Physics 351 — Monday, January 29, 2018

- For today you read §7.1–7.7 of Ch 7 (Lagrange's equations). Next weekend you'll read the rest of Ch 7.
- You can do the Mathematica extra credit any time you like (if at all), but the earlier you do it, the more you'll be able to make use of Mathematica to reduce tedious algebra in your own homework. The "hands on start" chapters are a good tutorial. I found them both helpful and painless.

Before class: use $\partial f/\partial y = \frac{\mathrm{d}}{\mathrm{d}x}\,\partial f/\partial y'$ to find a function $y(x)$ that "extremizes" (i.e. makes stationary w.r.t. small variations in $y(x)$)

$$F[y] = \int_{x=0}^{x=1} \mathrm{d}x\ \left((y')^2 + 2ye^x\right)$$

subject to $y(0) = 0$ and $y(1) = 1$. So $f(x, y, y') = (y')^2 + 2ye^x$ in the Euler-Lagrange equation.

Find a function $y(x)$ that minimizes

$$I[y] = \int_0^1 \mathrm{d}x \, \left((y')^2 + 2ye^x\right)$$

subject to $y(0) = 0$ and $y(1) = 1$.

$$f(x,y,y') = y'^2 + 2y\,e^x$$

$$\frac{\partial f}{\partial y} = 2e^x \qquad \frac{\partial f}{\partial y'} = 2y'$$

$$\frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) = 2y''$$

$$\frac{\partial f}{\partial y} = \frac{d}{dx}\frac{\partial f}{\partial y'} \Rightarrow e^x = y''$$

$$y' = e^x + A$$

$$y = e^x + Ax + B$$

$$y(0) = 0 \Rightarrow 1 + B = 0 \Rightarrow B = -1 \qquad y(1) = 1 \Rightarrow 1 = e + A - 1 \Rightarrow A = 2 - e$$

$$y(x) = e^x + (2-e)x - 1$$

In Calculus of Variations, we want to find the **function** $y(x)$ that "minimizes" (or at least makes "stationary") some integral

$$F[y] = \int_{x_i}^{x_f} \mathrm{d}x\, f(x, y, y')$$

w.r.t. small variations in $y(x)$. So we consider "nearby" paths

$$Y(x, \epsilon) = y(x) + \epsilon\eta(x)$$

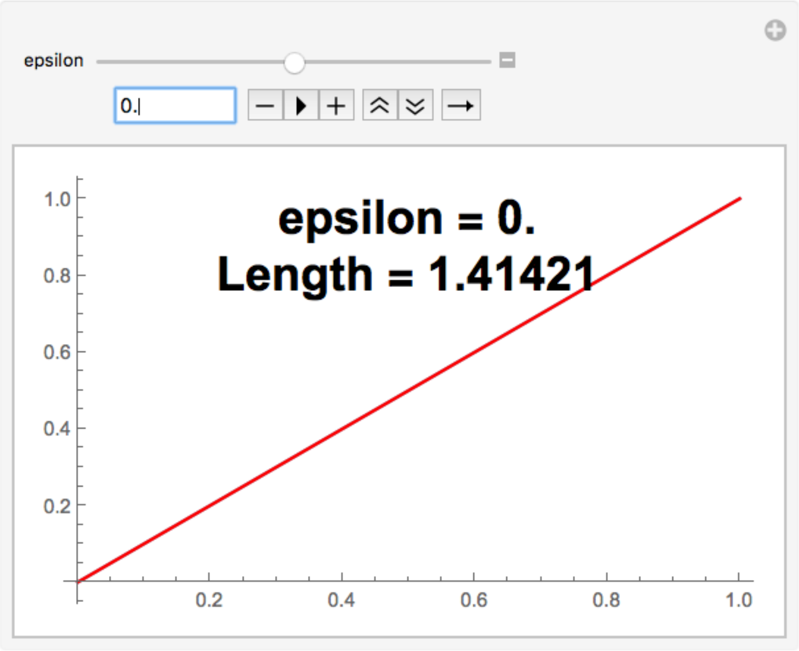
but we insist that $Y(x)$ coincide with $y(x)$ at the endpoints, i.e. $\eta(x_i) = \eta(x_f) = 0$. For varied paths $Y[x]$, the integral

$$F[y + \epsilon\,\eta] = \int_{x_i}^{x_f} \mathrm{d}x\, f(x,\, y(x) + \epsilon\,\eta(x),\, y' + \epsilon\,\eta'(x))$$

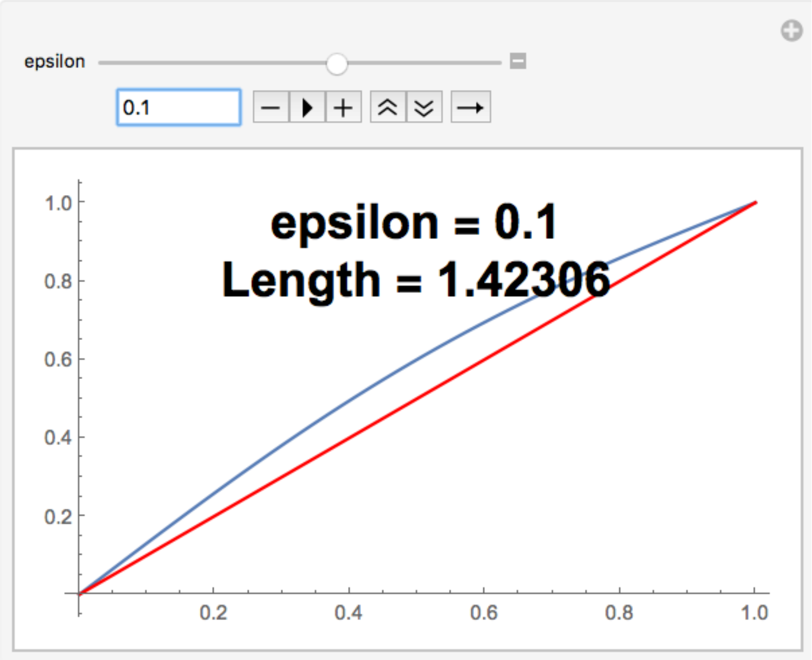
depends on ϵ. "Stationary" path $y(x)$ is where $\mathrm{d}F/\mathrm{d}\epsilon = 0$. If you Taylor-expand $F[y + \epsilon\eta] = F[y] + a\epsilon + b\epsilon^2 + \cdots$, you get $a = 0$.

Ex: $F[y] = \int_{x=0}^{1} \mathrm{d}x \sqrt{1 + (y')^2}$, such that $y(0) = 0$, $y(1) = 1$, and let's explicitly choose $\eta(x) = \sin(\pi x)$ to perturb $y(x)$.

$(x_1, y_1) = (0,0)$, $(x_2, y_2) = (1,1)$, $f(x, y, y') = \sqrt{1 + y'^2}$, $\eta(x) = \sin(\pi x)$



$$(x_1, y_1) = (0, 0),\ (x_2, y_2) = (1, 1), \quad f(x, y, y') = \sqrt{1 + y'^2}, \quad \eta(x) = \sin(\pi x)$$



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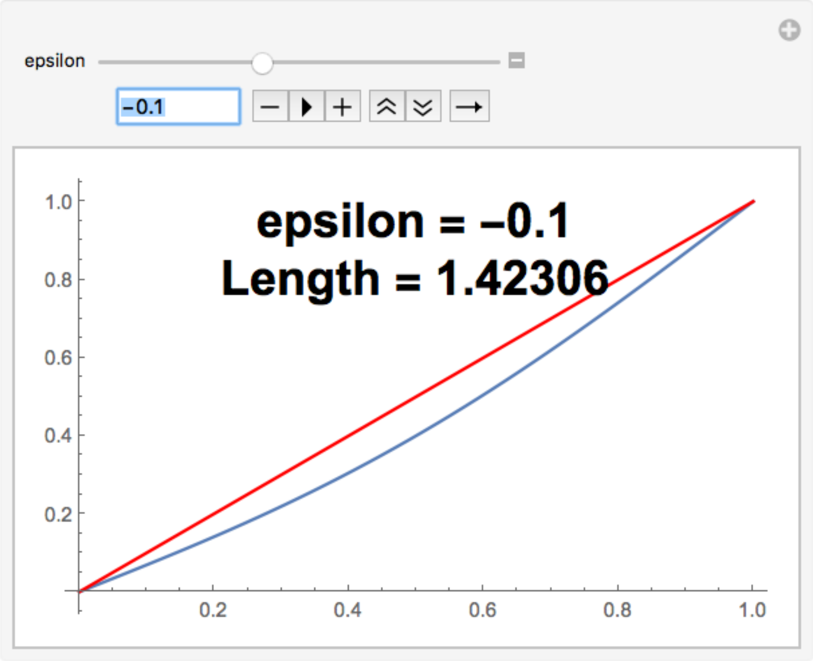

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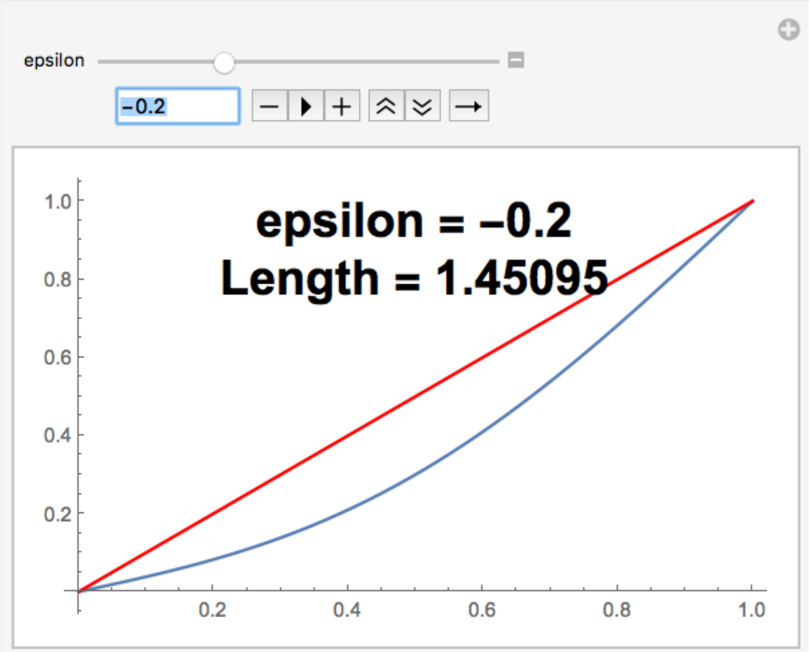
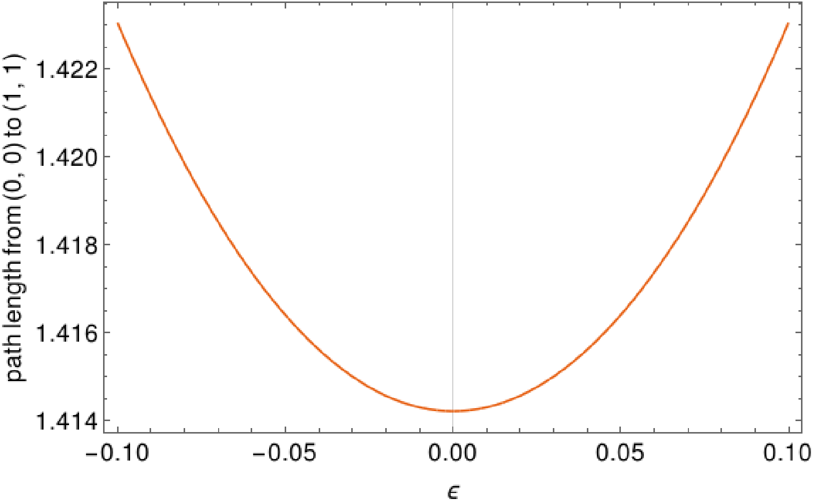


$(x_1, y_1) = (0, 0),\ (x_2, y_2) = (1, 1), \quad f(x, y, y') = \sqrt{1 + y'^2}, \quad \eta(x) = \sin(\pi x)$



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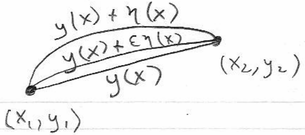


Consider paths $Y(x) = y(x) + \varepsilon\,\eta(x)$

$$F[Y(x)] = F[y(x) + \varepsilon\,\eta(x)]$$

$$\simeq F[y(x)] + \varepsilon \frac{dF}{d\varepsilon} + \cancel{\frac{1}{2}\varepsilon^2 \frac{d^2F}{d\varepsilon^2} + \cdots}$$

$$0 = \frac{dF}{d\varepsilon} = \frac{d}{d\varepsilon} \int_{x=x_1}^{x_2} f\left(x, y(x) + \varepsilon\eta(x), y'(x) + \varepsilon\eta'(x)\right) dx$$

If we had $f(x, u(x), v(x))$ instead, where $u(x)$ and $v(x)$ both depend on ε, then

$$\frac{d}{d\varepsilon} f(x, u(x), v(x)) = \frac{\partial f}{\partial u}\frac{du}{d\varepsilon} + \frac{\partial f}{\partial v}\frac{dv}{d\varepsilon}$$

In this case we have
$$u(x) = y(x) + \varepsilon\,\eta(x)$$
$$v(x) = y'(x) + \varepsilon\,\eta'(x)$$

$$0 = \frac{dF}{d\varepsilon} = \int_{x}^{x_2} dx \left(\frac{\partial f}{\partial y}\eta(x) + \frac{\partial f}{\partial y'}\eta'(x) \right)$$

$$0 = \frac{dF}{d\varepsilon} = \int_{x_1}^{x_2} dx \left(\frac{\partial f}{\partial y} \eta(x) + \frac{\partial f}{\partial y'} \eta'(x) \right)$$

Notice that $\frac{d}{dx}\left(\frac{\partial f}{\partial y'} \eta(x) \right) = \left(\frac{d}{dx} \frac{\partial f}{\partial y'} \right) \eta(x) + \frac{\partial f}{\partial y'} \eta'(x)$

so then $\frac{\partial f}{\partial y'} \eta'(x) = -\left(\frac{d}{dx} \frac{\partial f}{\partial y'} \right) \eta(x) + \frac{d}{dx} \left(\frac{\partial f}{\partial y'} \eta(x) \right)$

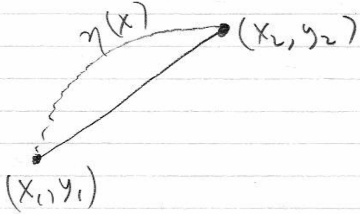
$$0 = \frac{dF}{d\varepsilon} = \int_{x_1}^{x_2} dx \, \eta(x) \left(\frac{\partial f}{\partial y} - \frac{d}{dx} \frac{\partial f}{\partial y'} \right) + \int_{x_1}^{x_2} dx \, \frac{d}{dx} \left(\frac{\partial f}{\partial y'} \eta(x) \right)$$

$$0 = \frac{dF}{d\varepsilon} = \int_{x_1}^{x_2} dx \, \eta(x) \left(\frac{\partial f}{\partial y} - \frac{d}{dx} \frac{\partial f}{\partial y'} \right) + \left[\frac{\partial f}{\partial y'} \eta(x) \right]_{x=x_1}^{x_2}$$

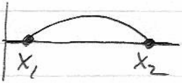
This is why we insist that
$\eta(x_1) = \eta(x_2) = 0$, to kill the 2nd term.

What remains is $0 = \int_{x_1}^{x_2} dx\, \eta(x) \left(\frac{\partial f}{\partial y} - \frac{d}{dx} \frac{\partial f}{\partial y'} \right)$

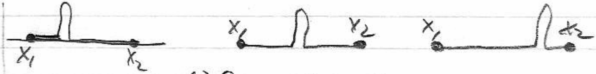
which must be true for any "reasonable" function $\eta(x)$ as long as $\eta(x_1) = \eta(x_2) = 0$.



Instead of consitering only an $\eta(x)$ that looks like this



Consiter these:



So we conclude that $\left(\frac{\partial f}{\partial y} - \frac{d}{dx} \frac{\partial f}{\partial y'} \right) = 0$

everywhere $x_1 < x < x_2$

Here's another example, but I think we may skip it to save time.

Determine curve $y = y(x)$ such that the integral

$$\int_{x_1}^{x_2} dx \sqrt{x} \sqrt{1+(y')^2}$$

is stationary.

When would this ever arise? Perhaps you want to find the path $y(x)$ followed by light when the index of refraction $n(x) = a\sqrt{x}$.

$$F[y] = \int dx \, \sqrt{x} \, \sqrt{1+(y')^2}$$

$$f(x,y,y') = \sqrt{x} \, \sqrt{1+(y')^2}$$

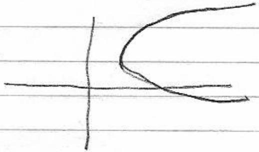
$$\frac{\partial f}{\partial y} = 0 \qquad \frac{\partial f}{\partial y'} = \frac{y' \sqrt{x}}{\sqrt{1+(y')^2}}$$

$$\frac{d}{dx}\frac{\partial f}{\partial y'} - \frac{\partial f}{\partial y} = 0 \Rightarrow \frac{\partial f}{\partial y'} = \text{const.}$$

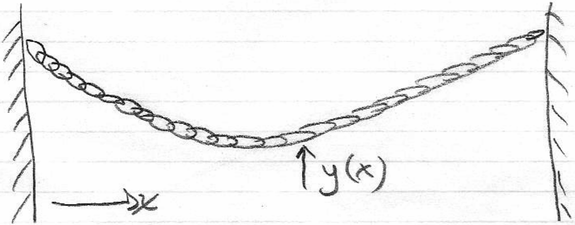
$$y'\sqrt{x} = c\sqrt{1+(y')^2} \Rightarrow (y')^2 x = c^2 + c^2 (y')^2$$

$$(y')^2 (x - c^2) = c^2 \Rightarrow y' = \frac{\pm c}{\sqrt{x-c^2}}$$

$$\Rightarrow y(x) = \pm c \sqrt{x-c^2} + d$$



Sideways parabola



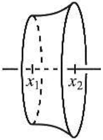
A chain $\left(\frac{\text{mass}}{\text{length}} = \mu\right)$ hanging between two posts has potential energy

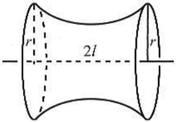
$$U[y] = \int ds\, \mu g y = \mu g \int y \sqrt{1+(y')^2}\, dx$$

What shape $y(x)$ minimizes U, for given boundary conditions?

We'll solve this problem 3 different ways, to illustrate two ways that "conserved quantities" (which will turn out in mechanics to be momentum and energy) can reduce the E-L equation to first-order.

(To do the hanging-chain problem properly, we should impose a constraint that the total length of the chain be fixed. Let's ignore that complication until we learn about Lagrange multipliers at the end of Chapter 7. Alternatively, you can view this integral as a (correct) formulation of the "soap film" problem: minimizes area of surface of revolution.)





$$U = \mu g \int y \, \mathrm{d}s = \mu g \int y \sqrt{1 + y'^2} \, \mathrm{d}x$$

Try writing down

$$\frac{\partial f}{\partial y} = \frac{\mathrm{d}}{\mathrm{d}x} \frac{\partial f}{\partial y'}$$

Warning — it's surprisingly messy! Downright gross, before some nice cancellations clean it up. See if you can get it to look like $1 + (y')^2 - y \, y'' = 0$ after a lot of cancelling.

$$f(x,y,y') = y\sqrt{1+y'^2}$$

$$\frac{\partial f}{\partial y} = \sqrt{1+y'^2} \quad \Bigg\} \quad \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) = \frac{d}{dx}\left(\frac{yy'}{\sqrt{1+y'^2}}\right)$$

$$= \frac{y'y'}{\sqrt{1+y'^2}} + \frac{yy''}{\sqrt{1+y'^2}} - \frac{yy'\,y'y''}{(1+y'^2)^{3/2}}$$

$$\sqrt{1+y'^2} = \frac{y'^2 + yy''}{\sqrt{1+y'^2}} - \frac{y(y'^2)y''}{(1+y'^2)^{3/2}}$$

$$(1+y'^2)^2 = (y'^2 + yy'')(1+y'^2) - y(y'^2)y''$$

$$1 + 2y'^2 + \underline{y'^4} = y'^2 + \underline{y'^4} + yy'' + \underline{yy''y'^2} - \underline{yy''y'^2}$$

$$1 + y'^2 - yy'' = 0$$

$$1 + y'^2 - yy'' = 0$$

If you happen to guess $y = A\cosh\left(\frac{x-B}{A}\right)$
then

$$y' = \sinh\left(\frac{x-B}{A}\right) \rightarrow y'^2 = \sinh^2\left(\frac{x-B}{A}\right)$$

$$y'' = \left(\frac{1}{A}\right)\cosh\left(\frac{x-B}{A}\right) \rightarrow yy'' = \cosh^2\left(\frac{x-B}{A}\right)$$

$$1 + y'^2 - yy'' = 1 + \sinh^2\left(\frac{x-B}{A}\right) - \cosh^2\left(\frac{x-B}{A}\right) = 1 - 1 = 0$$

which satisfies $1 + y'^2 - yy'' = 0$ ✓

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Plot[Cosh[x], {x, -1, 1}, LabelStyle → 18]
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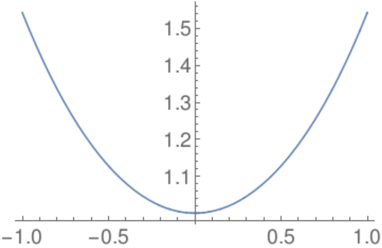

A second way to approach this same problem is to solve for $x(y)$ instead of solving for $y(x)$.

$$U = \mu g \int y \, \mathrm{d}s = \mu g \int y \sqrt{1 + x'^2} \, \mathrm{d}y$$

Now we have $\int \mathrm{d}y f(y, x, x')$, so the Euler-Lagrange equation becomes

$$\frac{\partial f}{\partial x} = \frac{\mathrm{d}}{\mathrm{d}y} \frac{\partial f}{\partial x'}$$

Now we can use the "momentum conservation" trick, because $f(y, x, x')$ is independent of x.

$$f(y, x, x') = y\sqrt{1+x'^2}$$

$$\frac{\partial f}{\partial x} = \frac{d}{dy}\frac{\partial f}{\partial x'} \Rightarrow \frac{\partial f}{\partial x} = 0 = \frac{d}{dy}\left(\frac{y\,x'}{\sqrt{1+x'^2}}\right)$$

$$\Rightarrow \frac{y\,x'}{\sqrt{1+x'^2}} = c$$

$$y^2 x'^2 = c^2(1+x'^2) \Rightarrow (y^2-c^2)\,x'^2 = c^2$$

$$x' = \frac{c}{\sqrt{y^2-c^2}}$$ which can be integrated to get

$$x = c\cosh^{-1}\left(\frac{y}{c}\right) + b$$

$$\cosh\left(\frac{x-b}{c}\right) = \frac{y}{c} \Rightarrow y = c\cosh\left(\frac{x-b}{c}\right)$$

So the "conserved momentum" trick — $f(y, x, x')$ indep. of x, which next time will be analogous to $\mathcal{L}(t, x, \dot{x})$ indep. of x — gave us a first-order ODE (compare with $1 + y'^2 - yy'' = 0$), which we could integrate instead of having to guess a solution.

There is a second trick for getting a first-order ODE from the E-L equation, which works if $f(x, y, y') = f(y, y')$ is independent of x. This trick will turn out next time to be analogous to the total energy being conserved, if $\mathcal{L}(t, x, \dot{x})$ is independent of t.

Claim:
If $\quad f(x, y, y') = f(y, y') \quad$ i.e. $f(x, y, y')$ is independent of x, then any function $y(x)$ that extremizes $\quad \int \mathrm{d}x\, f(y, y') \quad$ satisfies

$$h \;\equiv\; y' \frac{\partial f}{\partial y'} - f(y, y') \;=\; \text{constant}$$

In the context of mechanics, h will be the "Hamiltonian," which equals the (constant) total energy if $\mathcal{L}(t, x, \dot{x})$ is independent of t.

Given: $y(x)$ extremizes $\int \mathrm{d}x\, f(y,y')$

Claim:

$$h \;\equiv\; y'\frac{\partial f}{\partial y'} - f(y,y') \;=\; \text{constant}$$

Proof:

$$\frac{\mathrm{d}h}{\mathrm{d}x} \;=\; y''\frac{\partial f}{\partial y'} + y'\left(\frac{\mathrm{d}}{\mathrm{d}x}\frac{\partial f}{\partial y'}\right) - \frac{\partial f}{\partial y}\,\frac{\mathrm{d}y}{\mathrm{d}x} - \frac{\partial f}{\partial y'}\,\frac{\mathrm{d}y'}{\mathrm{d}x}$$

$$\frac{\mathrm{d}h}{\mathrm{d}x} \;=\; y''\frac{\partial f}{\partial y'} + y'\left(\frac{\mathrm{d}}{\mathrm{d}x}\frac{\partial f}{\partial y'}\right) - y'\frac{\partial f}{\partial y} - y''\frac{\partial f}{\partial y'}$$

$$\frac{\mathrm{d}h}{\mathrm{d}x} \;=\; y'\left(\frac{\mathrm{d}}{\mathrm{d}x}\frac{\partial f}{\partial y'}\right) - y'\frac{\partial f}{\partial y}$$

$$\frac{\mathrm{d}h}{\mathrm{d}x} \;=\; y'\left(\frac{\mathrm{d}}{\mathrm{d}x}\frac{\partial f}{\partial y'} - \frac{\partial f}{\partial y}\right) \;=\; 0$$

because $y(x)$ must satisfy the Euler-Lagrange equation. QED.

Back to our hanging-chain problem. $f(y, y') = y\sqrt{1 + y'^2}$.

$$h \;\equiv\; y' \frac{\partial f}{\partial y'} - f(y, y') \;=\; \text{constant}$$

Back to our hanging-chain problem. $f(y, y') = y\sqrt{1 + y'^2}$.

$$h \equiv y' \frac{\partial f}{\partial y'} - f(y, y') = \text{constant}$$

$$f(y, y') = y\sqrt{1+y'^2}$$

$$h = y' \frac{\partial f}{\partial y'} - f = y'\left(\frac{y y'}{\sqrt{1+y'^2}}\right) - y\sqrt{1+y'^2} = h$$

$$y y'^2 - y(1+y'^2) = h\sqrt{1+y'^2}$$

$$y y'^2 - y - y y'^2 = h\sqrt{1+y'^2}$$

$$-y = h\sqrt{1+y'^2} \Rightarrow \left(\frac{y}{h}\right)^2 = 1+y'^2$$

which you could solve e.g. by guessing

$$y = h\cosh\left(\frac{x-B}{h}\right)$$

$$y' = \sinh\left(\frac{x-B}{h}\right) \longrightarrow 1+y'^2 = 1+\sinh^2\left(\frac{x-B}{A}\right) = \cosh^2\left(\frac{x-B}{A}\right)$$

$$1+y'^2 = \left(\frac{y}{h}\right)^2 \checkmark$$

If you formally study the Calculus of Variations, the "momentum" trick is called the "first integral" of the E-L equation, and the "energy" trick is called the "Baltrami identity."

I mentioned them for two reasons: Most importantly, because we will see mechanical analogues of these tricks in the next two weeks. In the first case, "ignorable coordinates" (coordinates not appearing in the Lagrangian) lead to "conserved (generalized) momenta." In the second case, conservation of energy is expressed (in mechanics) by writing down a Hamiltonian function.

The second reason is that first-order ODEs are generally easier to solve than second-order ODEs, so these two tricks can save effort when using the E-L equation for optimization problems.

If you're a physicist, you'll often find that the easiest way to remember a given math result is to remember the analogous physics problem for which it is useful!

Now, a moment you've been waiting 2.5 weeks for!

In last weekend's reading (the non-asterisk parts of Ch7), you saw that the trajectory of a particle moving in potential $U(x)$ follows the "path of least action," i.e. it follows the path $x(t)$ for which the "action" $S[x(t)]$ is stationary:

$$S[x] \;=\; \int \mathcal{L}(t, x, \dot{x})\,\mathrm{d}t \;=\; \int_{t_i}^{t_f} (T - U)\,\mathrm{d}t$$

Feynman points out
  `http://www.feynmanlectures.caltech.edu/II_19.html`
that to be precise, one should really call the Lagrangian approach "the principle of stationary Hamilton's first principal function." But most people say, more concisely, "the principle of least action."

Lagrangian mechanics: Use the Euler-Lagrange equation to find the trajectory $x(t)$ for which the "action" $S[x]$ is stationary.

$$S[x] \;=\; \int \mathcal{L}(t, x, \dot{x})\,\mathrm{d}t \;=\; \int_{t_i}^{t_f} (T - U)\,\mathrm{d}t$$

Let's give it a try for a particle of mass m dropped vertically from a short distance x above Earth's surface. For notational simplicity, let the x axis point **vertically upward**. (I should draw this.)

First write down $\mathcal{L}(t, x, \dot{x})$. (Try it!)

Then write the E-L equation: (which variables are which here?)

Lagrangian mechanics: Use the Euler-Lagrange equation to find the trajectory $x(t)$ for which the "action" $S[x]$ is stationary.

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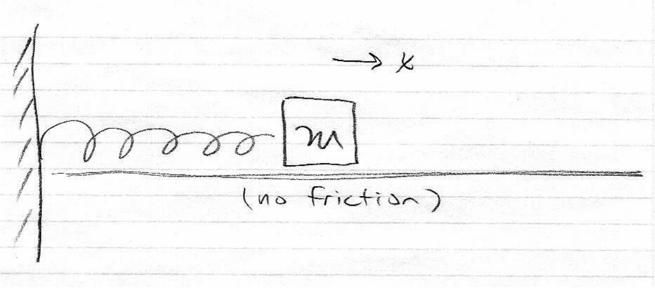
First write down $\mathcal{L}(t, x, \dot{x})$. (Try it!)

Then write the E-L equation: (which variables are which here?)

$$\frac{\partial \mathcal{L}}{\partial x} = \frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial \mathcal{L}}{\partial \dot{x}}$$

Then turn the crank and see what EOM pops out. (Try it!)

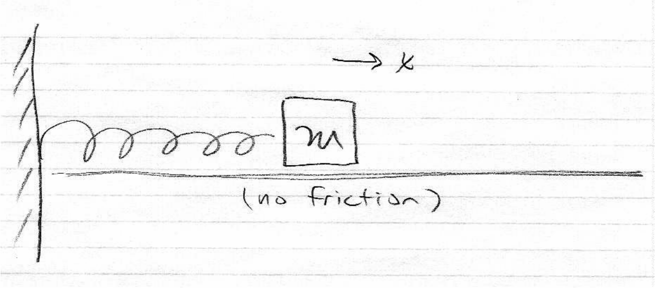


OK, how about a block of mass m moving horizontally on a frictionless table, under the influence of Hooke's-Law potential

$$U = \frac{1}{2}kx^2$$

so $x = 0$ when spring is at its equilibrium length.

Try writing down $\mathcal{L}$, then using $E - L$ equations to find EOM.



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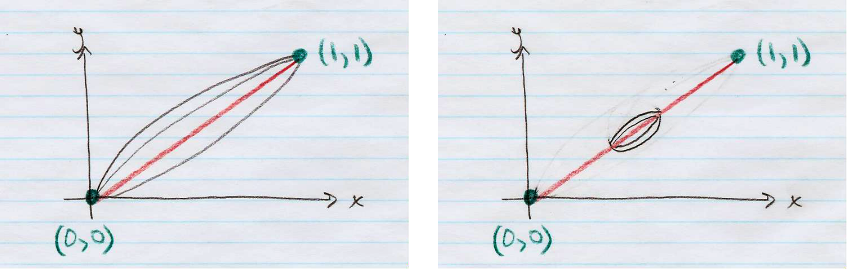
Try writing down $\mathcal{L}$, then using $E - L$ equations to find EOM.

What EOM do we get for a general 1D potential $U(x)$?

When you use the E-L equation to optimize $\int f(x, y, y')\,\mathrm{d}x$, you may be thinking globally, but the E-L equation is acting locally.



The $y(x)$ that makes path length $\int \sqrt{1 + y'^2}\,\mathrm{d}x$ optimal from $(0, 0)$ to $(1, 1)$ will also make the path length optimal from $(0.4, 0.4)$ to $(0.6, 0.6)$ or from $(0.49, 0.49)$ to $(0.51, 0.51)$.

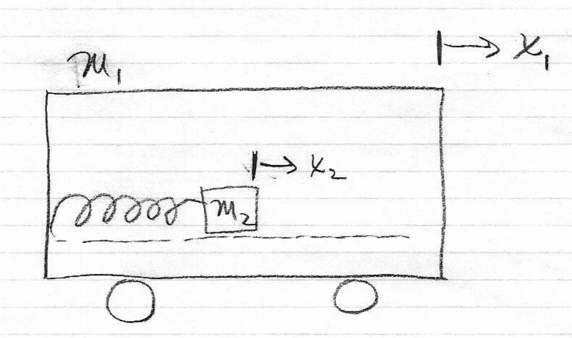
$$\frac{\partial f}{\partial y} = \frac{\mathrm{d}}{\mathrm{d}x} \frac{\partial f}{\partial y'}$$

is imposing a **local** requirement on $y(x)$. It's acting on each little segment of the curve separately.

Similarly,

$$\frac{\partial \mathcal{L}}{\partial x} = \frac{\mathrm{d}}{\mathrm{d}t} \frac{\partial \mathcal{L}}{\partial \dot{x}}$$

is enforcing $\vec{F} = m\vec{a}$ locally at each step in time. But the effect is that the overall path optimizes the action $S = \int \mathcal{L} \, \mathrm{d}t$.



A cart of mass m_1 rolls horizontally without friction. The cart's position is x_1. Inside the cart, a mass m_2 is attached to the wall of the cart with a spring (constant k). The position of m_2 w.r.t. the spring's relaxed position is x_2. So x_2 is w.r.t. the cart, not w.r.t. the ground. Write $\mathcal{L}(t, x_1, \dot{x}_1, x_2, \dot{x}_2)$.

$$L = \frac{1}{2} m_1 \dot{x}_1^2 + \frac{1}{2} m_2 (\dot{x}_1 + \dot{x}_2)^2 - \frac{1}{2} k x_2^2$$

$$0 = \frac{d}{dt} \frac{\partial L}{\partial \dot{x}_1} = \frac{d}{dt} \left[m_1 \dot{x}_1 + m_2 (\dot{x}_1 + \dot{x}_2) \right]$$

$$0 = (m_1 + m_2) \ddot{x}_1 + m_2 \ddot{x}_2$$

$$\frac{\partial L}{\partial x_2} = -k x_2 = \frac{d}{dt} \left[m_2 (\dot{x}_1 + \dot{x}_2) \right]$$

$$-k x_2 = m_2 \ddot{x}_1 + m_2 \ddot{x}_2$$

Physics 351 — Monday, January 29, 2018

- For today you read §7.1–7.7 of Ch 7 (Lagrange's equations). Next weekend you'll read the rest of Ch 7.
- You can do the Mathematica extra credit any time you like (if at all), but the earlier you do it, the more you'll be able to make use of Mathematica to reduce tedious algebra in your own homework. The "hands on start" chapters are a good tutorial. I found them both helpful and painless.

Before class: use $\partial f/\partial y = \frac{\mathrm{d}}{\mathrm{d}x}\,\partial f/\partial y'$ to find a function $y(x)$ that "extremizes" (i.e. makes stationary w.r.t. small variations in $y(x)$)

$$F[y] = \int_{x=0}^{x=1} \mathrm{d}x\ \left((y')^2 + 2ye^x\right)$$

subject to $y(0) = 0$ and $y(1) = 1$. So $f(x, y, y') = (y')^2 + 2ye^x$ in the Euler-Lagrange equation.