supplement by writing up many real-world examples drawn from Richard Farley's long experience in architecture/engineering practice.
- Also, Physics 8 is no longer a prerequisite for Physics 9.

Physics can give us new insights into the everyday world. We should go through this video a second time at end of semester.

https://www.youtube.com/watch?v=XfZFuw7a13E

https://www.youtube.com/watch?v=XfZFuw7a13E&t=35

- 0:35 — impulse
- 0:43 — rotational inertia, torque
- 0:51 — torque, periodic motion, velocity, projectile motion
- 2:53 — friction, circular motion, projectile motion (173 s)
- 3:30 — center of mass (210 s)
- 6:18 — friction, "normal force" (378 s)

https://www.youtube.com/watch?v=XfZFuw7a13E&t=378

Course recap

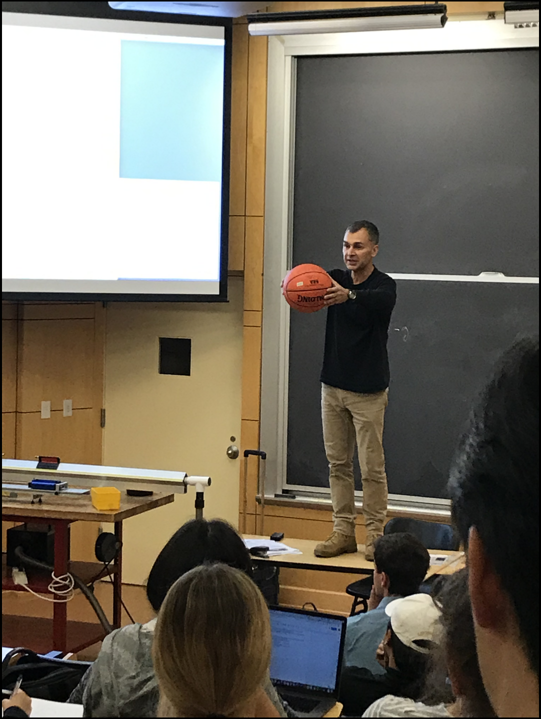
- We'll do a more technical recap at the review session. Here's my more philosophical recap of Physics 8.
- A key motivation for architects to study Newtonian mechanics is to enable you to go on to study architectural structures.
- Prof. Farley has told me several times that he wants students to enter his Structures course with a solid understanding of forces, vectors, and torque ("moments").
- Another motivation to study physics, which Richard Wesley likes to point out, is that undergraduate Architecture at Penn is set in a liberal-arts context. Breadth of knowledge is good.
- The "force" concept is notoriously difficult for students to learn. Even after you've learned how to solve homework problems using forces, it still takes a lot of thinking and practice to grasp Newton's three laws fully. Newton's laws are counter-intuitive: they defy your innate intuition.
- (1) Law of inertia. (2) $\vec{F} = m\vec{a}$. (3) $\vec{F}_{12} = -\vec{F}_{21}$.

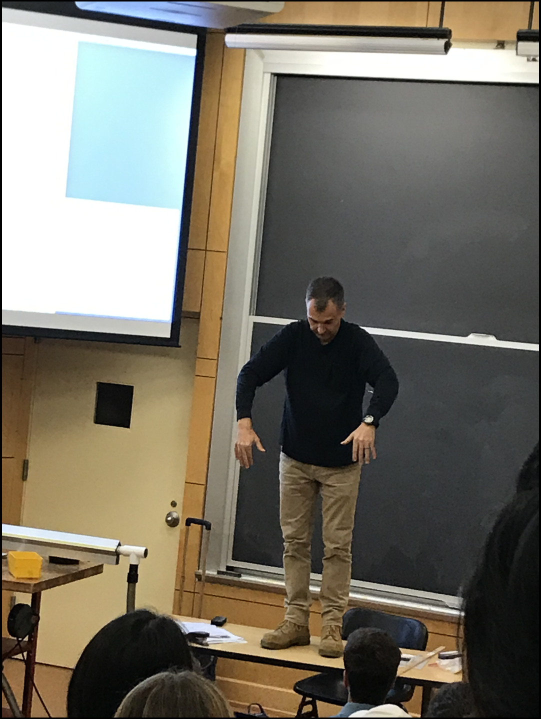
- (1) Law of inertia. (2) $\vec{F} = m\vec{a}$. (3) $\vec{F}_{12} = -\vec{F}_{21}$.
- If you told a little kid that if you do nothing to a moving object, it continues forever with the same direction and speed, the kid would not believe you.
- Even after you've learned calculus, it takes some time to get used to $\mathrm{d}^2x/\mathrm{d}t^2 = (1/m)\, F_x$.
- And you can find many examples of book authors who believe, incorrectly, that $\vec{F}_{12} = -\vec{F}_{21}$ stops being true if you put such a large weight on a table that the table collapses.
- The fact that force and acceleration are vectors makes all of the above even more complicated, since you're remembering trigonometry at the same time as you're learning forces.
- So we began by describing motion, in 1D, to grasp position, velocity, and acceleration. Then we studied colliding objects in 1D, to cement the idea that when two objects interact, the motion of both objects is affected. We studied momentum, then energy, then finally forces and work. Probably different order from your first physics course.

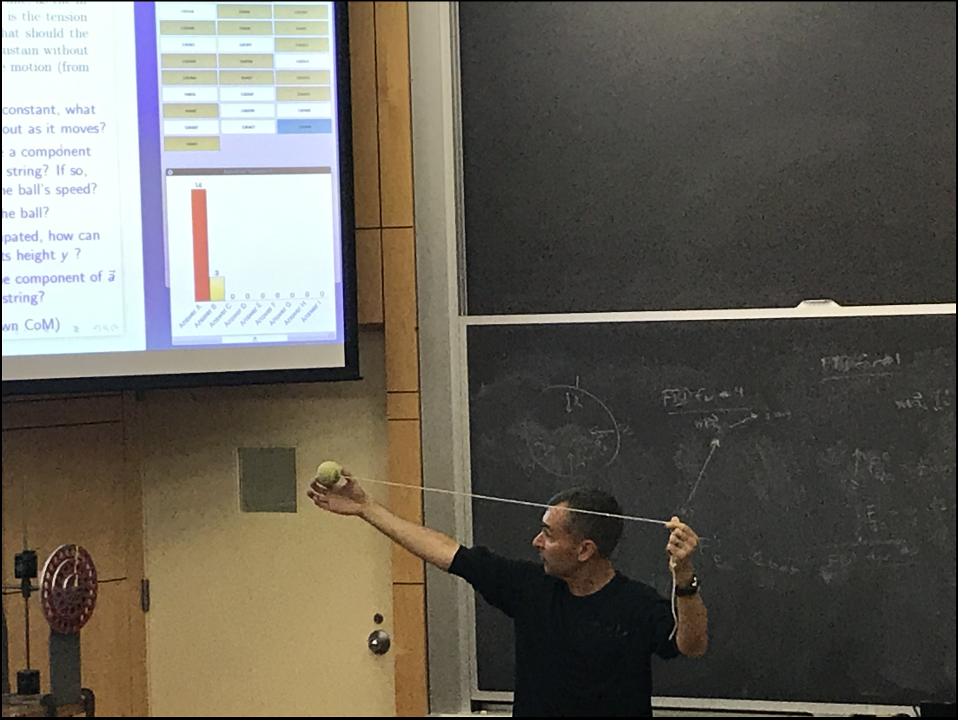
- Then we moved from 1D to 2D. Vectors/trigonometry!
- That let us model friction quantitatively. It let us solve some classic projectile-motion problems.
- Then motion in a circle, where we saw that “accelerating” does not have to mean “changing speed!” That let us understand the strange effects you feel on a highway offramp, or the tension in a string on which a ball twirls around.
- Finally we came to rotation and torque, which make acceleration and forces look easy by comparison.
- That gave us exactly the background we needed for our brief 5-week survey of architectural structures. We used the three conditions of equilibrium — again and again and again. We identified forces and their lines-of-action and drew EFBDs. We grew more and more accustomed to working with torques (“moments”), through example after example.
- Finally we finished up with periodic motion (“oscillation”), which is also relevant for structures.
- I hope that along the way, a lot of physics ideas have become much more comfortable and familiar to you.

- Teaching this course ($5\times$ so far) — discussing physics with you in person and by email, and working on solving physics problems with many of you — is a huge amount of fun for me. Doing this job doesn't feel like "work."
- I try to make this course interactive, so that I can adapt it to your interests, your questions, your learning styles. Student feedback has made this "your" physics course — more so than "mine." What I've learned from you will help to make this a better course for future students.
- I've tried to push you to learn as much as you reasonably could about the physics that I think will inform your intuition about the physical world in which your own creations will reside.
- My goal is to be a good "coach," rather than a kind of gatekeeper between you and grad school. Like other coaches, I can point you in the right direction, and offer help when you get stuck, but I can't do the learning for you.
- It has been a great honor to study physics with you this fall! Best wishes on your exams/reviews and in your careers!

If you're a senior, expect to see me applauding and shouting out your name from the Locust Walk gauntlet in the commencement procession. Email/text if you want a group graduation photo.







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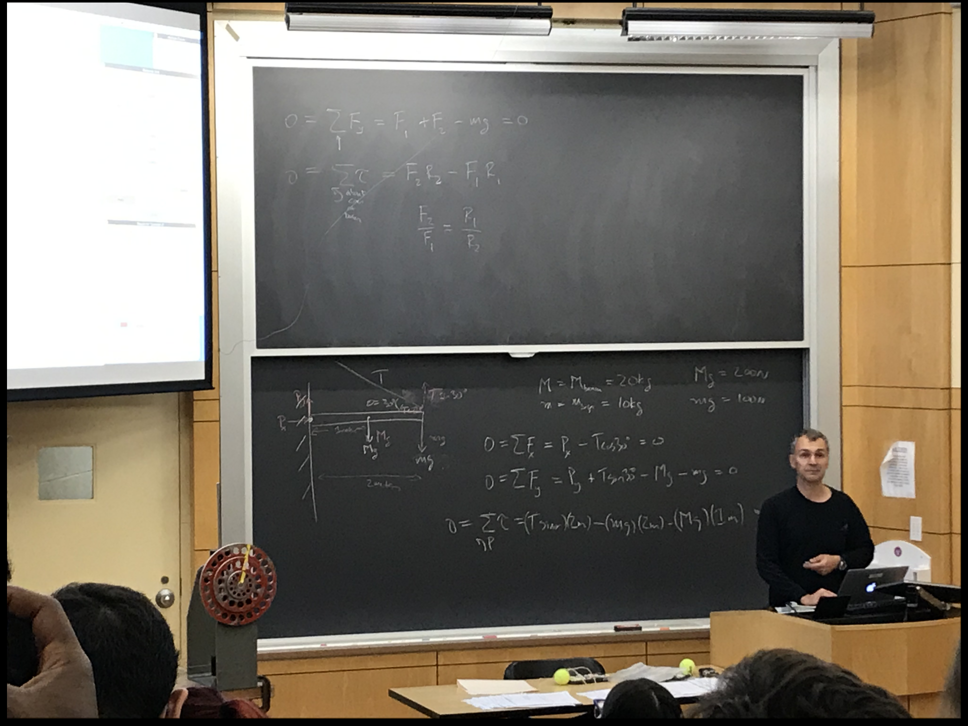
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F_2 / F_1 = R_1 / R_2
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M = M_beam = 20kg
Mg = 200N
m = m_sign = 10kg
mg = 100N
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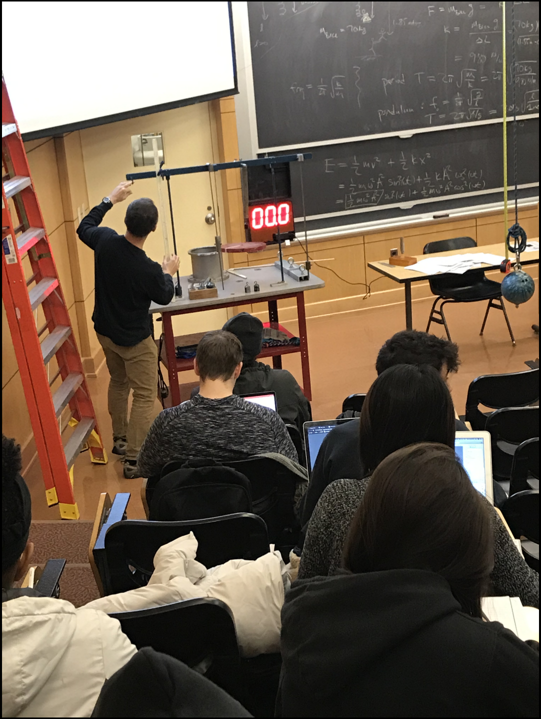
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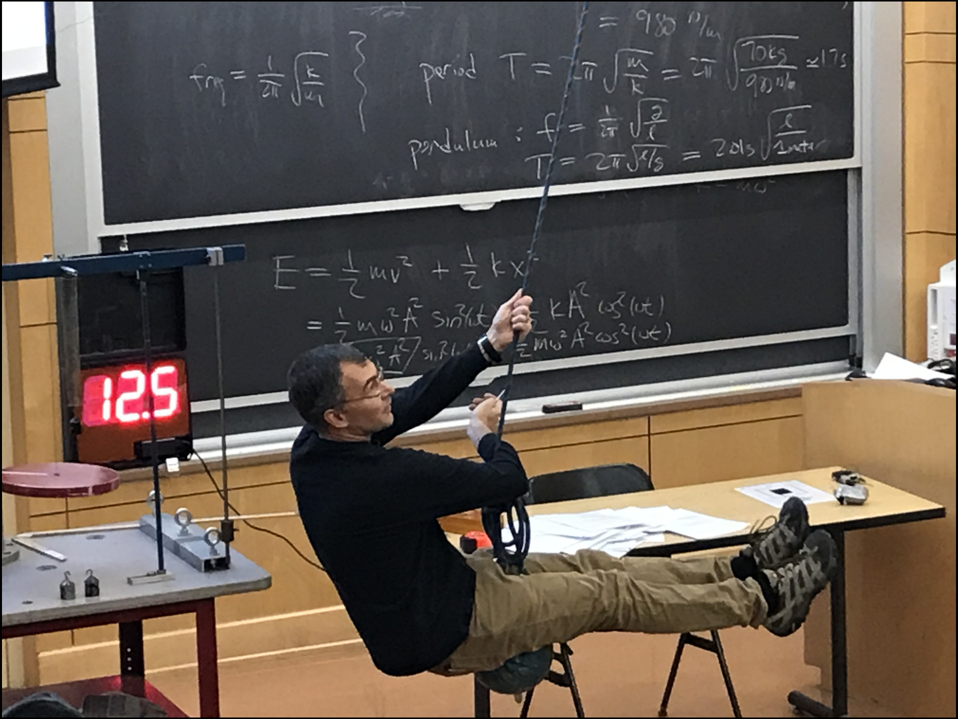

12.5
period T = 2π √(m/k)
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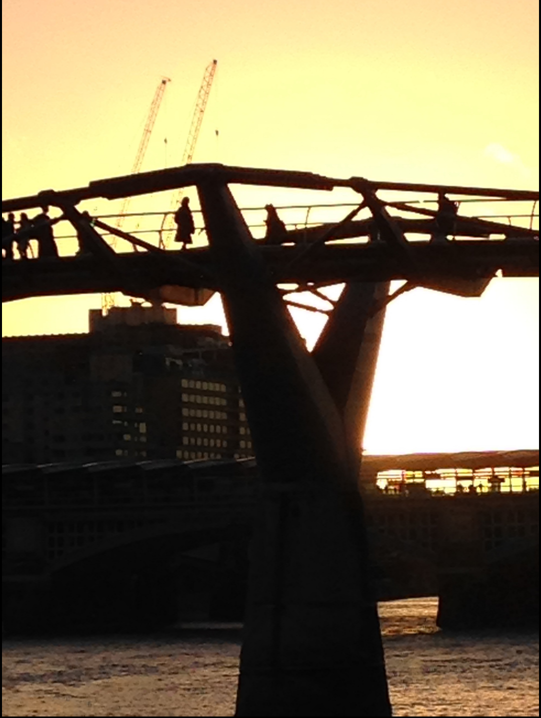

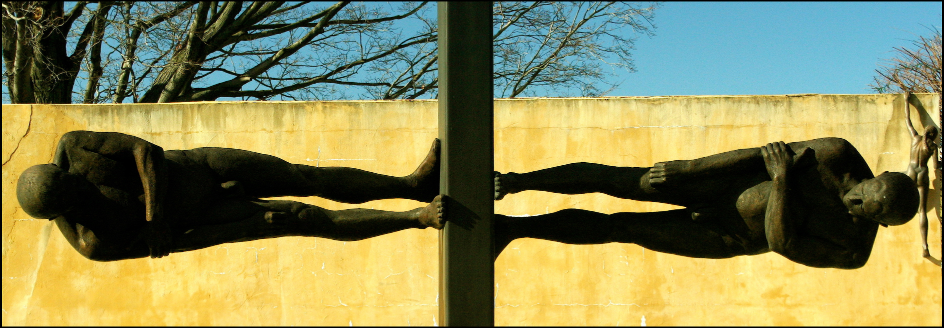

Physics 8 — Monday, December 9, 2019

- Practice exam: If you turned it in Friday, I'll email it back to you, graded, this evening. Otherwise, I'll return it to you on Wed. If I don't have your exam by 5pm today, your score is zero, with no exceptions, so that I can put solutions online tonight.
  http://www.hep.upenn.edu/~ashmansk/drl_1w15.jpg
- 3×5 card + "dumb" calculator on final exam (no internet, no reference materials except 3×5 card)
- Review session (optional) Wed 2–4pm DRL A6
- 4 previous years' exams & practice exams are at
  http://positron.hep.upenn.edu/p8/files/oldexams
- Extra credit options (until Thu, Dec 19):
  - O/K ch9 (columns)
  - Citigroup Center "structural integrity" podcast
  - Learn about Taipei 101 "Tuned Mass Damper"
  - Mazur ch13 (gravity), ch14 (Einstein relativity)
  - Code something in Processing or Py.Processing
  - Go through tutorials to learn Wolfram Mathematica
  - Go through Prof. Nelson's python data modeling book
  - Suggest something else: https://youtu.be/Wiln4BU0zDg
- If you have your clicker here, please turn it in after class.