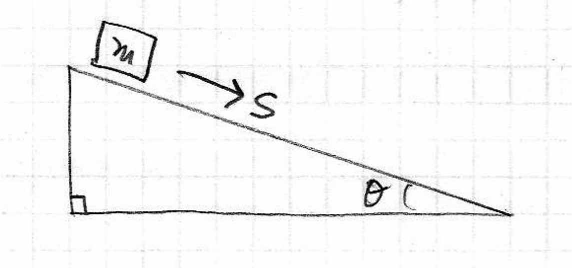
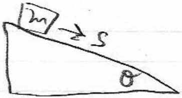
Physics 351 — Friday, January 12, 2018

To finish off Wednesday's preview of things to come, let's try the "trivial" version of the freshman-physics problem with the Lagrangian formalism. Work on this with your neighbor while we wait for everyone to arrive. Your result for $\ddot{s}$ should look familiar.



1. Choose coordinate: s points downhill.
2. Write down T in terms of s and $\dot{s}$.
3. Write down U in terms of s and $\dot{s}$.
4. Write down $\mathcal{L} = T - U$.
5. Equation of motion is given by

$$\frac{\partial \mathcal{L}}{\partial s} = \frac{\mathrm{d}}{\mathrm{d}t} \frac{\partial \mathcal{L}}{\partial \dot{s}}$$



$$T = \tfrac{1}{2} m \dot{s}^2 \qquad U = -mgs\sin\theta$$

$$L = T - U = \tfrac{1}{2} m \dot{s}^2 + mgs\sin\theta$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{s}}\right) = \frac{\partial L}{\partial s} \Rightarrow \frac{d}{dt}\left(m\dot{s}\right) = mg\sin\theta$$

$$m\ddot{s} = mg\sin\theta$$

$$\boxed{\ddot{s} = g\sin\theta}$$

as expected

The "magic incantation" is called the Euler-Lagrange equation. We'll meet it officially in Chapters 6 and 7.

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- Last year's students suggested that this year we go much more quickly through the review material, in order to get to the new material sooner. So this year we'll start ch7 (Lagrangian) a week earlier: about 2 weeks from today. But for the first two weeks you have a lot of review material to read through.

Schedule

Monday	Wednesday	Friday
	Jan 10 *first day of class* notes/slides	**Jan 12** read ch 1+2 (newton's laws, 30pp; projectiles & charged particles, 28pp): questions notes/slides
Jan 15 (holiday) read ch3 (momentum & angular momentum, 15pp): questions recommended: download Mathematica & watch/do screencast (35 min)	**Jan 17** read ch4 (energy, 43pp): questions notes/slides	**Jan 19** read ch5 (oscillations, 44pp): questions notes/slides
Jan 22 read ch6 (calculus of variations, 15pp): questions notes/slides	**Jan 24** reading/exercises from *Hands-on start to Mathematica*: questions notes/slides	**Jan 26** hw01 due: feedback notes/slides
Jan 29 read (start) ch7 (Lagrange's equations, first 30pp): questions notes/slides	**Jan 31** notes/slides	**Feb 02** hw02 due: feedback notes/slides

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Jan 22 read ch6 (calculus of variations, 15pp)	**Jan 24** reading/exercises from *Hands-on start to Mathematica*:	**Jan 26** hw01 due
Jan 29 read (start) ch7 (Lagrange's equations, first 30pp)	**Jan 31**	**Feb 02** hw02 due
Feb 05 read (finish) ch7 (Lagrange's equations, last 13pp)	**Feb 07**	**Feb 09** hw03 due

I generally **will not** spend class repeating the contents of the book. Instead, I'll assume you've done the **required reading**, so we can focus class time on assimilating the key ideas and working through example problems together. The 2015 and 2017 students generally agreed that this approach was a good fit for them:

- "I learned so much from this course, from the textbook, from the homework problems, from the strategy of teaching. The homework assignments require a lot of work and time, as does the reading, but you are rewarded by actually understanding the material."
- "I liked the approach of solving problems during class rather than just repeating the book. I did like Bill's emphasis on reading the textbook by Taylor, which is quite exemplary, and I think this focus was very helpful to my understanding."

But it only works if you actually read the textbook before class!

Several quotes from a past year's course evaluation:

- "Sometimes, lectures were too detail oriented but overall, I liked the approach of solving problems during class rather than just repeating the book. I did like Bill's emphasis on reading the textbook by Taylor, which is quite exemplary, and I think this focus was very helpful to my understanding. My only criticism is that I thought the class focused on a sheer volume of problems ...."
- "The class centered on interesting problems and lots of them. We learned physics in what I think was the best way: fun problem solving and lots of examples."
- "The course was a lot of work, but if you put in the effort you should be able to learn the material well and get a decent grade. Bill really tried his best to make sure we learned the material and was always available to offer help."
- "Far too often courses at Penn can leave half the students behind and I know I personally have been made to feel as though I don't know anything about physics. Bill clearly cares for his students very much and puts in the effort to make sure everyone knows what they're doing. I've learned more about physics in this course, than any other class."

From Richard Feynman (*Feynman Lectures*):

- "I think, however that there isn't any solution to this problem of education other than to realize that the best teaching can be done only when there is a direct individual relationship between a student and a good teacher — a situation in which the student discusses the ideas, thinks about the things, and talks about the things. It's impossible to learn very much by simply sitting in a lecture . . . ."

From Mary Boas (*Mathematical Methods in the Physical Sciences*):

- "One point about your study of this material cannot be emphasized too strongly: To use mathematics effectively in applications, you need not just knowledge but *skill*. Skill can be obtained only through practice. You can obtain a certain superficial *knowledge* of mathematics by listening to lectures, but you cannot obtain *skill* this way. How many students have I heard say, 'It looks so easy when you do it,' or 'I understand it but I can't do the problems!' Such statements show lack of practice . . . ."

To whatever degree is feasible, I'll try to focus the time you spend for this course on solving (worthwhile) problems. I'll also try to facilitate your "talking about the things" with me, with Grace, and with each other as much as possible.

I see my teaching role as analogous to the role of a good coach. I'll guide you through material that I think will benefit you. I'll motivate you to work hard — to try to make optimal use of our "budget" of about 10-12 (total) hours/week of your time. I'll be here to help when you get stuck, or something is unclear to you.

But for you to gain from our time together, **you** have to do the learning. I can't do it for you (though I often learn *with* you).

My class may be a bit less formal than you're used to. I count on substantial feedback from you, so that we can keep the workload challenging, but manageable and low-stress, and so that we make good use of the time that you devote to this course. As I hear from you, I'll do my best to incorporate your feedback.

Overall, I want this course to be challenging, fun, and low-stress. It's not a competition. I'll do my best to help each of you to gain as much you can from the time we spend on Physics 351. All you have to do is work diligently and put in the effort each week.

Let's try a slightly different derivation of Newton's 2nd law in 2D polar coords. (On the board, but I copied my notes to these slides.)

Newton's second law: $\underset{\sim}{F} = m \ddot{\underset{\sim}{r}}$ $\quad (\vec{F} = m\vec{a})$

Cartesian coordinates

$$\underset{\sim}{r} = x\hat{x} + y\hat{y} + z\hat{z}$$
$$\dot{\underset{\sim}{r}} = \dot{x}\hat{x} + \dot{y}\hat{y} + \dot{z}\hat{z}$$
$$\ddot{\underset{\sim}{r}} = \ddot{x}\hat{x} + \ddot{y}\hat{y} + \ddot{z}\hat{z}$$

(Unit vectors $\hat{x}, \hat{y}, \hat{z}$ do not depend on time.)

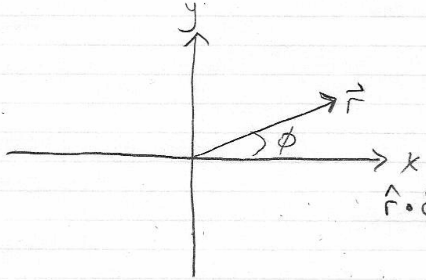
2D polar coordinates

$$\underset{\sim}{r} = r\hat{r}$$

(But $\hat{r}$ depends on ϕ, which in turn depends on time. Same is true for $\hat{\phi}$.)

$$\dot{\underset{\sim}{r}} = \dot{r}\hat{r} + r\left(\frac{d\hat{r}}{dt}\right)$$

The trick of differentiating the unit vectors is sometimes a handy thing to know how to do, so I think it's worth our going through it together here, step-by-step.



$$\hat{r} = (\cos\phi, \sin\phi)$$

$$\hat{\phi} = (-\sin\phi, \cos\phi)$$

check:

$$\hat{r}\cdot\hat{\phi} = -\cos\phi\sin\phi + \sin\phi\cos\phi$$
$$= 0 \quad \checkmark$$

$$\frac{d}{dt}\hat{r} = \frac{d}{dt}(\cos\phi, \sin\phi) = (-\sin\phi\,\dot{\phi}, \cos\phi\,\dot{\phi}) = \dot{\phi}\,\hat{\phi}$$

$$\frac{d}{dt}\hat{\phi} = \frac{d}{dt}(-\sin\phi, \cos\phi) = (-\cos\phi\,\dot{\phi}, -\sin\phi\,\dot{\phi}) = -\dot{\phi}\,\hat{r}$$

$$\dot{\underset{\sim}{r}} = \dot{r}\hat{r} + r\left(\frac{d\hat{r}}{dt}\right) = \dot{r}\hat{r} + r(\dot{\phi}\hat{\phi}) = \dot{r}\hat{r} + r\dot{\phi}\hat{\phi}$$

$$\dot{\underset{\sim}{r}} = \dot{r}\hat{r} + r\omega\hat{\phi}, \quad \text{or} \quad v_r = \dot{r}, \quad v_\phi = r\omega$$

$$\ddot{\underset{\sim}{r}} = \ddot{r}\hat{r} + \dot{r}\left(\frac{d\hat{r}}{dt}\right) + \dot{r}\dot{\phi}\hat{\phi} + r\ddot{\phi}\hat{\phi} + r\dot{\phi}\left(\frac{d\hat{\phi}}{dt}\right)$$

$$= \ddot{r}\hat{r} + \dot{r}(\dot{\phi}\hat{\phi}) + \dot{r}\dot{\phi}\hat{\phi} + r\ddot{\phi}\hat{\phi} + r\dot{\phi}(-\dot{\phi}\hat{r})$$

$$\ddot{\underset{\sim}{r}} = (\ddot{r} - r\dot{\phi}^2)\hat{r} + (r\ddot{\phi} + 2\dot{r}\dot{\phi})\hat{\phi}$$

$$F_r = m(\ddot{r} - r\dot{\phi}^2) = m\ddot{r} - m\omega^2 r$$

$$F_\phi = m(r\ddot{\phi} + 2\dot{r}\dot{\phi}) = mr\alpha + 2m\omega\dot{r}$$

Constant r case (familiar): $\mathbf{a} = \ddot{\mathbf{r}} = -\omega^2 r\hat{r} + \alpha r\hat{\phi}$

Constant ϕ case (line through origin): $\mathbf{a} = \ddot{\mathbf{r}} = \ddot{r}\hat{r}$

The two most interesting terms in the 2D polar form of Newton's 2nd law are called the "centripetal" acceleration and the "Coriolis" acceleration. We'll study the Coriolis effect formally in Ch 9.

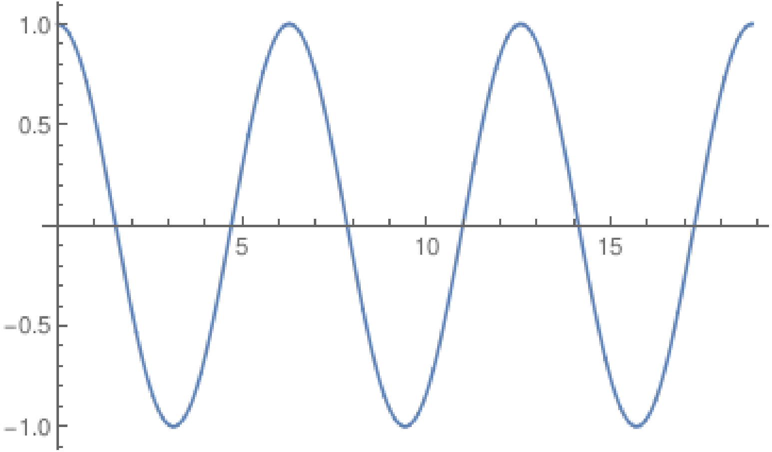
But now that we've mentioned the pseudo-forces that appear in non-inertial reference frames, let's watch a fun demonstration of the Coriolis effect, which also serves as an excuse to show you how you can take your first few steps in Mathematica.

It turns out to be surprisingly easy to use Mathematica to graph or animate what we see in this demo. See "notebook" at

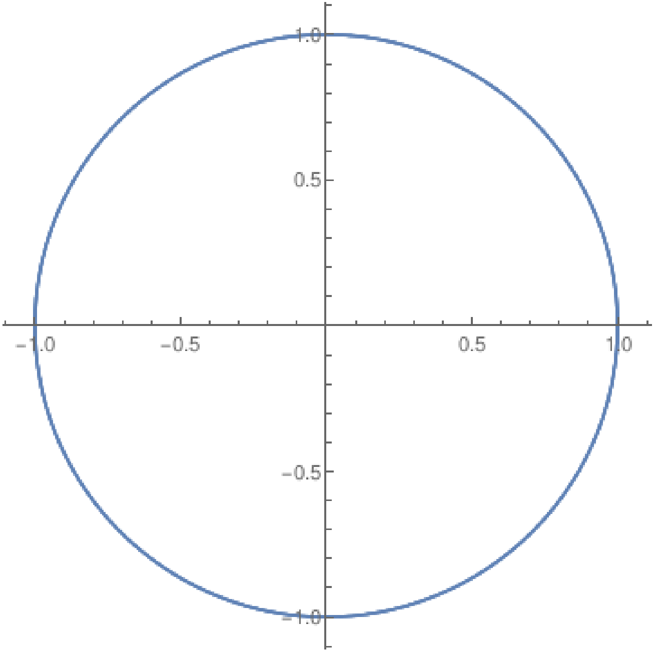
`positron.hep.upenn.edu/p351/files/0112_coriolis_pendulum.nb`

I point this out because I really think you'll find it helpful to force yourself to learn Mathematica this semester. A 2017 student comment: Learning to use Mathematica was worthwhile, and I think it would be advisable to assign the early Mathematica chapers as a regular assignment early in the course. The early chapters go very quickly, so it's not much of a burden. I put this off until later in the semester, and it would have been really helpful to do earlier. It's also one of those things, like the Lagrange equations, that I'll never forget how to do if/when I need it again.

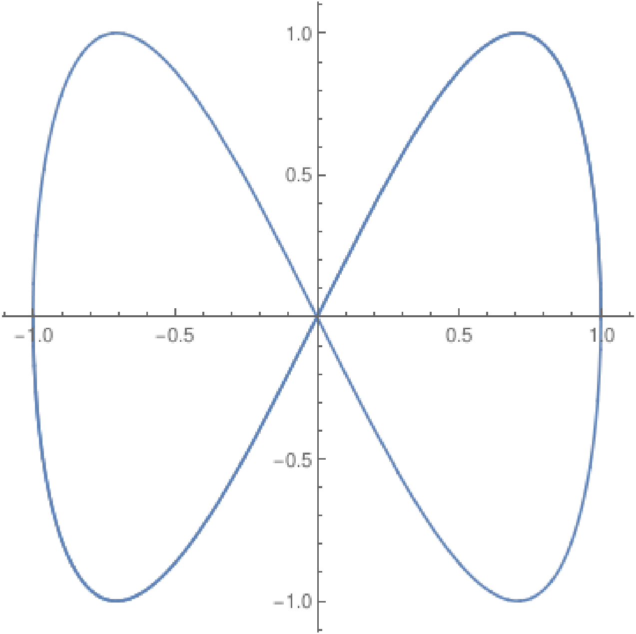
```
Plot[Cos[t], {t, 0, 6 Pi}]
```



```
ParametricPlot[{Cos[t], Sin[t]}, {t, 0, 6 Pi}]
```



```
ParametricPlot[{Cos[t / 2], Sin[t]}, {t, 0, 6 Pi}]
```



```
{x, y}
```

$\{x, y\}$

```
MatrixForm[{x, y}]
```

$$\begin{pmatrix} x \\ y \end{pmatrix}$$

```
{{Cos[phi], -Sin[phi]}, {Sin[phi], Cos[phi]}}
```

{{Cos[phi], -Sin[phi]}, {Sin[phi], Cos[phi]}}

```
MatrixForm[{{Cos[phi], -Sin[phi]}, {Sin[phi], Cos[phi]}}]
```

$$\begin{pmatrix} \text{Cos[phi]} & -\text{Sin[phi]} \\ \text{Sin[phi]} & \text{Cos[phi]} \end{pmatrix}$$

```
{{Cos[phi], -Sin[phi]}, {Sin[phi], Cos[phi]}} . {x, y}
```

{x Cos[phi] - y Sin[phi], y Cos[phi] + x Sin[phi]}

```
MatrixForm[{x Cos[phi] - y Sin[phi], y Cos[phi] + x Sin[phi]}]
```

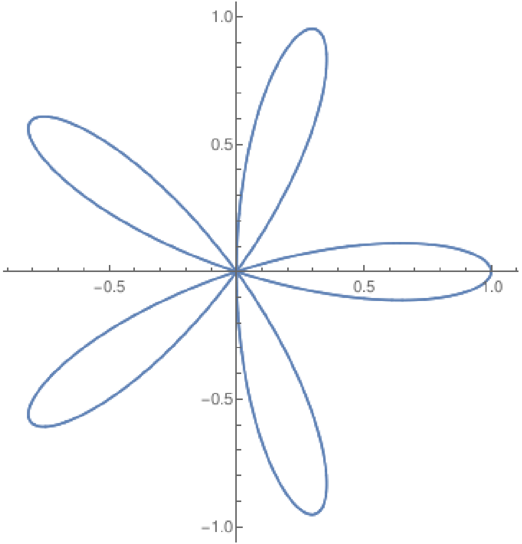
$$\begin{pmatrix} x\,\text{Cos[phi]} - y\,\text{Sin[phi]} \\ y\,\text{Cos[phi]} + x\,\text{Sin[phi]} \end{pmatrix}$$

```
rot[phi_] := {{Cos[phi], -Sin[phi]}, {Sin[phi], Cos[phi]}}
```

```
MatrixForm[rot[45 Degree]]
```

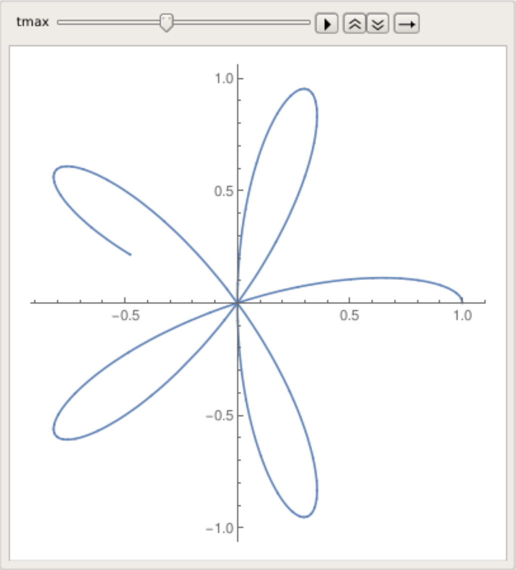
$$\begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

```
omegaPendulum = 1.0;
omegaEarth = 0.2;
xy[t_] := {Cos[omegaPendulum * t], 0};
xyRot[t_] := rot[omegaEarth * t] . xy[t];
ParametricPlot[xyRot[t], {t, 0, 10 Pi}]
```



In[18]:=
```
(* If this shows a blank graph, you need to do
   "Evaluation" → "Evaluate Notebook" so that the
   plot can use the definition of xyRot, etc. *)
Animate[ParametricPlot[xyRot[t], {t, 0, tmax}], {tmax, 0, 10 Pi}]
```

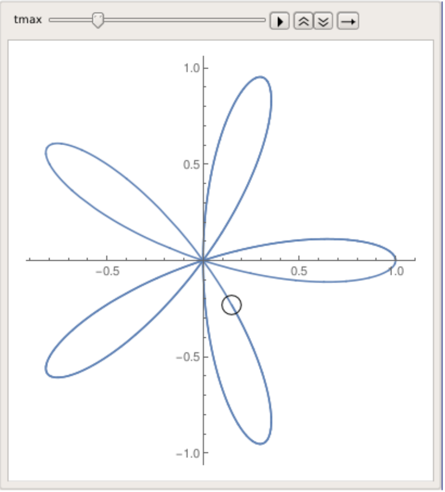
Out[18]=



In[19]:=

```
(* This adds a small circle to the plot, showing (x,y) at tmax *)
Animate[
 Show[
  ParametricPlot[xyRot[t], {t, 0, tmax}],
  Graphics[Circle[xyRot[tmax], 0.05]]
 ],
 {tmax, 0, 40 Pi},
 AnimationRate → 0.03
]
```

Out[19]=



One great feature of Taylor's book is the gentle math review presented alongside the physics. One worthwhile math trick from Ch2 is separation of varibles. (The many detailed drag-force results from Ch2 are not worth remembering, but the math methods he illustrates are valuable.) Let's try one separation-of-variables problem together.

A mass m has initial velocity v_0 at $t = 0$ and coasts along the x axis with drag force $F(v) = -cv^3$. Find $v(t)$.

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$$F(v) = -cv^3$$

$$m\ddot{x} = -c(\dot{x})^3 \rightarrow m\dot{v} = -cv^3 \rightarrow \frac{\dot{v}}{v^3} = -\frac{c}{m}$$

$$\int_{v'=v_0}^{v'=v} \frac{dv'}{(v')^3} = -\frac{c}{m}\int_{t'=0}^{t} dt' \rightarrow \left[\frac{-1}{2(v')^2}\right]_{v_0}^{v} = -\frac{c}{m}(t-0)$$

$$\frac{1}{v^2} - \frac{1}{v_0^2} = \frac{2ct}{m} \rightarrow \frac{1}{v^2} = \left(\frac{2ct}{m} + \frac{1}{v_0^2}\right)$$

$$v(t) = \frac{1}{\sqrt{\frac{2ct}{m} + \frac{1}{v_0^2}}} = \frac{\sqrt{\frac{m}{2c}}}{\sqrt{t + \frac{m}{2cv_0^2}}}$$

Incidentally, here's one way to solve the same problem using Mathematica. As you learn more and more of Mathematica's obscure syntax, you can solve problems with less and less typing.

```
In[1]:= solution = DSolve[{v'[t] / v[t]^3 == -c / m, v[0] == v0}, v, t]
```

Out[1]= $$\left\{\left\{v \to \text{Function}\left[\{t\}, -\frac{\sqrt{m}}{\sqrt{\frac{m+2\,c\,t\,v0^2}{v0^2}}}\right]\right\}, \left\{v \to \text{Function}\left[\{t\}, \frac{\sqrt{m}}{\sqrt{\frac{m+2\,c\,t\,v0^2}{v0^2}}}\right]\right\}\right\}$$

```
In[2]:= (v /. solution[[2]])[t]
```

Out[2]= $$\frac{\sqrt{m}}{\sqrt{\frac{m+2\,c\,t\,v0^2}{v0^2}}}$$

```
In[3]:= FullSimplify[Out[2]]
```

Out[3]= $$\frac{\sqrt{m}}{\sqrt{2\,c\,t+\frac{m}{v0^2}}}$$

Obscure: The mysterious `/.` stands for the `ReplaceAll[]` command, which can “plug in” solutions or values to an expression.

```
In[236]:= a + 2 b + 10

Out[236]= 10 + a + 2 b

In[237]:= a + 2 b + 10 /. {a → 1}

Out[237]= 11 + 2 b

In[238]:= a + 2 b + 10 /. {a → 1, b → 2}

Out[238]= 15
```

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