pipe. The range in depth of the water was less than 0.025 mm, but it was measured with a microscope to within 1 percent. The experiment was performed with pipes running both east and west and north and south.

The tides produced in the pipes were only 69 percent as great as would be expected from calculations based on the assumption that the earth is completely rigid. It is from this experiment that the extent of the tides produced in the solid earth could be inferred.

(f) Other tides

The tides produced by the moon and the sun upon the earth are not the only tides in nature. The earth exerts a tidal force upon the moon that is stronger than the one the moon exerts upon the earth. Over many millions of years the earth's tidal force has acted as a brake on the lunar rotation, so that today the moon keeps the same face turned toward the earth.

In fact, all bodies in the universe exert a tidal force on all other bodies, just as they exert a general gravitational attraction. In most cases these tidal forces are too small to produce observable effects. For example, the tides produced by planets on each other, and on the sun, are entirely negligible. On the other hand, we find instances of binary stars, in which the two stars are so close together as to produce substantial tidal distortion, many times greater than that produced by the earth and moon upon each other (see Chapter 23).

(g) Criteria for a satellite

We can now delineate the criterion for the maximum and minimum distances that a satellite can have from a planet. The former depends on the differential gravitational force of the sun and the latter on the tidal force of the planet itself. For a satellite to remain always in a closed orbit about a planet, its orbital velocity, with respect to that planet, must always be less than its velocity of escape, or the parabolic velocity. The formula for velocity of escape, given in Section 5.4, assumes that the only force between two bodies is their mutual gravitational attraction. However, this attraction must be corrected for the differential gravitational force between the two if a third body is present.

A related problem is to find the minimum distance a satellite can be from its planet. At smaller distances the satellite could not withstand the differential, or tidal, forces exerted on it by the planet and would be torn apart. E. Roche investigated the problem in 1850 and found that if the constituent parts of a satellite are held together only by their mutual gravitation, as, for example, in a liquid body, and if the satellite has the same density as its planet, the critical distance is 2.44 times the planet's radius. At a greater distance, the satellite suffers only tidal distortion, but holds together. At a smaller distance it is torn apart by the tidal forces, for they are greater than the gravitational forces holding the satellite together. If the satellite has high rigidity, so that cohesive forces add to gravitational ones in binding it together, it could survive at a somewhat smaller distance from the planet. The critical distance at which a satellite can survive tidal destruction is called *Roche's limit.* The rings of Saturn are particles that are closer to the planet than the distance at which a large solid body can survive—that is, they are within Roche's limit.

6.5 Precession

The earth, because of its rapid rotation, is not perfectly spherical but has taken on the approximate shape of an oblate spheroid; its equatorial diameter is 43 km greater than its polar diameter. As we have seen, the plane of the earth's equator, and thus of its equatorial bulge, is inclined at about $23\frac{1}{2}°$ to the plane of the ecliptic, which, in turn, is inclined at 5° to the plane of the moon's orbit. The differential gravitational forces of the sun and moon upon the earth not only cause the tides but also attempt to pull the equatorial bulge of the earth into coincidence with the ecliptic.

The latter pull is illustrated in Figure 6.18. The solid arrows are vectors that represent the attraction of the moon on representative parts of the earth. The part of the earth's equatorial bulge nearest the moon is pulled more strongly than the part farthest from the moon, and the earth's center is pulled with an intermediate force. The dashed arrows show the differential forces with respect to the earth's

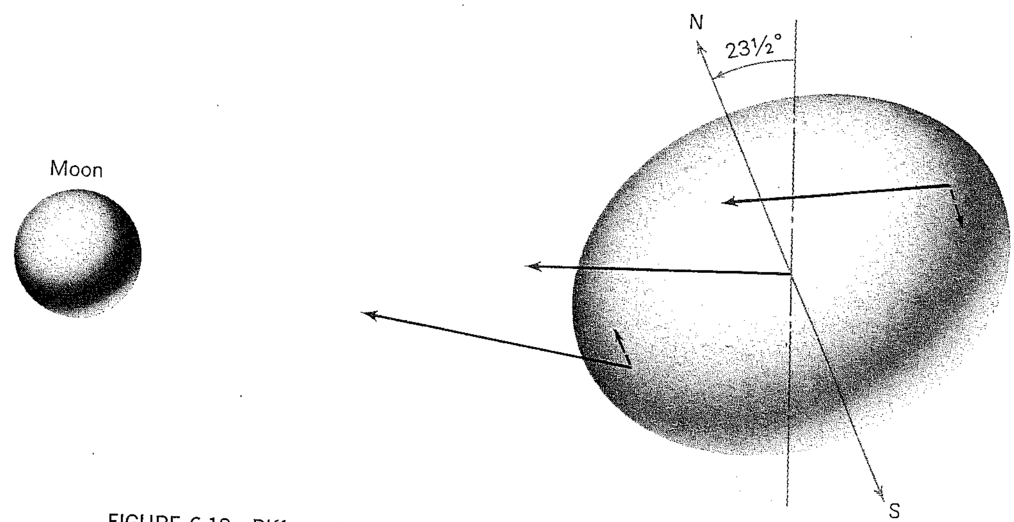

FIGURE 6.18 Differential force of the moon on the oblate earth tends to "erect" its axis.

center. Note how they tend not only to "stretch" the earth toward the moon, but also to pull the equatorial bulge into the plane of the ecliptic. The differential force of the sun, although less than half as effective, does the same thing. Thus, the gravitational attractions of the sun and the moon on the earth act in such a way as to attempt to *change the direction of the earth's axis of rotation,* so that it would stand perpendicular to the orbital plane of the earth. To understand what actually takes place, we must digress for a moment to consider what happens when a similar force acts upon a top or gyroscope.

(a) Precession of a gyroscope

Consider the top (a simple form of gyroscope) pictured in Figure 6.19. If the top's axis is not perfectly vertical, its weight (the force of gravity between it and the earth) tends to topple it over. The actual force that acts to change the orientation of the axis of rotation of the top is that component of the top's weight that is perpendicular to its axis. We know from watching a top spin that the axis of the top does not fall toward the horizontal, but rather moves off in a direction *perpendicular to the plane defined by the axis and the force tending to change its orientation.* Until the spin of the top is slowed down by friction the axis does not change its angle of inclination to the vertical (or to the floor), but rather describes a conical motion (a cone about the vertical line passing through the pivot point of the top). This conical motion of the top's axis is called *precession.*

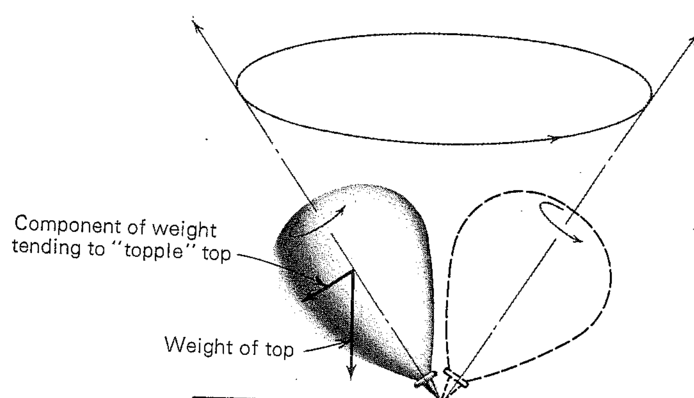

FIGURE 6.19 Precession of a top.

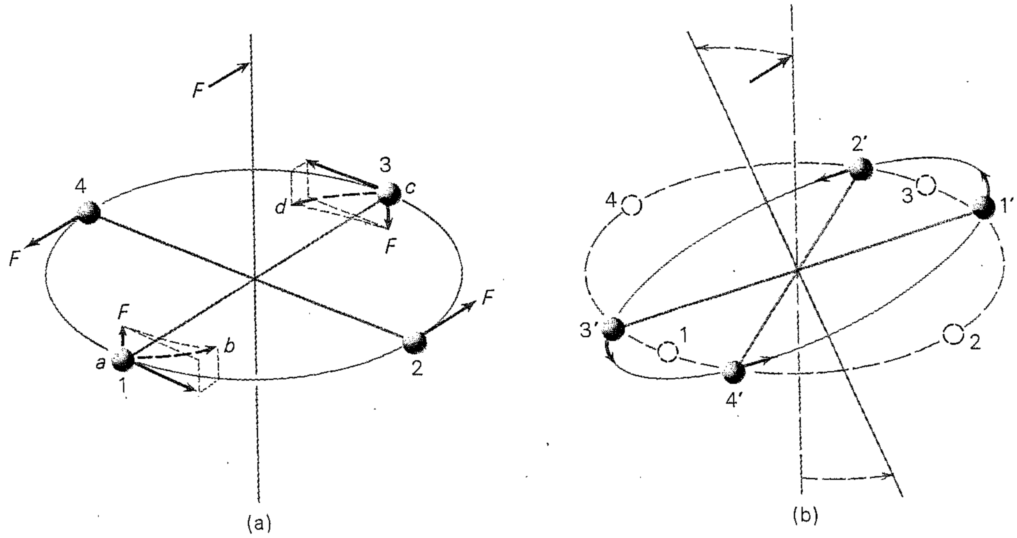

FIGURE 6.20 (a) Force applied to the axis of a simple gyroscope; (b) the new orientation taken by the gyroscope.

(b) Qualitative explanation of precession

The surprising phenomenon of precession can be understood in terms of Newton's laws of motion. Consider, for simplicity, the jack-shaped gyroscope in Figure 6.20(a), consisting of four masses supported at the ends of rigid light rods perpendicular to each other and to the axis of rotation. As the gyroscope spins, the masses move in the plane indicated. Suppose now that a force *F* is applied to the axis in a direction perpendicular to the plane defined by the axis of the jack and the line between masses 2 and 4. The force is transmitted through the rods to each of the four masses. Mass 1 feels a force tending to raise it (in the orientation of the diagram), and mass 3 feels a force tending to lower it; only masses 2 and 4 do not feel forces in the vertical direction. Masses 2 and 4 tend to continue moving in the same plane as before the force was applied. Mass 1 accelerates upward, but because of its forward motion it moves along the path *ab*. Similarly, mass 3 accelerates downward, but because of its forward motion follows path *cd*. Thus, after a part of a revolution, the masses are in the positions shown in Figure 6.20(b). The axis of rotation has changed, not in the direction of the applied force, but at right angles to it.

The above discussion is not a very rigorous description of precession; it is intended only to give the reader some feeling for the fact that the axis of a spinning top does not yield in the direction of a force acting on it. When we consider how each of the constituent parts of the top should behave under the influence of the applied force we can understand the apparently strange motion of the axis of the whole spinning body in terms of Newton's laws. It can be shown, however, by a rigorous mathematical treatment, that if a force is applied to the axis of any spinning body, the axis itself will move in a plane perpendicular to that defined by the force and the instantaneous axis of rotation.

(c) Precession of the earth

The differential gravitational force of the sun on the earth tends to pull the earth's equatorial bulge into the plane of the ecliptic, and that of the moon tends to pull the bulge into the plane of the moon's orbit, which is nearly in the ecliptic. These forces, in other words, tend to pull the earth's axis into a direction approximately perpendicular to the ecliptic plane. Like a top, however, the earth's axis does not yield in the direction of these forces, but precesses. The obliquity of the ecliptic remains approximately $23\frac{1}{2}°$. The earth's axis slides along the surface of an imaginary cone, perpendicular to the ecliptic, and with a half-angle at its apex of $23\frac{1}{2}°$ (see Figure 6.21). The precessional motion is exceedingly slow; one

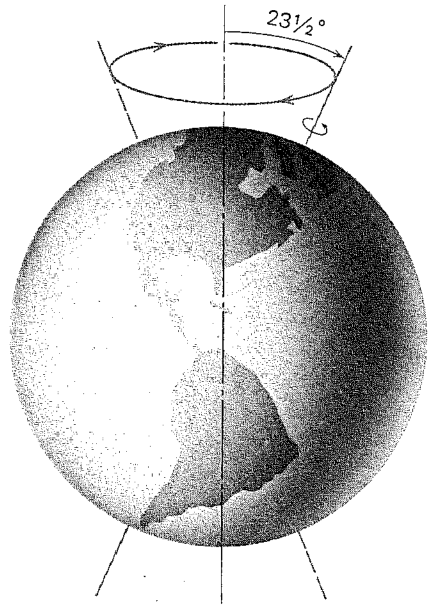

FIGURE 6.21 Precession of the earth.

complete cycle of the axis about the cone requires about 26,000 years.

Precession is this motion of the axis of the earth. It must not be confused with *variation in latitude* (Chapter 7), which is caused by a slight wandering of the terrestrial poles with respect to the earth's surface. Precession does not affect the cardinal directions on the earth nor the positions of geographical places that are measured with respect to the earth's rotational axis, but only the orientation of the axis with respect to the celestial sphere.

Precession does, however, affect the positions among the stars of the celestial poles, those points where extensions of the earth's axis intersect the celestial sphere. In the twentieth century, for example, the north celestial pole is very near Polaris. This was not always so. In the course of 26,000 years, the north celestial pole will move on the celestial sphere along an approximate circle of about $23\frac{1}{2}°$ radius, centered on the pole of the ecliptic (where the perpendicular to the earth's orbit intersects the celestial sphere). This motion of the pole is shown in Figure 6.22. In about 12,000 years, the celestial pole will be fairly close to the bright star Vega.

As the positions of the poles change on the celestial sphere, so do the regions of the sky that are circumpolar; that is, that are perpetually above (or below) the horizon for an observer at any particular place on earth. The Little Dipper, for example, will not always be circumpolar as seen from north temperate latitudes. Moreover, 2000 years ago, the Southern Cross was sometimes visible from parts of the United States. It was by noting the very gradual changes in the positions of stars with respect to the celestial poles that Hipparchus discovered precession in the second century B.C. (Section 2.3c).

(d) Nutation

If the differential gravitational attractions of the sun and moon upon the earth's equatorial bulge were always exactly the same, precession of the earth's axis would be the smooth conical motion we have described in the preceding sections. However, the effect of the differential forces on the orientation of the earth's axis depends on the directions of the sun and moon with respect to the direction of its $23\frac{1}{2}°$ tilt. These directions change as the

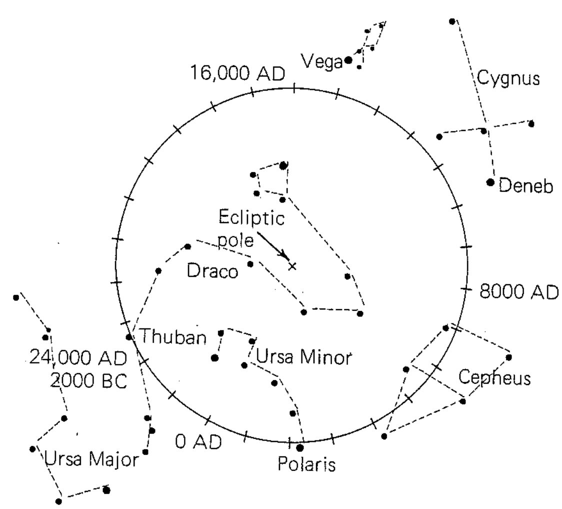

FIGURE 6.22 Precessional path of the north celestial pole among the northern stars.

earth and moon move in their respective orbits. Moreover, the moon's orbit is inclined at about 5° to the ecliptic. Not only is that 5° inclination slightly variable itself, but the intersections of the moon's orbit with the ecliptic slide around the ecliptic in 18.6-year intervals (the regression of the nodes).

The average effect of the sun and moon on the earth's equatorial bulge is to produce the relatively smooth precession we have described. We define the *mean pole* of rotation of the celestial sphere as a fictitious one that describes this smooth precessional motion. The motion of the actual celestial pole varies slightly around the motion of the mean pole. These variations, which are quite small, can be fairly well represented by an elliptical orbit of the actual pole about the mean pole with a semimajor axis of 9″.2, and a period of about 19 years. In other words, the motion of the celestial pole about the ecliptic pole is not quite a perfect circle, but a slightly wavy circle, with the "waves" having amplitudes of about 9 seconds of arc (″)—small compared to the $23\frac{1}{2}°$ radius of the precessional orbit of the pole in the sky. This slight "nodding" of the pole about a smooth circle is called *nutation.*

(e) Planetary precession

Up to now we have implied that the plane of the earth's orbit is fixed in space. The earth's orbit, however, is constantly being perturbed by the gravitational attractions of the other planets upon the earth. These perturbations are very slight, but they do measurably alter the plane of the earth's orbit and hence the position of the pole of the ecliptic on the celestial sphere. This motion of the pole of the ecliptic, only a fraction of a second of arc per year, adds to the complications of precession.

The motion of the mean celestial pole with respect to the ecliptic pole is called *lunisolar precession.* The motion of the ecliptic pole, because of planetary perturbations of the earth's orbital motion, is called *planetary precession;* the ecliptic pole moves only about one-fortieth as fast as the celestial pole. The two kinds of motion combined give *general precession.*

EXERCISES

1. Find the separation d between two small bodies, each of unit mass, lined up with a large body of mass M, at a distance R from the nearest of the small bodies, such that the gravitational attraction between the small bodies is just equal to the differential gravitational force between them caused by their attraction to the large body. The answer should be in terms of G, M, and R.

2. If the three bodies described in the last exercise are free to move and no other bodies or forces are present, how may their motion be described? How do the various forces change as the bodies move?

3. Strictly speaking, should it be a 24-hour period during which there are two "high tides"? If not, what should the interval be?

4. Compute the relative tide-raising effectiveness of the sun and the moon. For this approximate calculation, assume that the earth is 80 times as massive as the moon, that the sun is 300,000 times as massive as the earth, and that the sun is 400 times as distant as the moon.
Answer: Moon is $\frac{8}{3}$ times as effective

5. Explain why the north celestial pole moves in the sky along a circle centered on the pole of the ecliptic, rather than some other point.

6. What will be the principal north circumpolar constellations as seen from Los Angeles (latitude 34° north) in the year 18,000?

7. In the year 13,000, will Orion be circumpolar as seen from the North Pole? Explain.

8. What would be the annual motion of the equinoxes along the ecliptic if the entire precessional cycle required only 360 years?

9. Describe how perturbations of the earth's motion by Mars can be considered as due to differential gravitational force.

10. Does a bicycle offer another example of precession? Explain. (*Hint:* Consider how a rider can steer by leaning to one side.)

11. If the precessional rate is about 50″ per year, show that the complete cycle is about 26,000 years.

12. The radius of curvature of the earth at a particular point is the radius of a sphere whose surface matches the curvature of the earth at that point (see Figure 6.3). How much greater is the radius of curvature of the earth at the poles than at the equator?