Physics 351 — Friday, January 26, 2018

- Turn in HW1. We prefer for you to write your name only on the back page of your homework, so that we can avoid knowing whose paper we're grading, until the end.
- Pick up HW2 handout. It's also online as a PDF.
- Read first 30pp (§7.1–7.7) of Chapter 7 (Lagrange's equations) for Monday, and answer the usual questions.
- You can do the Mathematica extra credit any time you like (if at all), but the earlier you do it, the more you'll be able to make use of Mathematica to reduce tedious algebra in your own homework. The "hands on start" chapters are a good tutorial. I found them both helpful and painless.

Before class: use $\partial f/\partial y = \frac{\mathrm{d}}{\mathrm{d}x}\,\partial f/\partial y'$ to show that $y = mx + b$ "extremizes" $\int_{x_0}^{x_1} \mathrm{d}x\, f(x, y, y')$ with $f(x, y, y') = \sqrt{1 + (y')^2}$.

```
ClearAll["Global`*"];
omega0 = 2.5; omega = 1.0; beta = 0.1; fampl = 1.0; fphi = 0.0; x0 = 3.0;
v0 = 0.0; tmax = 100.0;
soln = NDSolveValue[{x''[t] + 2 beta x'[t] + omega0^2 x[t] == fampl Cos[omega t + fphi],
    x'[0] == v0, x[0] == x0}, x[t], {t, 0, tmax}];
Plot[{soln, fampl Cos[omega t + fphi]}, {t, 0, tmax}, PlotRange → All]
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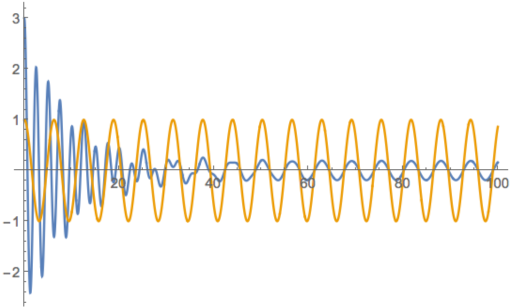
752]=



In[5]:=
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ClearAll["Global`*"];
g = 9.8;
l = 1.0;
soln = NDSolveValue[
  {θ''[t] + (g / l) Sin[θ[t]] == 0, θ'[0] == 0,
   θ[0] == 0.1}, θ[t], {t, 0, 10}]
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Out[8]= InterpolatingFunction[Domain: {{0., 10.}} Output: scalar][t]

In[9]:=
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Plot[soln, {t, 0, 10}]
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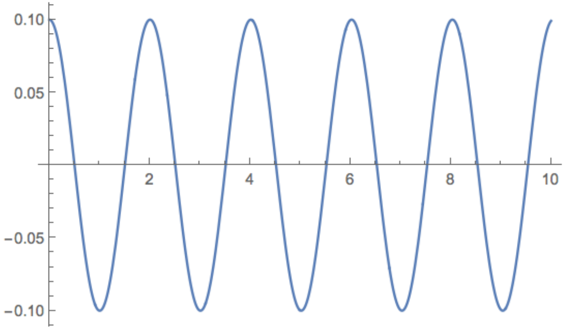
Out[9]=



Let's go back to the complex-number driving force

For driving force $F_0 e^{i\omega t}$, we found

$$x(t) = e^{-\beta t}(Ae^{+\Omega t} + Be^{-\Omega t}) + Ce^{i\omega t}$$

Once the transients have died away (after $\sim Q$ periods of ω_0),

$$x(t) = Ce^{i\omega t}$$

with

$$C = \frac{F_0}{-\omega^2 + 2i\beta\omega + \omega_0^2}$$

Now suppose you have a more complicated driving force:

$$\ddot{x} + 2\beta\dot{x} + \omega_0^2 x = F_a\, e^{i\omega_a t} + F_b\, e^{i\omega_b t}$$

Since $\mathcal{D}$ is linear,

$$\mathcal{D}\left[\frac{F_a\, e^{i\omega_a t}}{-\omega_a^2 + 2i\beta\omega_a + \omega_0^2}\right] = F_a e^{i\omega_a t}$$

$$\mathcal{D}\left[\frac{F_b\, e^{i\omega_b t}}{-\omega_b^2 + 2i\beta\omega_b + \omega_0^2}\right] = F_b e^{i\omega_b t}$$

$$\mathcal{D}\left[\frac{F_a\, e^{i\omega_a t}}{-\omega_a^2 + 2i\beta\omega_a + \omega_0^2} + \frac{F_b\, e^{i\omega_b t}}{-\omega_b^2 + 2i\beta\omega_b + \omega_0^2}\right] = F_a e^{i\omega_a t} + F_b e^{i\omega_b t}$$

So the general solution is

$$x(t) = \frac{F_a\, e^{i\omega_a t}}{-\omega_a^2 + 2i\beta\omega_a + \omega_0^2} + \frac{F_b\, e^{i\omega_b t}}{-\omega_b^2 + 2i\beta\omega_b + \omega_0^2} + e^{-\beta t}(Ae^{+\Omega t} + Be^{-\Omega t})$$

where again the transient terms are irrelevant for $t \gg 1/\beta$.

Now consider the more general case

$$\ddot{x} + 2\beta\dot{x} + \omega_0^2 x = f(t)$$

and suppose we're able to write

$$f(t) = \sum_n F_n \, e^{i\omega_n t}$$

Then it's clear that the solution would be

$$x(t) = (\text{transient}) + \sum_n \frac{F_n \, e^{i\omega_n t}}{-\omega_n^2 + 2i\beta\omega_n + \omega_0^2}$$

If $f(t)$ is periodic (period $T \equiv 2\pi/\omega$), then Prof. Fourier tells us

$$f(t) = \sum_{n=-\infty}^{+\infty} F_n \, e^{in\omega t}$$

$$f(t) = \sum_{n=-\infty}^{+\infty} F_n \, e^{in\omega t}$$

$$\frac{1}{T}\int_{-T/2}^{+T/2} f(t)\, e^{-im\omega t}\, \mathrm{d}t = \sum_n \frac{F_n}{T}\int_{-T/2}^{+T/2} \mathrm{d}t \, e^{i(n-m)\omega t} = \sum_n F_n \delta_{mn} = F_m$$

So the Fourier coefficient F_m is

$$F_m = \frac{1}{T}\int_{-T/2}^{+T/2} f(t) e^{-im\omega t}\, \mathrm{d}t$$

Note: for $f(t)$ real, $F_{-m} = F_m^*$, i.e. the negative-frequency coefficients are the complex conjugates of the corresponding positive-frequency coefficients.

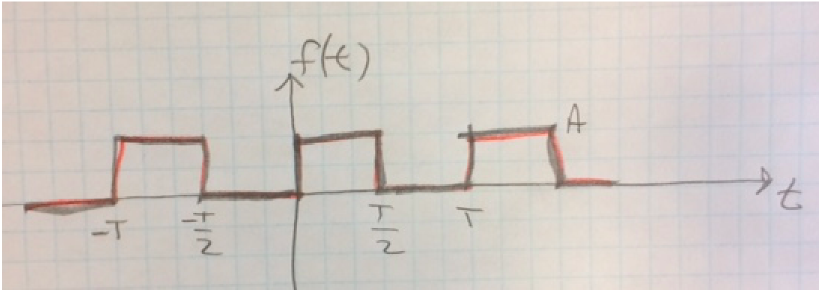
$$f(t) = \sum_{n=-\infty}^{+\infty} F_n \, e^{in\omega t}$$

with Fourier coefficient F_n given by

$$F_n = \frac{1}{T} \int_{-T/2}^{+T/2} f(t) e^{-in\omega t} \, \mathrm{d}t$$

Exercise: use this complex-number Fourier formalism to find the Fourier series for a square wave $f(t)$ of period $T = 2\pi/\omega$, with

$f(t) = 0$ for $-T/2 < t < 0$
$f(t) = A$ for $0 < t < T/2$



Square wave: $f(t) = \begin{matrix} 0 \text{ if } -T/2 < t < 0 \\ A \text{ if } 0 < t < T/2 \end{matrix}$ $T = \frac{2\pi}{\omega}$

$$F_n = \frac{1}{T}\int_{-T/2}^{T/2} f(t) e^{-in\omega t} dt = \frac{A}{T}\int_0^{T/2} e^{-in\omega t} dt = \frac{A}{-in\omega T}\left[e^{-in\omega T/2} - 1\right]$$

$$= \frac{A}{-in2\pi}\left(e^{-in\pi} - 1\right) = \frac{A}{in2\pi}\left(1 - (-1)^n\right)$$

$F_n = 0$ for even $n \neq 0$ $\quad F_n = \frac{A}{in\pi}$ for odd n

$$F_0 = \frac{A}{T}\int_0^{T/2} dt = \frac{A}{2} = \langle f(t) \rangle$$

$$f(t) = \frac{A}{2} + \sum_{n=1,3,5,\ldots} \frac{A}{in\pi}\left(\underbrace{e^{+in\omega t} - e^{-in\omega t}}_{2i\sin(n\omega t)}\right)$$

$$= \frac{A}{2} + \frac{2A}{\pi}\left(\sin(\omega t) + \frac{1}{3}\sin(3\omega t) + \frac{1}{5}\sin(5\omega t) + \cdots\right)$$

699]:=

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f[t_] := 1/2 + (2/Pi) (Sin[t] + Sin[3 t]/3 + Sin[5 t]/5);
Plot[f[t], {t, -10, 10}]
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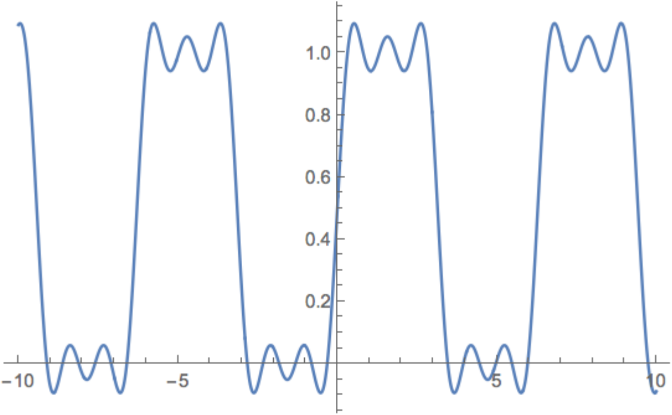
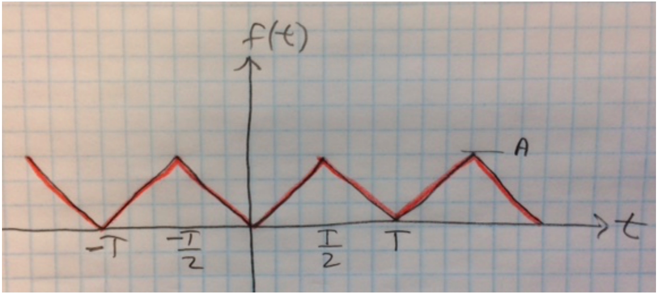
700]=



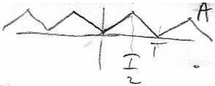
$$f(t) = \sum_{n=-\infty}^{+\infty} F_n \, e^{in\omega t} \qquad\qquad F_n = \frac{1}{T}\int_{-T/2}^{+T/2} f(t) e^{-in\omega t} \, \mathrm{d}t$$

Exercise: use this complex-number Fourier formalism to find the Fourier series for a triangle wave $f(t)$ of period $T = 2\pi/\omega$, with

$f(t) = -2At/T$ for $-T/2 < t < 0$
$f(t) = 2At/T$ for $0 < t < T/2$



$$\text{hint :} \quad \int t e^{-in\omega t} \mathrm{d}t \;=\; \frac{1 + in\omega t}{(n\omega)^2} \, e^{-in\omega t}$$



$$f(t) = \begin{cases} \dfrac{2At}{T} & 0<t<\dfrac{T}{2} \\ -\dfrac{2At}{T} & -\dfrac{T}{2}<t<0 \end{cases}$$

$$T = \frac{2\pi}{\omega}$$
$$\omega T = 2\pi$$
$$\omega T/2 = \pi$$

$$F_m = \frac{2A}{T^2}\left(\int_0^{T/2} t e^{-im\omega t}\,dt - \int_{-T/2}^{0} t e^{-im\omega t}\,dt\right)$$

$$= \frac{2A}{T^2}\left(\left[\frac{e^{-im\omega t}(1+im\omega t)}{(m\omega)^2}\right]_0^{T/2} - \left[\frac{e^{-im\omega t}(1+im\omega t)}{(m\omega)^2}\right]_{-T/2}^{0}\right)$$

$$= \frac{2A}{(m\omega T)^2}\left(e^{-im\pi}(1+im\pi) - 1 - 1 + e^{im\pi}(1-im\pi)\right)$$

$$= \frac{2A}{(m2\pi)^2}\left((-1)^m \left(1 + \cancel{im\pi}\right) - 2 + (-1)^m \left(1 - \cancel{im\pi}\right)\right)$$

$$= \frac{A}{(m\pi)^2}\left((-1)^m - 1 \right) \qquad \begin{aligned} &= 0 \text{ for even } m \neq 0 \\ &= \frac{-2A}{(m\pi)^2} \text{ for odd } m \end{aligned}$$

$$F_0 = \frac{2A}{T^2}\left(2\int_0^{T/2} t\,dt \right) = \frac{4A}{T^2}\left[\frac{1}{2}\left(\frac{T}{2}\right)^2 \right] = \frac{A}{2} \quad ✓$$

$$f(t) = \frac{A}{2} - \sum_{n=1,3,5,\ldots} \frac{2A}{(n\pi)^2}\left(\underbrace{e^{in\omega t} + e^{-in\omega t}}_{2\cos(n\omega t)} \right)$$

$$= \frac{A}{2} - \frac{4A}{\pi^2}\left(\cos(\omega t) + \frac{1}{9}\cos(3\omega t) + \frac{1}{25}\cos(5\omega t) \cdots \right)$$

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f[t_] := 1/2 - (4/Pi^2) (Cos[t] + Cos[3 t]/9 + Cos[5 t]/25);
Plot[f[t], {t, -10, 10}]
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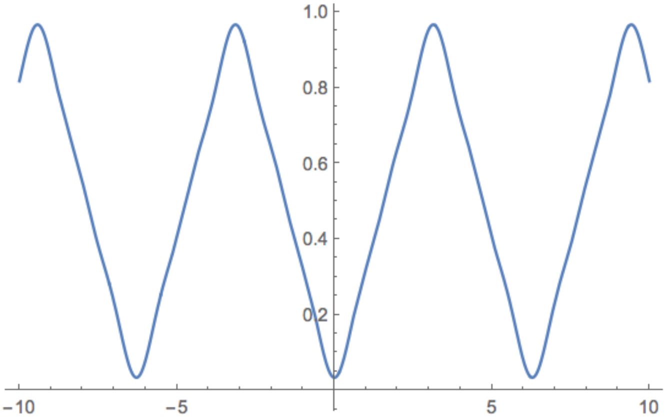
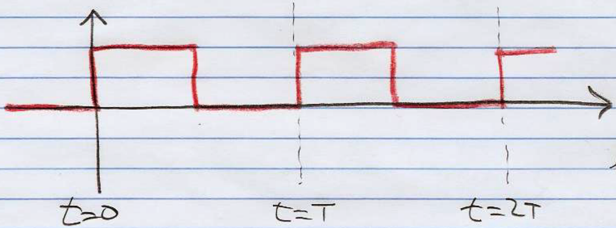


Example: periodic square wave



$$f(t) = \begin{cases} f_A & 0<t<\frac{T}{2} \\ 0 & -\frac{T}{2}<t<0 \end{cases}$$

let $\omega = \dfrac{2\pi}{T}$

$$F_m = \frac{1}{T}\int_{-T/2}^{+T/2} f(t)\, e^{-im\omega t} = \frac{f_A}{T}\int_0^{T/2} e^{-im\left(\frac{2\pi}{T}\right)t}\, dt$$

$$= \frac{f_A}{T}\left(\frac{T}{-2\pi i m}\right)\left[e^{-im\left(\frac{2\pi}{T}\right)t}\right]_0^{T/2}$$

$$= -\frac{f_A}{2\pi i m}\left(\left(e^{-i\pi}\right)^m - 1\right) = \frac{f_A}{2\pi i m}\left(1-(-1)^m\right)$$

$f(t) = \sum_{n=-\infty}^{+\infty} F_n\, e^{in\omega t}$

For m odd, $F_m = \dfrac{2f_A}{2\pi i m} = \dfrac{f_A}{i\pi m}$

For even $m \neq 0$, $F_m = 0$.

For $m = 0$, $F_0 = \dfrac{f_A}{T}\int_0^{T/2} dt = \dfrac{f_A}{T}\left(\dfrac{T}{2}\right) = \boxed{\dfrac{f_A}{2} = F_0}$

(F_0 is just the average $\langle f(t) \rangle$)

So $x(t) = (\text{transient}) + \sum_n \dfrac{F_n\, e^{in\omega t}}{(\omega_0^2 - n^2\omega^2) + 2in\beta\omega}$

Dropping uninteresting transient,

$$x(t) = \sum_n \frac{\left(F_n\, e^{in\omega t}\right)\left((\omega_0^2 - n^2\omega^2) - 2in\beta\omega\right)}{(\omega_0^2 - n^2\omega^2)^2 + (2n\beta\omega)^2}$$

$$x(t) = \frac{F_0}{\omega_0^2} + \sum_{\substack{n= \\ 1,3,5,\ldots}} \frac{\left(\frac{f_A}{i\pi n}\right)\left(e^{in\omega t} - e^{-in\omega t}\right)\left(\omega_0^2 - n^2\omega^2\right)}{\left(\omega_0^2 - n^2\omega^2\right)^2 + \left(2n\beta\omega\right)^2}$$

$$+ \sum_{\substack{n= \\ 1,3,5,\ldots}} \frac{\left(\frac{f_A}{i\pi n}\right)\left(e^{in\omega t} + e^{-in\omega t}\right)\left(-2in\beta\omega\right)}{\left(\omega_0^2 - n^2\omega^2\right)^2 + \left(2n\beta\omega\right)^2}$$

$$x(t) = \frac{f_A}{2\omega_0^2} + \sum_{\substack{n= \\ 1,3,5,\ldots}} \frac{\left(\frac{f_A}{i\pi n}\right)\left(2i\sin(n\omega t)\right)\left(\omega_0^2 - n^2\omega^2\right)}{\left(\omega_0^2 - n^2\omega^2\right)^2 + \left(2n\beta\omega\right)^2}$$

$$+ \sum_{\substack{n= \\ 1,3,5,\ldots}} \frac{\left(\frac{f_A}{i\pi n}\right)\left(2\cos(n\omega t)\right)\left(-2in\beta\omega\right)}{\left(\omega_0^2 - n^2\omega^2\right)^2 + \left(2n\beta\omega\right)^2}$$

$$x(t) = \frac{f_A}{2\omega_0^2} + \sum \frac{\left(\frac{2f_A}{\pi n}\right) \sin(n\omega t)\,(\omega_0^2 - n^2\omega^2)}{(\omega_0^2 - n^2\omega^2)^2 + (2n\beta\omega)^2}$$

$$+ \sum \frac{\left(\frac{2f_A}{\pi n}\right) \cos(n\omega t)\,(-2n\beta\omega)}{(\omega_0^2 - n^2\omega^2)^2 + (2n\beta\omega)^2}$$

Now let $C_n = \dfrac{\omega_0^2 - n^2\omega^2}{\sqrt{(\omega_0^2 - n^2\omega^2)^2 + (2n\beta\omega)^2}} \equiv \cos(\delta_n)$

$$S_n = \frac{-2n\beta\omega}{\sqrt{(\omega_0^2 - n^2\omega^2)^2 + (2n\beta\omega)^2}} \equiv \sin(\delta_n)$$

notice that $C_n^2 + S_n^2 = 1$

$$x(t) = \frac{f_A}{2\omega_0^2} + \sum_{n=1,3,5,\ldots} \sin(n\omega t)\cos(\delta_n)\frac{2f_A}{\pi n\sqrt{(\omega_0^2 - n^2\omega^2)^2 + (2n\beta\omega)^2}}$$

$$+ \sum_{n=1,3,5,\ldots} \cos(n\omega t)\sin(-\delta_n)\frac{2f_A}{\pi n\sqrt{(\omega_0^2 - n^2\omega^2)^2 + (2n\beta\omega)^2}}$$

$$x(t) = \frac{f_A}{2\omega_0^2} + \sum_{n=1,3,5,\dots} A_n \sin(n\omega t - \delta_n)$$

where $$A_n = \frac{2f_A}{\pi n\sqrt{(\omega_0^2 - n^2\omega^2)^2 + (2n\beta\omega)^2}}$$

$$\sin(n\omega t - \delta_n) = \sin(\omega t)\cos(\delta) + \cos(\omega t)\sin(-\delta)$$

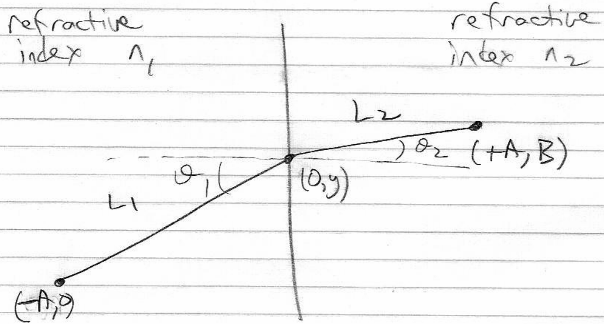
$$\delta_n = \arctan\left(\frac{2n\beta\omega}{\omega_0^2 - n^2\omega^2}\right)$$

Notice that the answer came out entirely real, even though we used complex exponentials. Also notice that this looks just like the result from the book using sines and cosines:

$x_n(t) = A_n \cos(n\omega t - \delta_n)$ $\quad A_n = \frac{f_n}{\sqrt{(\omega_0^2 - n^2\omega^2)^2 + (2\beta n\omega)^2}}$ $\quad$ same δ_n

and $2f_A/(\pi n)$ is just f_n. ($\sin$ vs. $\cos$ depends on chosen time offset of square wave.)

Chapter 6



$$L_1 = (A^2 + y^2)^{1/2} \qquad L_2 = (A^2 + (y-B)^2)^{1/2}$$

$$t = \frac{L_1}{v_1} + \frac{L_2}{v_2} = \frac{n_1 L_1}{c} + \frac{n_2 L_2}{c}$$

$$t = \frac{n_1}{c}\left(A^2+y^2\right)^{1/2} + \frac{n_2}{c}\left(A^2+(y-B)^2\right)^{\frac{1}{2}}$$

"principle of least time"

$$0 = \frac{dt}{dy} = \frac{n_1}{c}\frac{1}{2}\left(A^2+y^2\right)^{-\frac{1}{2}}(2y) + \frac{n_2}{c}\frac{1}{2}\left(A^2+(y-B)^2\right)^{-\frac{1}{2}}(2(y-B)),$$

$$0 = \frac{n_1 y}{\sqrt{A^2+y^2}} + \frac{n_2 (y-B)}{\sqrt{A^2+(y-B)^2}}$$

$$n_1 \frac{y}{L_1} = n_2 \frac{(B-y)}{L_2}$$

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

What if instead $n(x)$ were some continuous function? hmmm. Keep in back of mind.

Then for example you would want to find path $y(x)$ that minimizes time given by $t = \int_{x_1}^{x_2} \frac{ds}{v} = \int_{x_1}^{x_2} \frac{n(s)\,ds}{c}$

$$= \int_{x_1}^{x_2} \frac{n(x)}{c} \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx \equiv F[y]$$

Calculus of variations

$$F[y] = \int_{x_1}^{x_2} dx\, f(x, y(x), y'(x))$$

seek function $y(x)$ such that $F[y]$ has no first-order dependence on variations in y

ordinary calculus: minimum or maximum requires that

$$\lim_{\varepsilon \to 0} \frac{g(x+\varepsilon) - g(x)}{\varepsilon} = 0$$

(no first-order variation w.r.t. changes in x)

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simpler example of variational problem:

what path $y(x)$ minimizes arc length

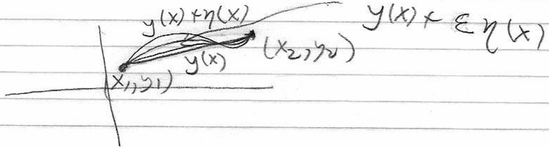
$$F[y] = \int_{x_1}^{x_2} ds = \int_{x_1}^{x_2} dx \sqrt{1 + \left(\frac{dy}{dx}\right)^2} = \int_{x_1}^{x_2} dx \sqrt{1 + (y')^2}$$

in this case $f(x, y, y') = \sqrt{1 + y'^2}$

in earlier case $f(x, y, y') = n(x)\sqrt{1 + y'^2}$

Let's postpone going over the derivation of the Euler-Lagrange equation until Monday. For now we'll skip to the result.

(skip for now)



we seek path $y(x)$ such that $F[y]$ is an extremum. In analogy with calculus, this occurs when

$F[y+\varepsilon\eta]$ has no first-order dependence on ε

$$F[y+\varepsilon\eta] = \int_{x_1}^{x_2} dx \, f\left(x, y(x)+\varepsilon\eta(x), y'(x)+\varepsilon\eta'(x)\right)$$

$$0 = \frac{dF}{d\varepsilon} = \int_{x_1}^{x_2} dx \left(\frac{\partial f}{\partial y}\eta(x) + \underline{\frac{\partial f}{\partial y'}\eta'(x)}\right)$$

work on this term to eliminate $\eta'(x)$

(skip for now)

Consider the expression

$$\frac{d}{dx}\left(\frac{\partial f}{\partial y'}\eta(x)\right) = \eta(x)\frac{d}{dx}\frac{\partial f}{\partial y'} + \frac{\partial f}{\partial y'}\eta'(x)$$

$$\Rightarrow \frac{\partial f}{\partial y'}\eta'(x) = -\eta(x)\frac{d}{dx}\frac{\partial f}{\partial y'} + \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\eta(x)\right)$$

$$0 = \frac{dF}{d\varepsilon} = \int_{x_1}^{x_2} dx\left(\eta(x)\frac{\partial f}{\partial y} - \eta(x)\frac{d}{dx}\frac{\partial f}{\partial y'} + \underline{\frac{d}{dx}\left(\frac{\partial f}{\partial y'}\eta(x)\right)}\right)$$

$$0 = \frac{dF}{d\varepsilon} = \left[\frac{\partial f}{\partial y'}\eta(x)\right]_{x_1}^{x_2} + \int_{x_1}^{x_2} dx\,\eta(x)\left(\frac{\partial f}{\partial y} - \frac{d}{dx}\frac{\partial f}{\partial y'}\right)$$

this term we can just integrate

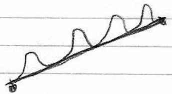
(skip for now)

only consider variations where $\eta(x_1) = \eta(x_2) = 0$

$\Rightarrow$ kills ✓ term in [ ] brackets.

boundary

We want $\displaystyle\int_{x_1}^{x_2} dx\, \eta(x) \left(\frac{\partial f}{\partial y} - \frac{d}{dx}\frac{\partial f}{\partial y'}\right)$ for otherwise arbitrary "reasonable" variations $\eta(x)$



etc.

$$\Rightarrow \quad \frac{\partial f}{\partial y} - \frac{d}{dx}\frac{\partial f}{\partial y'}$$

everywhere along the path $y(x)$

You already know how the story will end:

$$\frac{\partial f}{\partial y} = \frac{\mathrm{d}}{\mathrm{d}x}\frac{\partial f}{\partial y'}$$

Let's try it.

$$f(x, y, y') = (1+(y')^2)^{1/2}$$

$$\frac{\partial f}{\partial y} = 0 \qquad \frac{\partial f}{\partial y'} = \frac{1}{2}(1+(y')^2)^{-\frac{1}{2}}(2y')$$

$$\frac{\partial f}{\partial y} = 0 = \frac{d}{dx}\frac{\partial f}{\partial y'} \Rightarrow \frac{y'}{\sqrt{1+(y')^2}} = \text{constant}$$

$$\Rightarrow y' = \text{constant}$$

$$\Rightarrow y = mx + b$$

Here's another example:

Determine curve $y = y(x)$ such that the integral

$$\int_{x_1}^{x_2} dx \sqrt{x} \sqrt{1+(y')^2}$$

is stationary.

When would this ever arise? Perhaps you want to find the path $y(x)$ followed by light when the index of refraction $n(x) = a\sqrt{x}$.

$$F[y] = \int dx\, \sqrt{x}\, \sqrt{1+(y')^2}$$

$$f(x,y,y') = \sqrt{x}\, \sqrt{1+(y')^2}$$

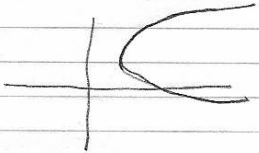
$$\frac{\partial f}{\partial y} = 0 \qquad \frac{\partial f}{\partial y'} = \frac{y'\sqrt{x}}{\sqrt{1+(y')^2}}$$

$$\frac{d}{dx}\frac{\partial f}{\partial y'} - \frac{\partial f}{\partial y} = 0 \Rightarrow \frac{\partial f}{\partial y'} = \text{const.}$$

$$y'\sqrt{x} = c\sqrt{1+(y')^2} \Rightarrow (y')^2 x = c^2 + c^2 (y')^2$$

$$(y')^2 (x - c^2) = c^2 \Rightarrow y' = \frac{\pm c}{\sqrt{x-c^2}}$$

$$\Rightarrow y(x) = \pm c\sqrt{x-c^2} + d$$



Sideways parabola

Find a function $y(x)$ that minimizes

$$I[y] = \int_0^1 \mathrm{d}x \, \left((y')^2 + 2ye^x\right)$$

subject to $y(0) = 0$ and $y(1) = 1$.

$$f(x, y, y') = y'^2 + 2y\,e^x$$

$$\frac{\partial f}{\partial y} = 2e^x \qquad \frac{\partial f}{\partial y'} = 2y'$$

$$\frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) = 2y''$$

$$\frac{\partial f}{\partial y} = \frac{d}{dx}\frac{\partial f}{\partial y'} \Rightarrow e^x = y''$$

$$y' = e^x + A$$

$$y = e^x + Ax + B$$

$$y(0) = 0 \Rightarrow 1 + B = 0 \qquad y(1) = 1 \Rightarrow 1 = e + A - 1$$

$$\Rightarrow B = -1 \qquad \Rightarrow A = 2 - e$$

$$y(x) = e^x + (2-e)x - 1$$

Physics 351 — Friday, January 26, 2018

- Turn in HW1. We prefer for you to write your name only on the back page of your homework, so that we can avoid knowing whose paper we're grading, until the end.
- Pick up HW2 handout. It's also online as a PDF.
- Read first 30pp (§7.1–7.7) of Chapter 7 (Lagrange's equations) for Monday, and answer the usual questions.
- You can do the Mathematica extra credit any time you like (if at all), but the earlier you do it, the more you'll be able to make use of Mathematica to reduce tedious algebra in your own homework. The "hands on start" chapters are a good tutorial. I found them both helpful and painless.