Physics 351, Spring 2018, Homework #7.

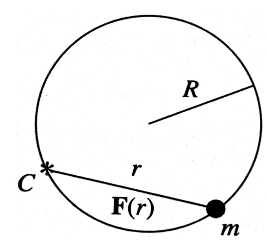
Due at start of class, Friday, March 16, 2018

Please write your name on the LAST PAGE of your homework submission, so that we don't notice whose paper we're grading until we get to the very end.

When you finish this homework, remember to tell me how the homework went for you, by visiting the feedback page at

`positron.hep.upenn.edu/q351`

1. A particle of mass m moves in a circular orbit of radius R under the influence of a central force $F(r)$. The center of force C lies at a point **on** the circle, as shown in the figure. Show that the force law has the form $F(r) = -k/r^5$, and find the constant k in terms of R, m, and angular momentum l.



2. A small spherical rock covered with sand falls in radially toward a planet. Let the planet have radius R and density ρ_p, and let the rock have density ρ_r. It turns out that when the rock gets close enough to the planet, the tidal force ripping the sand off the rock will be larger than the gravitational force attracting the sand to the rock. The cutoff distance is called the Roche limit. Show that it is given by

$$d = R\left(\frac{2\rho_p}{\rho_r}\right)^{1/3}$$

(Note the lack of dependence on the rock's radius. The Roche limit gives the radial distance below which loose objects won't collect into larger blobs. Our moon (a sphere of rock and sand) lies outside the earth's Roche limit. But Saturn's rings (which consist of loose ice particles) lie inside its Roche limit.)

3. A puck slides with speed v on frictionless ice. The surface is "level" in the sense that it is orthogonal to the effective (gravitational + centrifugal) $\boldsymbol{g}$ at all points. Show that the puck moves in a circle, as seen in the earth's rotating frame. (Assume that v is small enough that the radius of the circle is much smaller than the radius of the earth, so that the colatitude θ is essentially constant throughout the motion.) What is the radius of the circle? What is the frequency of the motion?

4. Consider (unrealistically) a perfectly spherical rotating earth whose g_0 value is constant over the surface. A bead lies on a frictionless wire that lies in the north-south direction across the equator. The wire takes the form of an arc of a circle; all points are the same distance from the center of the earth. The bead is released from rest, a short distance from the equator. Because the effective $\boldsymbol{g}$ does not point directly toward the earth's center, the bead will head toward the equator and undergo oscillatory motion. What is the frequency of these oscillations?

5. At a polar angle θ, a projectile is fired eastward with speed v_0 at an angle α above the ground. Show that the southward (in the northern hemisphere) and eastward deflections due to the Coriolis force are (to first order in Ω)

$$d_{\text{south}} = \frac{4\Omega v_0^3}{g^2}\cos\theta\cos\alpha\sin^2\alpha$$
$$d_{\text{east}} = \frac{4\Omega v_0^3}{g^2}\sin\theta\left(\cos^2\alpha\sin\alpha - \frac{1}{3}\sin^3\alpha\right).$$

Hint: The first term in d_{east} arises because the flight time is modified due to the vertical component of the Coriolis force.

6. To illustrate the result $T = \frac{1}{2}\sum m_\alpha \dot{\boldsymbol{r}}_\alpha'^{\,2}$ (Eq. 10.18) that the total KE of a body is just the rotational KE relative to any point that is instantaneously at rest, do the following: Write down the KE of a uniform wheel (mass M, radius R) rolling with speed v along a flat road, as the sum of the energies of the CM motion and the rotation about the CM. Now instead write it as the energy of rotation about the instantaneous point of contact with the road and show that you get the same answer. (Recall that the energy of rotation is $\frac{1}{2}I\omega^2$, that the moment of inertia of a uniform wheel about its center is $I = \frac{1}{2}MR^2$, and that the moment of inertia of the wheel about a point on the rim is $I' = \frac{3}{2}MR^2$.) [This problem is assigned just to remind you of useful result (Eq. 10.18).]

7. Five equal point masses are placed at the five corners of a square pyramid whose square base is centered on the origin in the xy plane, with side L, and whose apex is on the z axis at a height H above the origin. Find the CM of the five-mass system.

8. Find the CM of a uniform hemispherical shell of inner radius a, outer radius b, and mass M, positioned with its flat base in the xy plane, with the base centered on the origin. Remember that $\mathrm{d}V = r^2\,\mathrm{d}r\,\sin\theta\,\mathrm{d}\theta\,\mathrm{d}\phi$, or sometimes more conveniently, $\mathrm{d}V = r^2\,\mathrm{d}r\,\mathrm{d}(\cos\theta)\,\mathrm{d}\phi$.

9. Verify that the components of the vector $\boldsymbol{r}\times(\boldsymbol{\omega}\times\boldsymbol{r})$ are given by (Eq. 10.35),

$$\begin{aligned}
[\boldsymbol{r}\times(\boldsymbol{\omega}\times\boldsymbol{r})]_x &= (y^2+z^2)\,\omega_x - xy\,\omega_y - xz\,\omega_z \\
[\boldsymbol{r}\times(\boldsymbol{\omega}\times\boldsymbol{r})]_y &= -\,yx\,\omega_x + (z^2+x^2)\,\omega_y - yz\,\omega_z \\
[\boldsymbol{r}\times(\boldsymbol{\omega}\times\boldsymbol{r})]_z &= -\,zx\,\omega_x - zy\,\omega_y + (x^2+y^2)\,\omega_z
\end{aligned}$$

Do this both by working with components and by using the so-called BAC-CAB rule, that is $\boldsymbol{A}\times(\boldsymbol{B}\times\boldsymbol{C}) = \boldsymbol{B}(\boldsymbol{A}\cdot\boldsymbol{C}) - \boldsymbol{C}(\boldsymbol{A}\cdot\boldsymbol{B})$. Instead of doing all of the work three times, it is probably easiest just to work out the z component and then to argue by cyclic permutation of x, y, z indices that the x and y components must have the above form.

Remember **online feedback** at `positron.hep.upenn.edu/q351`