$1\to 2$
$2\to 3$
$3\to 1$

$$0 = \vec{\Gamma} = \cancel{\left(\frac{d\vec{L}}{dt}\right)_{space}}^{\,0} = \left(\frac{d\vec{L}}{dt}\right)_{body} + \vec{\omega}\times\vec{L}$$

$$(\vec{A}\times\vec{B})_3 = A_1B_2 - A_2B_1$$

$$0 = (\dot{L}_1, \dot{L}_2, \dot{L}_3) + \vec{\omega}\times\vec{L}$$

$$(\lambda_1\dot{\omega}_1, \lambda_2\dot{\omega}_2, \lambda_3\dot{\omega}_3) + (\vec{\omega}\times\vec{L})$$

$$0 = \lambda_3\dot{\omega}_3 + \omega_1 L_2 - \omega_2 L_1 = \lambda_3\dot{\omega}_3 + \omega_1\omega_2\lambda_2 - \omega_2\omega_1\lambda_1$$

$$\dot{\omega}_3 = \cancel{\frac{\lambda_1-\lambda_2}{\lambda_3}}\,\omega_1\omega_2 \to 0$$

$$\dot{\omega}_1 = \frac{\lambda_2-\lambda_3}{\lambda_1}\omega_2\omega_3$$

$$\dot{\omega}_2 = \frac{\lambda_3-\lambda_1}{\lambda_2}\omega_3\omega_1$$

$\lambda_1 = \lambda_2$

$$\Omega_b = \frac{\lambda_1-\lambda_3}{\lambda_1}\omega_3$$

$$\dot{\omega}_1 = +\Omega_b\omega_2 \qquad \dot{\omega}_2 = -\Omega_b\omega_1$$

$\boxed{\Omega_b>0}$ prolate (US) football

$\otimes\vec{\Omega}_{body}\sim -\hat{e}_3$

$\Omega_b<0$ oblate frisbee

$\odot\vec{\Omega}_{body}\sim +\hat{e}_3$

$$\vec{\Omega}_{body} = -\Omega_b\hat{e}_3$$

$$\vec{\Omega}_{body} = \frac{\lambda_3-\lambda_1}{\lambda_1}\omega_3\hat{e}_3$$

$$\vec{L} = \lambda_1\omega_1\hat{e}_1 + \lambda_1\omega_2\hat{e}_2 + \lambda_3\omega_3\hat{e}_3$$

$$\frac{\vec{L}}{\lambda_1} = \omega_1\hat{e}_1 + \omega_2\hat{e}_2 + \omega_3\hat{e}_3 + \frac{\lambda_3}{\lambda_1}\omega_3\hat{e}_3 - \omega_3\hat{e}_3\frac{\lambda_1}{\lambda_1}$$

$$\frac{\vec{L}}{\lambda_1} = \vec{\omega} + \frac{\lambda_3-\lambda_1}{\lambda_1}\omega_3\hat{e}_3 = \vec{\omega} + \vec{\Omega}_{body} \equiv \vec{\Omega}_{space} = \frac{\vec{L}}{\lambda_1}$$

$$\vec{\Omega}_{space} = \vec{\Omega}_{body} + \vec{\omega}$$

$\nearrow \sim \vec{L}$ $\quad \nearrow \pm e_3$

We can write $\vec{\omega} = \vec{\Omega}_{space} - \vec{\Omega}_{body}$

$$\left(\frac{d\hat{e}_3}{dt}\right)_{space} = \cancel{\left(\frac{d\hat{e}_3}{dt}\right)_{body}}^{\,0} + \vec{\omega} \times \hat{e}_3$$

$$\left(\frac{d\hat{e}_3}{dt}\right)_{space} = 0 + \vec{\Omega}_{space} \times \hat{e}_3 \qquad \Big\} \; \vec{\Omega}_{space} = \frac{\vec{L}}{\lambda_1}$$

$$\cancel{\left(\frac{d\vec{L}}{dt}\right)_{space}}^{\,0} = \left(\frac{d\vec{L}}{dt}\right)_{body} + \vec{\omega} \times \vec{L}$$

$$0 = \left(\frac{d\vec{L}}{dt}\right)_{body} + \left(-\vec{\Omega}_{body}\right) \times \vec{L}$$

$$\left(\frac{d\vec{L}}{dt}\right)_{body} = \vec{\Omega}_{body} \times \vec{L}$$

So $\vec{\Omega}_{space} = \dfrac{\vec{L}}{\lambda_1}$ describes the rotation (precession) of $\hat{e}_3, \hat{\omega}$ about the fixed $\vec{L}$ as seen in space axes

and $\vec{\Omega}_{body} = \dfrac{\lambda_3 - \lambda_1}{\lambda_1} \omega_3 \hat{e}_3$ describes the rotation (precession) of $\vec{L}, \vec{\omega}$ about the fixed $\hat{e}_3$ as seen in body axes

This all assumes $\lambda_1 = \lambda_2$, $\vec{\Gamma} = 0$

If $\vec{L} \approx \lambda_3 \omega_3 \hat{e}_3$ then $\vec{\Omega}_{space} \approx \dfrac{\lambda_3}{\lambda_1} \omega_3 \hat{e}_3$

So frisbee wobbles 2x as fast as it spins

and US football wobbles less quickly than it spins

$\frac{Ma^2}{12}$

$I_{zz} = \frac{M}{12}(a^2+b^2)$

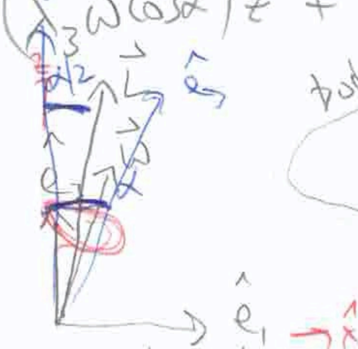
$\frac{M}{12}(2a^2) = \frac{Ma^2}{6}$

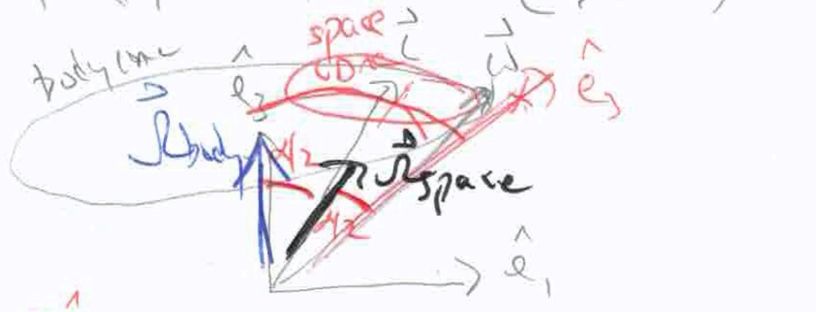
at $t=0$ $\quad \vec{\omega} = (\omega\cos\alpha)\hat{z} + \omega\sin\alpha\,\hat{x}$

$\hat{e}_3 = \hat{z}$ $\quad \vec{\omega} = (\omega\cos\alpha)\hat{e}_3 + (\omega\sin\alpha)\hat{e}_1$

$\hat{e}_1 = \hat{x}$ $\quad$ ① $\vec{L} = \overset{2I_0}{\lambda_3}(\omega\cos\alpha)\hat{e}_3 + \overset{I_0}{\lambda_1}(\omega\sin\alpha)\hat{e}_1$ continues to be

② $\vec{L} = (\lambda_3\omega\cos\alpha)\hat{z} + (\lambda_1\omega\sin\alpha)\hat{x}$ (true for $t>0$)





$\vec{\Omega}_{body} = -\Omega_b\hat{e}_3 = \frac{\lambda_3-\lambda_1}{\lambda_1}\omega_3\hat{e}_3 = \frac{2I_0 - I_0}{I_0}\omega_3\hat{e}_3 = \omega_3\hat{e}_3$

$\vec{\Omega}_{space} = \frac{\vec{L}}{\lambda_1} = \frac{\lambda_3}{\lambda_1}(\omega\cos\alpha)\hat{z} + \left(\frac{\lambda_1}{\lambda_1}\omega\sin\alpha\right)\hat{x}$

$\vec{\Omega}_{space} = (2\omega\cos\alpha)\hat{z} + (\omega\sin\alpha)\hat{x}$

$\vec{\omega}(t=0) = (\omega\cos\alpha)\hat{e}_3 + (\omega\sin\alpha)\hat{e}_1$

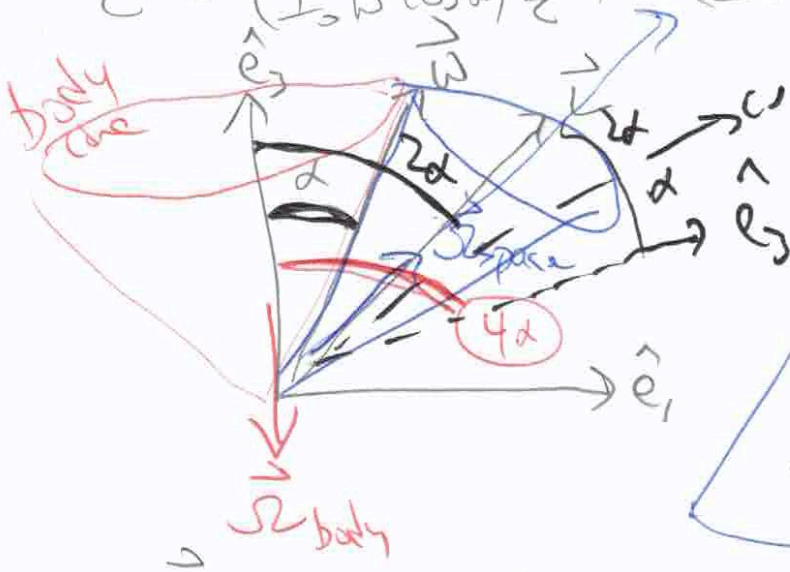
$\vec{\Omega}_{body}(t=0) = \omega_3\hat{e}_3 = (\omega\cos\alpha)\hat{e}_3$

$\vec{\Omega}_{space}(t=0) = (2\omega\cos\alpha)\hat{z} + (\omega\sin\alpha)\hat{x}$

$t = \frac{\pi}{\Omega_{space}} = \frac{\pi}{2\omega}$

$I_0 = ma^2/6 \quad \lambda_1 = \lambda_2 = 2I_0 \quad \lambda_3 = I_0$ (4)

at $t=0$,
$$\vec{\omega} = (\omega\cos\alpha)\hat{z} + (\omega\sin\alpha)\hat{x}$$
$$\vec{\omega} = (\omega\cos\alpha)\hat{e}_3 + (\omega\sin\alpha)\hat{e}_1$$
$$\vec{L} = (\lambda_3\omega\cos\alpha)\hat{e}_3 + (\lambda_1\omega\sin\alpha)\hat{e}_1$$
$$\vec{L} = (I_0\omega\cos\alpha)\hat{e}_3 + (2I_0\omega\sin\alpha)\hat{e}_1$$
$$\vec{L} = (I_0\omega\cos\alpha)\hat{z} + (2I_0\omega\sin\alpha)\hat{x}$$
✓ remains true $t>0$



$$t = \frac{\pi}{\Omega_{space}}$$
$$t = \frac{\pi}{(\omega/2)}$$
$$t = \frac{2\pi}{\omega}$$

$$\vec{\Omega}_{space} = \frac{\vec{L}}{\lambda_1} = \frac{1}{2}\omega\cos\alpha\,\hat{z} + \omega\sin\alpha\,\hat{x}$$
$$\vec{\Omega}_{body} = \frac{\lambda_3 - \lambda_1}{\lambda_1}\omega_3\hat{e}_3 = \frac{I_0 - 2I_0}{2I_0}\omega_3\hat{e}_3 = -\frac{1}{2}\omega_3\hat{e}_3$$
$$\omega_3 = \omega\cos\alpha \qquad \vec{\Omega}_{body} = -\frac{1}{2}\omega\cos\alpha\,\hat{e}_3$$

at $t=0$
$$\vec{\Omega}_{body} = -\frac{1}{2}\omega\cos\alpha\,\hat{e}_3$$
$$\vec{\omega} = (\omega\cos\alpha)\hat{e}_3 + (\omega\sin\alpha)\hat{e}_1$$
$$\vec{\Omega}_{space} = +\frac{1}{2}\omega\cos\alpha\,\hat{z} + \omega\sin\alpha\,\hat{x}$$
✓

$$\vec{\Omega}_{space} = \vec{\Omega}_{body} + \vec{\omega}$$