Physics 351, Spring 2018, Midterm Exam.

This closed-book exam has (only) 15% weight in your course grade. You can use one 3×5 card of your own hand-written notes. **Turn in your 3×5 card (if any) with your exam.** Please show your work on these sheets. The last page of the exam is blank, in case you run out of space. Try to work in a way that makes your reasoning obvious to me, so that I can give you credit for correct reasoning even in cases where you might have made a careless error. Correct answers without clear reasoning may not receive full credit.

Write your name on this page now. You should leave the exam closed until everyone is ready to begin. The exam contains three questions. The first and third questions are worth 35% each. The second question is worth 30%.

Because I believe that most of the learning in a physics course comes from your investing the time to work through homework problems, these exam problems are similar or identical to problems that you have already solved. The only point of the exams, in my opinion, is to motivate you to take the weekly homework seriously. So you should find nothing surprising in this exam.

Possibly useful equations.

$$U(r) = -\frac{\gamma}{r} \qquad E = \frac{\gamma^2\mu}{2\ell^2}(\epsilon^2 - 1) \qquad r(\phi) = \frac{c}{1+\epsilon\cos\phi} \qquad c = \frac{\ell^2}{\gamma\mu}$$

Remember that the four different orbit shapes are circle, ellipse, parabola, hyperbola.

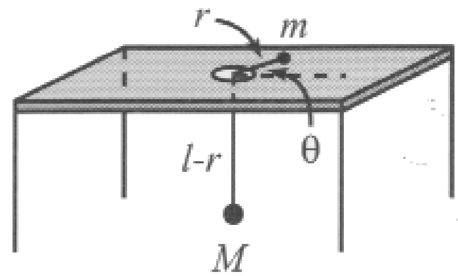
$$\left(\frac{\mathrm{d}\boldsymbol{Q}}{\mathrm{d}t}\right)_{\text{space}} = \left(\frac{\mathrm{d}\boldsymbol{Q}}{\mathrm{d}t}\right)_{\text{body}} + \boldsymbol{\Omega}\times\boldsymbol{Q}$$

$$m\ddot{\boldsymbol{r}} = \boldsymbol{F} + 2m\,\dot{\boldsymbol{r}}\times\boldsymbol{\Omega} + m\,(\boldsymbol{\Omega}\times\boldsymbol{r})\times\boldsymbol{\Omega}$$

$$\boldsymbol{r} = \boldsymbol{r}_1 - \boldsymbol{r}_2 \qquad \boldsymbol{r}_1 = \boldsymbol{R}_{\text{cm}} + \frac{m_2}{M}\boldsymbol{r} \qquad \boldsymbol{r}_2 = \boldsymbol{R}_{\text{cm}} - \frac{m_1}{M}\boldsymbol{r} \qquad \mu = \frac{m_1 m_2}{M} \qquad M = m_1 + m_2$$

Name: ______________________________

Problem 1. (35%)
A mass m is free to slide on a frictionless table and is connected, via a string that passes through a hole in the table, to a mass M that hangs below. Assume that M moves in a vertical line only, and assume that the string always remains taut.



(a) Write down the Lagrangian, then find the Lagrange equations of motion for r and for θ.

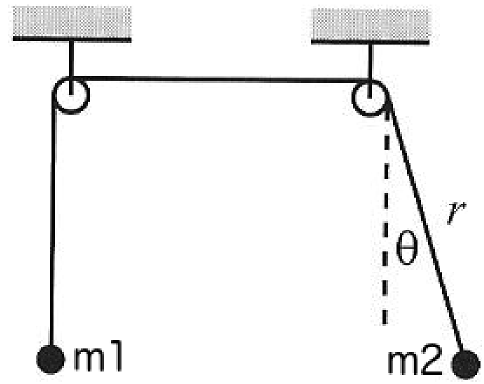
(b) One of these equations of motion identifies a conserved quantity (a constant of the motion), which you can plug into the other equation of motion. Name the conserved quantity. What feature of the Lagrangian (sometimes stated as a property of the corresponding coordinate) led us directly to this conserved quantity?

(c) If the mass m moves in a circle of radius $r = r_0$, what is the angular frequency ω of this motion? Express ω in terms of m, M, g, and r_0.

(d) If mass m is then perturbed slightly from this circular motion, what is the angular frequency Ω of the oscillations about the radius $r = r_0$? Express Ω in terms of ω, m, and M, where ω is your answer from part (c). [If your answer for part (d) makes no sense (e.g. if Ω appears to be imaginary), then you may have forgotten to make proper use of your answer for part (b).]

Problem 2. (30%)

Point masses m_1 and m_2 are connected by a massless string and hang over two pulleys (of negligible size). The left mass, m_1, moves in a vertical line, but the right mass, m_2, is free to swing back and forth in the plane of the masses and pulleys (i.e. the plane of the diagram). m_1 and m_2 may be unequal.



(a) Using generalized coordinates r and θ as shown in the diagram, write the Lagrangian for the two-particle system in terms of masses m_1 and m_2 and coordinates r and θ.

(b) Write the Lagrange equation of motion for r and write the Lagrange equation of motion for θ. (That's all.)

Problem 3. (35%)
At a polar angle θ (colatitude), a projectile is thrown with initial velocity $\mathbf{v}_0 = (v_{x0}, v_{y0}, v_{z0})$ from a point $(x, y, z) = (0, 0, 0)$ on Earth's surface. Use a coordinate system in which $\hat{x}$ points east, $\hat{y}$ points north, $\hat{z}$ points up, and whose origin is the point from which the projectile is thrown.

Working to first order in Earth's rotational velocity Ω, show that the projectile's orbit is

$$\begin{aligned} x &= v_{x0}t + A\Omega t^2 + B\Omega t^3 \\ y &= v_{y0}t + C\Omega t^2 \\ z &= v_{z0}t - \frac{1}{2}gt^2 + D\Omega t^2 \end{aligned}$$

and find the coefficients A, B, C, and D, which may be positive or negative, or may even involve more than one term. Write your results for A,B,C,D at the lower-right corner of this page.

[Assume that air resistance is negligible and that $\boldsymbol{g}$ is a constant throughout the flight. I recommend first neglecting the Coriolis force and writing the "zeroth order" solution for ordinary projectile motion; then use this zeroth-order trajectory to calculate the first-order Coriolis deflection, after evaluating $\boldsymbol{\Omega}$ in terms of the local $\hat{\boldsymbol{x}}$ (east), $\hat{\boldsymbol{y}}$ (north), $\hat{\boldsymbol{z}}$ (up) axes.] If you happen to know the right answer, you **cannot** just write it down. You **must** show how to work it out step by step.

$A =$

$B =$

$C =$

$D =$

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