Physics 351, Spring 2018, Homework #8.

Due at start of class, Friday, March 23, 2018

Please write your name on the $\boxed{\textbf{LAST PAGE}}$ of your homework submission, so that we don't notice whose paper we're grading until we get to the very end.

When you finish this homework, remember to tell me how the homework went for you, by visiting the feedback page at

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1. [Here's a problem from a midterm exam from 4 or 5 years ago.] Consider a particle of mass m moving in a plane in a circular orbit under the influence of an attractive central Hooke's-law-like force $F(r) = -kr$, where k is a constant. The angular momentum of the particle is ℓ. (a) Find the effective potential energy of the equivalent one-dimensional problem. (b) Show that the radius R of the circular orbit is

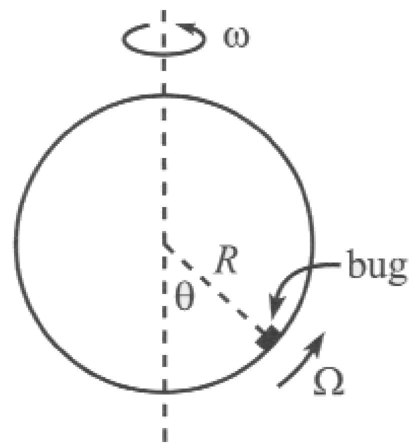
$$R = \left(\frac{\ell^2}{mk}\right)^{1/4}$$

(c) Now consider orbits that are slightly perturbed from the circular orbit, i.e. orbits where the radius can be written $r(t) = R + \epsilon(t)$, where $\epsilon(t) \ll R$. Show that the motion is stable and calculate the **period** of the oscillations about the circular orbit.

2. A high-speed train is traveling at a constant 150 m/s (about 335 mph) on a straight, horizontal track across the South Pole. Find the angle between a plumb line suspended from the ceiling inside the train and another inside a hut on the ground. In what direction is the plumb line on the train deflected?

3. If a negative charge $-q$ (an electron, for example) in an elliptical orbit around a fixed positive charge Q is subjected to a weak uniform magnetic field $\boldsymbol{B}$, the effect of $\boldsymbol{B}$ is to make the ellipse precess slowly — an effect known as **Larmor precession**. To prove this, write down the EOM of the negative charge in the field of Q and $\boldsymbol{B}$. Now rewrite it for a frame rotating with angular velocity $\boldsymbol{\Omega}$. [Remember that this changes both $\mathrm{d}^2\boldsymbol{r}/\mathrm{d}t^2$ and $\mathrm{d}\boldsymbol{r}/\mathrm{d}t$.] Show that by suitable choice of $\boldsymbol{\Omega}$ you can arrange that the terms involving $\dot{\boldsymbol{r}}$ cancel out, but that you are left with one term involving $\boldsymbol{B} \times (\boldsymbol{B} \times \boldsymbol{r})$. If $\boldsymbol{B}$ is weak enough this term can certainly be neglected. Show that in this case the orbit in the rotating frame is an ellipse (or hyperbola). Describe the appearance of the ellipse as seen in the original nonrotating frame.

4. A hoop of radius R is made to rotate at constant angular speed ω around a diameter, as shown in the figure. A small bug of mass m walks at constant angular speed Ω around the hoop. Let $\boldsymbol{F}$ be the total force that the hoop applies to the bug when the bug is at the angle θ shown, and let $F_\perp$ be the component of $\boldsymbol{F}$ that is perpendicular to the plane of the hoop. Find $F_\perp$ in two ways (ignore gravity in this problem): (a) Work in the lab frame: at the angle θ, find the rate of change of the bug's angular momentum around the rotation axis, and then consider the torque on the bug. (b) Work in the rotating frame of the hoop: at the angle θ, find the relevant fictitious force, and then take it from there.



5. A thin rod (of width zero, but not necessarily uniform) is pivoted freely at one end about the horizontal z axis, being free to swing in the xy plane (x horizontal, y vertically down). Its mass is m, its CM is a distance a from the pivot, and its moment of inertia (about the z axis) is I. (a) Write down the EOM $\dot{L}_z = \Gamma_z$ and, assuming the motion is confined to small angles (measured from the downward vertical), find the period of this compound pendulum. ("Compound pendulum" is traditionally used to mean any pendulum whose mass is distributed — as contrasted with a "simple pendulum," whose mass is concentrated at a single point on a massless arm.) (b) What is the length of the "equivalent" simple pendulum, that is, the simple pendulum with the same period?

6. Consider the rod of Problem 5. The rod is struck sharply with a horizontal force F which delivers an impulse $F\,\Delta t = \xi$ a distance b below the pivot. (a) Find the rod's angular momentum about the pivot, and hence the rod's momentum, just after the impulse. (b) Find the impulse η delivered to the pivot. (c) For what value of b (call it b_0) is $\eta = 0$? (The distance b_0 defines the so-called "sweet spot." If the rod were a tennis racquet and the pivot your hand, then if the ball hits the sweet spot, your hand would experience no impulse.)

7. A rigid body comprises 8 equal masses m at the corners of a cube of side a, held together by massless struts. (a) Use the definitions (Eq. 10.37 and 10.38) $I_{xx} = \sum m_\alpha(y_\alpha^2 + z_\alpha^2)$ and $I_{xy} = -\sum m_\alpha x_\alpha y_\alpha$ (and cyclic permutations) to find the moment of inertia tensor $\boldsymbol{I}$ for rotation about a corner O of the cube. (Use axes along the three edges through O.) (b) Find the inertia tensor of the same body but for rotation about the center of the cube. (Again use axes parallel to the edges.) Explain why in this case certain elements of $\boldsymbol{I}$ could be expected to be zero.

8. Consider a rigid plane body (a "lamina"), such as a flat piece of sheet metal, rotating about a point O on the body. If we choose axes so that the lamina lies in the xy plane, which elements of the inertia tensor $\boldsymbol{I}$ are automatically zero? Prove that $I_{zz} = I_{xx} + I_{yy}$.

9. A thin, flat, uniform metal triangle lies in the xy plane with its corners at $(1, 0, 0)$, $(0, 1, 0)$, and the origin. Its surface density (mass/area) is $\sigma = 24$. (Let's measure distances and masses in unspecified units, with the number 24 chosen to make the answer come out nicely.) In each case, the rotation axis passes through the origin. (a) Find the triangle's inertia tensor $\boldsymbol{I}$. (b) What are its principal moments and the corresponding axes? [Since the object is flat, the z axis is automatically a principal axis. (Do you see why?) As you proved in problem 8, $I_{zz} = I_{xx} + I_{yy}$ for a flat object in the xy plane.]

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XC00. Optional/extra-credit. If there are extra-credit problems from earlier assignments that you didn't have time to do sooner, you can feel free to turn them in with HW08 for full credit. Just clearly indicate which problem you're solving.

XC1. Optional/extra-credit. (a) Show that if an object is thrown with initial velocity $\boldsymbol{v}_0$ from a point O on Earth's surface at colatitude θ, then (as you worked out last week, in less generality) to first order in Ω its orbit is

$$
\begin{aligned}
x &= v_{x0}t + \Omega(v_{y0}\cos\theta - v_{z0}\sin\theta)t^2 + \frac{1}{3}\Omega g t^3 \sin\theta \\
y &= v_{y0}t - \Omega(v_{x0}\cos\theta)t^2 \\
z &= v_{z0}t - \frac{1}{2}gt^2 + \Omega(v_{x0}\sin\theta)t^2
\end{aligned}
$$

Use this result to do the following: A naval gun shoots a shell at colatitude θ in a direction that is an angle α above the horizontal and due east, with muzzle speed v_0. (b) Ignoring Earth's rotation (and air resistance), find how long (t) the shell would be in the air and how far away (R) it would land. If $v_0 = 500$ m/s and $\alpha = 20°$, what are t and R? (c) A naval gunner spots an enemy ship due east at the range R of part (b) and, forgetting about the Coriolis effect, aims his/her gun exactly as in part (b). Find by how far north or south, and in which direction, the shell will miss the target, in terms of Ω, v_0, α, θ, and g. (It will also miss in the east-west direction, but we save this complication for XC2.) (d) If the incident occurs at latitude 50° north ($\theta = 40°$), what is this distance? (e) What if the latitude is 50° south ($\theta = 140°$)? This problem is a serious issue in long-range gunnery: In a battle near the Falkland

Islands (a.k.a. Islas Malvinas) in World War I, the British navy consistently missed German ships by many tens of yards because they apparently forgot that the Coriolis effect in the southern hemisphere is opposite to that in the north.

XC2. Optional/extra-credit. For problem XC1(d) and (e), find the distance by which the shell misses its target in both the north-south and east-west directions. [In this case, you should account for the fact that the time of flight is affected by the Coriolis effect.] How much is your calculated deflection affected by whether or not you account for the time-of-flight correction? (Note: While the TOF correction to the N-S deflection is a correction to a correction, i.e. $\mathcal{O}(\Omega^2)$, the TOF correction to the eastward range is a correction ($\mathcal{O}(\Omega)$) to a value that starts off with no Ω dependence, so it is significant.)

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