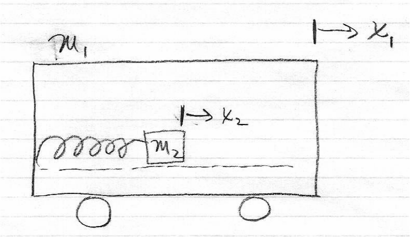
Physics 351 — Friday, February 2, 2018

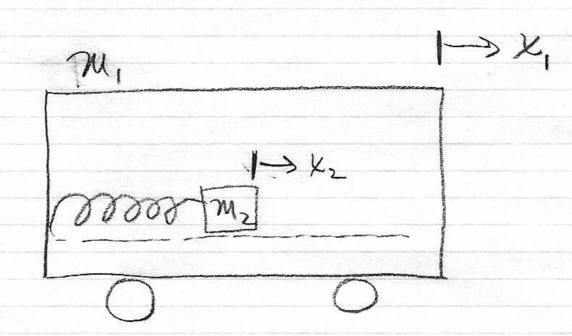
- Pick up HW3 handout. It's also online as a PDF.
- Turn in HW2. We prefer for you to write your name only on the back page of your homework, so that we can avoid knowing whose paper we're grading, until the end.
- This weekend you'll read the rest of Ch 7.



Before class: A cart of mass m_1 rolls horizontally without friction. The cart's position is x_1. Inside the cart, a mass m_2 is attached to the wall of the cart with a spring (constant k). The position of m_2 w.r.t. the spring's relaxed position is x_2. So x_2 is w.r.t. the cart, not w.r.t. the ground. Write $\mathcal{L}(t, x_1, \dot{x}_1, x_2, \dot{x}_2)$.

Reading question: “Does the Lagrangian method still work if one chooses generalized coordinates relative to a non-inertial reference frame? If so, is there some precaution one needs to take in writing down the Lagrangian?”

Yes: Lagrange’s equations are true for any choice of generalized coordinates, even if they are relative to a non-inertial frame. One just has to be careful to write the Lagrangian L=T-U in an inertial frame.



A cart of mass m_1 rolls horizontally without friction. The cart's position is x_1. Inside the cart, a mass m_2 is attached to the wall of the cart with a spring (constant k). The position of m_2 w.r.t. the spring's relaxed position is x_2. So x_2 is w.r.t. the cart, not w.r.t. the ground. Write $\mathcal{L}(t, x_1, \dot{x}_1, x_2, \dot{x}_2)$.

$$L = \tfrac{1}{2} m_1 \dot{x}_1^2 + \tfrac{1}{2} m_2 (\dot{x}_1 + \dot{x}_2)^2 - \tfrac{1}{2} k x_2^2$$

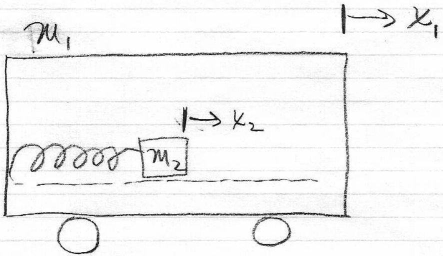
$$0 = \frac{d}{dt} \frac{\partial L}{\partial \dot{x}_1} = \frac{d}{dt}\left[m_1 \dot{x}_1 + m_2 (\dot{x}_1 + \dot{x}_2) \right]$$

$$0 = (m_1 + m_2) \ddot{x}_1 + m_2 \ddot{x}_2$$

$$\frac{\partial L}{\partial x_2} = -k x_2 = \frac{d}{dt}\left[m_2 (\dot{x}_1 + \dot{x}_2) \right]$$

$$-k x_2 = m_2 \ddot{x}_1 + m_2 \ddot{x}_2$$

By the way, notice that x_1 is an "ignorable" (a.k.a. "cyclic") coordinate, i.e. $\partial \mathcal{L} / \partial x_1 = 0$. The corresponding conserved quantity is the momentum of the CM, $m_1 \dot{x}_1 + m_2 (\dot{x}_1 + \dot{x}_2)$.

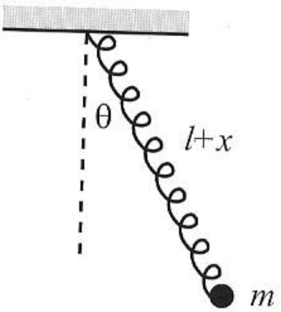


$$x_{cm} = \frac{m_1 x_1 + m_2(x_1 + x_2)}{m_1 + m_2}$$

$$\begin{aligned}(m_1 + m_2)\, x_{cm} &= m_1 x_1 + m_2(x_1 + x_2) \\ &= (m_1 + m_2)\, x_1 + m_2 x_2\end{aligned}$$

$$(m_1 + m_2)\, \ddot{x}_{cm} = (m_1 + m_2)\, \ddot{x}_1 + m_2 \ddot{x}_2 = 0$$

$$\Rightarrow \ddot{x}_{cm} = 0$$



Consider a pendulum made of a spring with a mass m on the end. The spring is arranged to lie in a straight line (e.g. by wrapping the spring around a massless rod). The equilibrium length of the spring is ℓ. Let the spring have length $\ell + x(t)$, and let its angle w.r.t. vertical be $\theta(t)$. Assuming the motion takes place in a vertical plane, write Lagrangian and find EOM for x and θ.

$$T = \frac{1}{2} m (l+x)^2 \dot{\theta}^2 + \frac{1}{2} m \dot{x}^2$$

$$U = \frac{1}{2} k x^2 - mg (x+l) \cos\theta$$

$$L = \frac{1}{2} m (l+x)^2 \dot{\theta}^2 + \frac{1}{2} m \dot{x}^2 + mg (x+l) \cos\theta - \frac{1}{2} k x^2$$

$$\frac{\partial L}{\partial x} = mg\cos\theta - kx + m(l+x)\dot{\theta}^2$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) = \frac{d}{dt}(m\dot{x}) = m\ddot{x} \Rightarrow \boxed{m\ddot{x} = mg\cos\theta - kx + m(l+x)\dot{\theta}^2}$$

$$\frac{\partial L}{\partial \theta} = -mg(x+l)\sin\theta$$

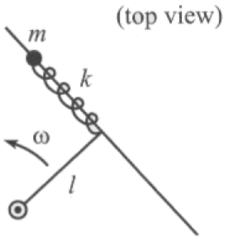
$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) = \frac{d}{dt}\left(m(l+x)^2\dot{\theta}\right) = m(l+x)^2\ddot{\theta} + 2m(l+x)\dot{x}\dot{\theta}$$

$$\Rightarrow m(l+x)^2\ddot{\theta} + 2m(l+x)\dot{x}\dot{\theta} = -mg(x+l)\sin\theta$$

$$\Rightarrow \boxed{(l+x)\ddot{\theta} = -g\sin\theta - 2\dot{x}\dot{\theta}}$$

(Notice "$\omega^2 r$" centripetal term and "$2\omega\dot{r}$" Coriolis term.)

8. A rigid T consists of a long rod glued perpendicular to another rod of length l that is pivoted at the origin. The T rotates around in a horizontal plane with constant frequency ω. A mass m is free to slide along the long rod and is connected to the intersection of the rods by a spring with spring constant k and relaxed length zero (see figure). Find $r(t)$, where r is the position of the mass along the long rod. There is a special value of ω. What is it, and why is it special?



[If you're feeling clever, you might want to solve Problem 9 first and then just set $g = 0$ to solve Problem 8.]

9. Consider the setup in Problem 8, but now let the T swing around in a vertical plane with constant frequency ω. Find $r(t)$. There is a special value of ω. What is it, and why is it special? (You may assume $\omega < \sqrt{k/m}$.) [You will find that whereas Problem 8 was a "free oscillation" problem, instead

Problem 9 is a "forced oscillation" problem. So you get to use what we learned in Chapter 5, after using the Lagrangian approach to write the equation of motion.]

Note: in problems like this, if you find a clever way to evaluate the kinetic energy, you can save yourself a huge amount of algebra.

```
ClearAll["Global`*"];
y[t_] := r[t] Cos[ω t] + l Sin[ω t];
x[t_] := l Cos[ω t] - r[t] Sin[ω t];
y'[t]
```

$$l\,\omega \operatorname{Cos}[t\,\omega] - \omega\, r[t] \operatorname{Sin}[t\,\omega] + \operatorname{Cos}[t\,\omega]\, r'[t]$$

```
x'[t]
```

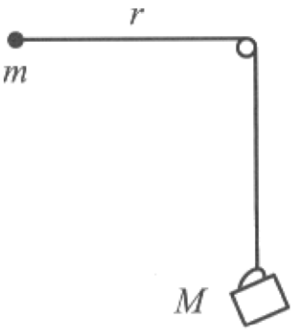
$$-\omega \operatorname{Cos}[t\,\omega]\, r[t] - l\,\omega \operatorname{Sin}[t\,\omega] - \operatorname{Sin}[t\,\omega]\, r'[t]$$

```
FullSimplify[x'[t]^2 + y'[t]^2]
```

$$\omega^2\, r[t]^2 + (l\,\omega + r'[t])^2$$

One problem from HW1 (Q10) illustrated an interesting idea that reappears this week on HW3 (Q10): work done against (or by) the centripetal force of an object in circular motion of changing radius.

A coffee cup of mass M is connected to a mass m by a string. The coffee cup hangs over a frictionless pulley of negligible size, and the mass m is initially held with the string horizontal, as shown in the figure. The mass m is then released. (a) Find the EOM for r (the length of string between m and the pulley) and θ (the angle that the string to m makes with the horizontal). Assume that m somehow doesn't run into the string holding the cup up. The coffee cup will initially fall, but it turns out that it will reach a lowest point and then rise back up. (b) Use Mathematica (or similar) to determine numerically the ratio of the r at this lowest point to the r at the start, as a function of the value of m/M. (To check your computation, a value of $m/M = 1/10$ yields a ratio of about 0.208.)

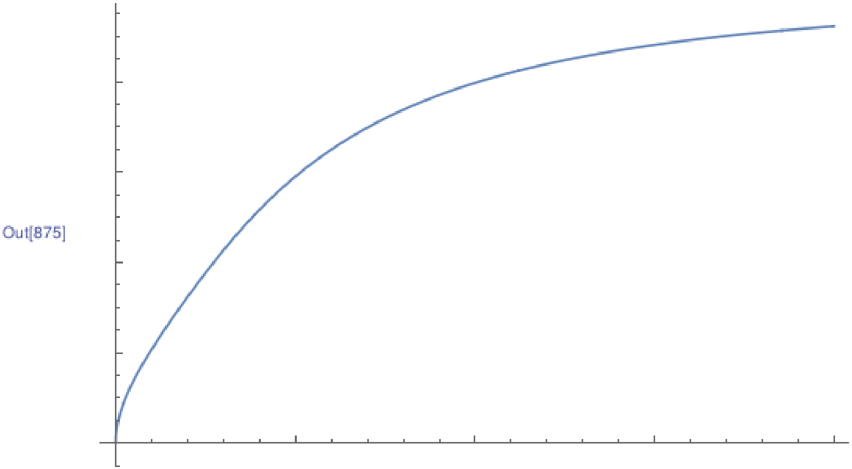


Crucial hint: the two coupled EOM can't be solved analytically. Use `NDSolveValue` then `FindMinimum` in Mathematica.

I defined $\mu = m/M$, let $r_0 = 1$, then let eq1 and eq2 be the EOM for $\ddot{r}$ and $\ddot{\theta}$ respectively, in terms of μ. Then `NDSolveValue` to numerically solve for $r(t)$ and $\theta(t)$, then `FindMinimum` (with a starting point of $t \approx 0.01$) to find r (which is same r/r_0, since $r_0 = 1$) at its turn-around point. It's also fun to graph $r(t)$.

```
ClearAll["global`*"];
mu = 0.1;
g = 9.8;
eq1 := (mu + 1) r''[t] == mu r[t] theta'[t]^2 +    (censored)    ;
eq2 :=  (censored)  + r[t] theta''[t] == g Cos[theta[t]];
Plot[
 rsoln =
  NDSolveValue[{eq1, eq2, theta[0] == 0, theta'[0] == 0, r[0] == 1, r'[0] == 0},
    {r[t], theta[t]}, {t, 0, 2}][[1]];
 rmin = FindMinimum[rsoln, {t, 0.01}][[1]],
 {mu, 0, 2}]
```

Here's my graph of r/r_0 (at turnaround point) vs. m/M (with axis scales censored).

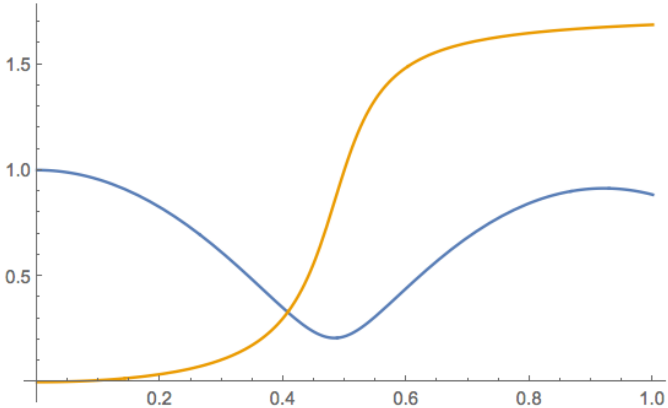


Checking that $r_{\mathrm{min}}/r_0 = 0.208$ for $m/M = 1/10$

```
ClearAll["global`*"];
mu = 0.1;
g = 9.8;
eq1 := (mu + 1) r''[t] == mu r[t] theta'[t]^2 + (censored) ;
eq2 := (censored) + r[t] theta''[t] == g Cos[theta[t]];
bothsolns =
 NDSolveValue[{eq1, eq2, theta[0] == 0, theta'[0] == 0, r[0] == 1, r'[0] == 0},
  {r[t], theta[t]}, {t, 0, 2}]

{InterpolatingFunction[ Domain: {{0., 2.}} Output: scalar ][t],
 InterpolatingFunction[ Domain: {{0., 2.}} Output: scalar ][t]}

rsoln = bothsolns[[1]]

InterpolatingFunction[ Domain: {{0., 2.}} Output: scalar ][t]

FindMinimum[rsoln, {t, 0.01}]

{0.207629, {t → 0.483041}}
```

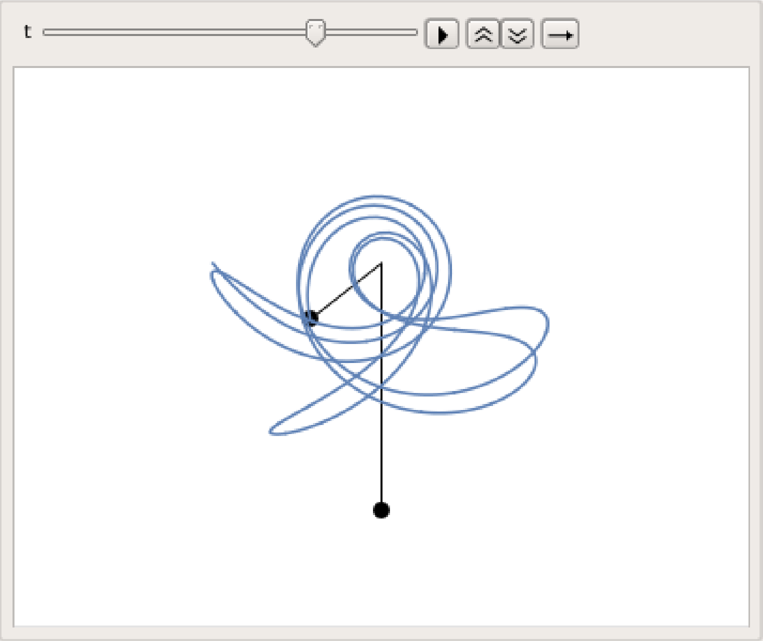
Graphing $r(t)$ and $\frac{1}{2\pi}\theta(t)$ for the $m/M = 1/10$ case

```
bothsolns =
  NDSolveValue[{eq1, eq2, theta[0] == 0, theta'[0] == 0,
    r[0] == 1, r'[0] == 0}, {r[t], theta[t]}, {t, 0, 2}];
rsoln = bothsolns[[1]];
thetasoln = bothsolns[[2]];
Plot[{rsoln, thetasoln / (2 Pi)}, {t, 0, 1}]
```



Non-linear behavior is evident at large amplitude!!

Out[10]=



Here was HW1/q10, containing a similar idea:

A particle of mass m is moving on a frictionless horizontal table and is attached to a massless string, whose other end passes through a hole in the table, where I am holding it. (a) Initially the particle is moving in a circle of radius r_0 with angular velocity ω_0, but I now pull the string down through the hole until a length r remains between the hole and the particle. What is the particle's angular velocity now? (b) Now let's see what happens during the pull described in part (a). Initially the particle is moving in a circle of radius r_0 with angular velocity ω_0. Starting at $t = 0$, I pull the string with constant velocity v so that the radial distance (r) to the mass decreases. Draw a force diagram for the mass and find a differential equation for $\omega(t)$. Find $\omega(t)$ and also find the force $F(t)$ that I need to exert on the string. [Hint: one component of the force exerted on m by the string is always zero.]

(a) $L_{final} = L_{initial}$

$I_f \omega_f = I_i \omega_i \Rightarrow m r^2 \omega = m r_0^2 \omega_0$

$\Rightarrow \omega = \dfrac{\omega_0 r_0^2}{r^2}$

"force diagram"

(b) $r(t) = r_0 - vt$

Since angular momentum L is constant, I wrote

$$0 = \frac{dL}{dt} = \frac{d}{dt}(mr^2\omega)$$

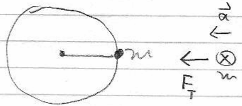
$$\Rightarrow 2r\dot{r}\omega + r^2\dot{\omega} = 0$$

$$\frac{\dot{\omega}}{\omega} = -\frac{2\dot{r}}{r}$$

$$\log\left(\frac{\omega}{\omega_0}\right) = -2\log\left(\frac{r}{r_0}\right)$$

$$\frac{\omega}{\omega_0} = \left(\frac{r_0}{r}\right)^2$$

$$\frac{\omega}{\omega_0} = \left(\frac{r_0}{r_0 - vt}\right)^2$$



The only force is tension, which acts radially.

$F_r = \cancel{m\ddot{r}} - m\omega^2 r$
0

$F_T = m\omega^2 r$ (tension)

$F_\phi = 0$

Same equation we got in part (a)

(We're not going to go through this again! But here it is.)

The force I exert on string equals tension, which equals centripetal force $m\omega^2 r$.

$$F_T = m\omega^2 r = m\left[\omega_0\left(\frac{r_0}{r}\right)^2\right]^2 r = \frac{m\omega_0^2 r_0^4}{r^3} = \frac{m\omega_0^2 r_0^4}{(r_0-vt)^3}$$

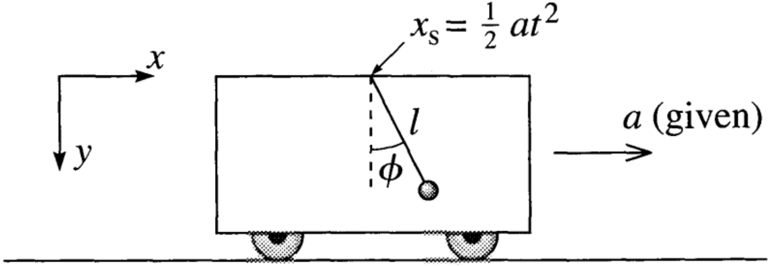
check: $K.E. = \frac{1}{2} I\omega^2 = \frac{1}{2}(mr^2)\left[\omega_0\left(\frac{r_0}{r}\right)^2\right]^2$

$$= \frac{m\omega_0^2 r_0^4}{2r^2} \qquad -F_T = \frac{d(\text{work})}{dr} = \frac{d(K.E.)}{dr} = -\frac{m\omega_0^2 r_0^4}{r^3} \quad \checkmark$$

(You can look at this if you're interested, but we're not going to go through it. I just thought it was interesting to notice that the change in K.E. of the mass-on-string equals the work done by whatever force is pulling the string beneath the table.) The relevance for the coffee-cup problem is that as the mass-on-string gains angular velocity, the tension in the string increases.

I have a fun mechanical demonstration of the coffee-cup problem (using pulleys), we could probably do Monday!

This problem will reappear in the text of Taylor's Ch9 ("mechanics in non-inertial frames"), so let's work through it by writing the Lagrangian w.r.t. an inertial frame.



(7.30) A pendulum is suspended inside a railroad car that is forced to accelerate at constant acceleration a.

(a) Write down $\mathcal{L}$ and find EOM for ϕ.

(b) Let $\tan\beta \equiv a/g$, so $g = \sqrt{g^2+a^2}\cos\beta$, $a = \sqrt{g^2+a^2}\sin\beta$. Simplify using $\sin(\phi+\beta) = \cos\beta\sin\phi + \sin\beta\cos\phi$.

(c) Find equilibrium angle ϕ_0. Use EOM to show $\phi = \phi_0$ is stable. Find frequency of small oscillations about ϕ_0.

$$x_m = \frac{1}{2}at^2 + l\sin\phi \qquad y_m = l\cos\phi \qquad U = -mgy_m = -mgl\cos\phi$$

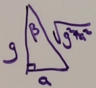
$$\dot{x}_m = at + l\dot\phi\cos\phi \qquad \dot{y}_m = -l\dot\phi\sin\phi$$

$$\dot{x}_m^2 + \dot{y}_m^2 = (at)^2 + (l\dot\phi)^2(\cos^2\phi + \sin^2\phi) + 2atl\dot\phi\cos\phi$$

$$\mathcal{L} = \frac{1}{2}ma^2t^2 + \frac{1}{2}ml^2\dot\phi^2 + matl\dot\phi\cos\phi + mgl\cos\phi$$

$$\frac{\partial\mathcal{L}}{\partial\phi} = -(\cancel{mat\,l\dot\phi} + mgl)\sin\phi = \frac{d}{dt}\left(\frac{\partial\mathcal{L}}{\partial\dot\phi}\right) = \frac{d}{dt}\left(ml^2\dot\phi + matl\cos\phi\right) = ml^2\ddot\phi + mal\cos\phi - \cancel{matl\dot\phi}\sin\phi$$

$$ml^2\ddot\phi + mal\cos\phi + mgl\sin\phi = 0 \Rightarrow \ddot\phi + \frac{a}{l}\cos\phi + \frac{g}{l}\sin\phi = 0$$



$$g = (\cos\beta)\sqrt{g^2+a^2}$$
$$a = (\sin\beta)\sqrt{g^2+a^2}$$

$$\ddot\phi + \frac{(\sqrt{g^2+a^2})}{l}(\sin\beta\cos\phi + \cos\beta\sin\phi) = 0$$

$$\ddot\phi + \frac{\sqrt{g^2+a^2}}{l}\sin(\phi+\beta) = 0$$

let $\varepsilon = \phi + \beta \Rightarrow \ddot\varepsilon = \ddot\phi \Rightarrow \ddot\varepsilon + \frac{\sqrt{g^2+a^2}}{l}\sin\varepsilon = 0$

for $\varepsilon \ll 1$ we get $\ddot\varepsilon = -\frac{\sqrt{g^2+a^2}}{l}\varepsilon$

$$x_m = \frac{1}{2}at^2 + l\sin\phi \qquad y_m = l(1-\cos\phi)$$

$$\dot{x}_m = at + l\cos\phi\,\dot{\phi} \qquad \dot{y}_m = l\sin\phi\,\dot{\phi}$$

$$\dot{x}_m^2 + \dot{y}_m^2 = a^2t^2 + l^2\cos^2\phi\,\dot{\phi}^2 + 2atl\cos\phi\,\dot{\phi} + l^2\sin^2\phi\,\dot{\phi}^2$$

$$= a^2t^2 + l^2\dot{\phi}^2 + 2atl\cos\phi\,\dot{\phi}$$

$$\boxed{U = mgl(1-\cos\phi)}$$

$$T = \frac{1}{2}m\left(a^2t^2 + l^2\dot{\phi}^2\right) + matl\cos\phi\,\dot{\phi}$$

$$L = \frac{m}{2}\left(a^2t^2 + l^2\dot{\phi}^2\right) + matl\cos\phi\,\dot{\phi} + mgl(\cos\phi - 1)$$

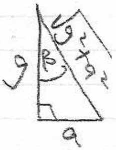
$$\frac{\partial L}{\partial \phi} = -matl\sin\phi\,\dot{\phi} - mgl\sin\phi$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\phi}}\right) = \frac{d}{dt}\left(ml^2\dot{\phi} + matl\cos\phi\right) = ml^2\ddot{\phi} + mal\cos\phi - matl\sin\phi\,\dot{\phi}$$

$$ml^2\ddot{\phi} + mal\cos\phi = -mgl\sin\phi \implies l\ddot{\phi} = -g\sin\phi - a\cos\phi$$

$$ml^2\ddot{\phi} + mal\cos\phi = -mgl\sin\phi \Rightarrow l\ddot{\phi} = -g\sin\phi - a\cos\phi$$

$$l\ddot{\phi} = -\sqrt{g^2+a^2}\left(\frac{g}{\sqrt{g^2+a^2}}\sin\phi + \frac{a}{\sqrt{g^2+a^2}}\cos\phi\right)$$



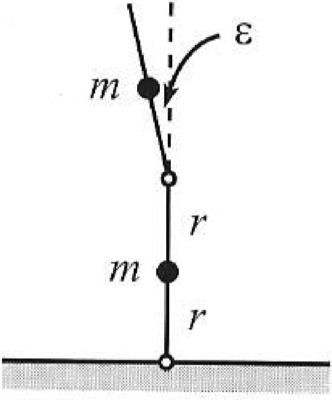
$$= -\sqrt{g^2+a^2}\left(\cos\beta\sin\phi + \sin\beta\cos\phi\right)$$

$$l\ddot{\phi} = -\sqrt{g^2+a^2}\sin(\phi+\beta)$$

let $\varepsilon = \phi - (-\beta) = \phi - \phi_0$

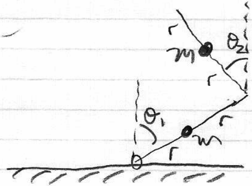
$$\ddot{\varepsilon} = \ddot{\phi} \qquad \ddot{\varepsilon} = -\frac{\sqrt{g^2+a^2}}{l}\sin\varepsilon$$

$$\rightarrow \omega_0 = \sqrt{\frac{\sqrt{g^2+a^2}}{l}}$$



HW3 XC7 is the "three sticks" generalization of this problem. Let's try the "two sticks" version.

Two massless sticks of length $2r$, each with a mass m fixed at its middle, are hinged at an end. One stands on top of the other. The bottom end of the lower stick is hinged on the ground. They are held such that the lower stick is vertical, and the upper one is tilted at a small angle ε w.r.t. vertical. They are then released. At the instant after release, what are the angular accelerations of the two sticks? Work in the approximation where $\varepsilon \ll 1$.



$$y_1 = r\cos\theta_1$$

$$y_2 = 2r\cos\theta_1 + r\cos\theta_2$$

$$U = mg(y_1+y_2) = mgr\left(3\cos\theta_1 + \cos\theta_2\right)$$

$$x_1 = r\sin\theta_1 \rightarrow \dot{x}_1 = r\cos\theta_1\,\dot{\theta}_1$$

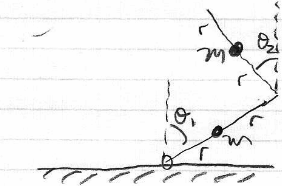
$$x_2 = 2r\sin\theta_1 - r\sin\theta_2 \rightarrow \dot{x}_2 = 2r\cos\theta_1\,\dot{\theta}_1 - r\cos\theta_2\,\dot{\theta}_2$$

$$\dot{y}_1 = -r\sin\theta_1\,\dot{\theta}_1 \qquad \dot{y}_2 = -2r\sin\theta_1\,\dot{\theta}_1 - r\sin\theta_2\,\dot{\theta}_2$$

$$T = \frac{m}{2}\left(\dot{x}_1^2 + \dot{y}_1^2 + \dot{x}_2^2 + \dot{y}_2^2\right) =$$

$$= \frac{mr^2}{2}\Big[\cos^2\theta_1\,\dot{\theta}_1^2 + \sin^2\theta_1\,\dot{\theta}_1^2 + 4\cos^2\theta_1\,\dot{\theta}_1^2 + \cos^2\theta_2\,\dot{\theta}_2^2 - 4\cos\theta_1\cos\theta_2\,\dot{\theta}_1\dot{\theta}_2$$
$$+ 4\sin^2\theta_1\,\dot{\theta}_1^2 + \sin^2\theta_2\,\dot{\theta}_2^2 + 4\sin\theta_1\sin\theta_2\,\dot{\theta}_1\dot{\theta}_2\Big]$$

$$T = \frac{mr^2}{2}\left[5\dot{\theta}_1^2 + \dot{\theta}_2^2 - 4\dot{\theta}_1\dot{\theta}_2\cos(\theta_1+\theta_2)\right] \simeq \frac{mr^2}{2}\left[5\dot{\theta}_1^2 + \dot{\theta}_2^2 - 4\dot{\theta}_1\dot{\theta}_2\left(1 - \frac{(\theta_1+\theta_2)^2}{2}\right)\right]$$



$$y_1 = r\cos\theta_1$$

$$y_2 = 2r\cos\theta_1 + r\cos\theta_2$$

$$U = mg(y_1 + y_2) = mgr(3\cos\theta_1 + \cos\theta_2)$$

$$x_1 = r\sin\theta_1 \rightarrow \dot{x}_1 = r\cos\theta_1\,\dot{\theta}_1$$

$$x_2 = 2r\sin\theta_1 - r\sin\theta_2 \rightarrow \dot{x}_2 = 2r\cos\theta_1\,\dot{\theta}_1 - r\cos\theta_2\,\theta_2$$

$$\dot{y}_1 = -r\sin\theta_1\,\dot{\theta}_1 \qquad \dot{y}_2 = -2r\sin\theta_1\,\dot{\theta}_1 - r\sin\theta_2\,\dot{\theta}_2$$

$$U = mgr(3\cos\theta_1 + \cos\theta_2) \simeq mgr\left(3 - \frac{3\theta_1^2}{2} + 1 - \frac{\theta_2^2}{2}\right)$$

$$\mathcal{L} = T - U = \frac{mr^2}{2}\left(5\dot{\theta}_1^2 + \dot{\theta}_2^2 - 4\dot{\theta}_1\dot{\theta}_2\cos(\theta_1 + \theta_2)\right) - mgr(3\cos\theta_1 + \cos\theta_2)$$

$$\simeq \frac{mr^2}{2}\left(5\dot{\theta}_1^2 + \dot{\theta}_2^2 - 4\dot{\theta}_1\dot{\theta}_2\right) + mgr\left(\frac{3\theta_1^2}{2} + \frac{\theta_2^2}{2} - 4\right)$$

$$L \simeq \frac{mr^2}{2}\left(5\dot\theta_1^2 + \dot\theta_2^2 - 4\dot\theta_1\dot\theta_2\right) + mgr\left(\frac{3\theta_1^2}{2} + \frac{\theta_2^2}{2} - 4\right)$$

$$\frac{\partial L}{\partial \theta_1} = 3mgr\theta_1 \qquad \frac{\partial L}{\partial \theta_2} = mgr\theta_2$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot\theta_1}\right) = \frac{d}{dt}\left(\frac{mr^2}{2}\left(10\dot\theta_1 - 4\dot\theta_2\right)\right) = mr^2\left(5\ddot\theta_1 - 2\ddot\theta_2\right)$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot\theta_2}\right) = \frac{d}{dt}\left(\frac{mr^2}{2}\left(2\dot\theta_2 - 4\dot\theta_1\right)\right) = mr^2\left(\ddot\theta_2 - 2\ddot\theta_1\right)$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot\theta_1} = \frac{\partial L}{\partial \theta_1} \Rightarrow \quad 5\ddot\theta_1 - 2\ddot\theta_2 = \frac{3g\theta_1}{r}$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot\theta_2} = \frac{\partial L}{\partial \theta_2} \Rightarrow \quad \ddot\theta_2 - 2\ddot\theta_1 = \frac{g\theta_2}{r}$$

Now plug in, at $t = 0$, given conditions $\theta_1 = 0$, $\theta_2 = \varepsilon$, and find initial angular accelerations $\ddot\theta_1$ and $\ddot\theta_2$.

$$\frac{d}{dt}\frac{\partial L}{\partial \dot\theta_1} = \frac{\partial L}{\partial \theta_1} \Rightarrow \boxed{5\ddot\theta_1 - 2\ddot\theta_2 = \frac{3g\theta_1}{r}}$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot\theta_2} = \frac{\partial L}{\partial \theta_2} \Rightarrow \boxed{\ddot\theta_2 - 2\ddot\theta_1 = \frac{g\theta_2}{r}}$$

at $\boxed{t=0,}$ plug in $\theta_1 = 0$, $\theta_2 = \varepsilon$

$$5\ddot\theta_1 = 2\ddot\theta_2 \qquad \ddot\theta_2 - 2\left(\frac{2}{5}\ddot\theta_2\right) = \frac{g\varepsilon}{r} = \frac{\ddot\theta_2}{5}$$

$$\rightarrow \ddot\theta_1 = \frac{2}{5}\ddot\theta_2 \qquad \rightarrow \boxed{\text{at } t=0,\ \ddot\theta_2 = 5g\varepsilon/r, \quad \ddot\theta_1 = 2g\varepsilon/r}$$

Physics 351 — Friday, February 2, 2018

- Pick up HW3 handout. It's also online as a PDF.
- Turn in HW2. We prefer for you to write your name only on the back page of your homework, so that we can avoid knowing whose paper we're grading, until the end.
- This weekend you'll read the rest of Ch 7.