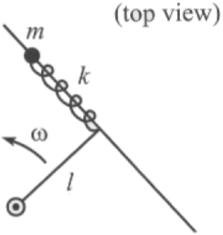
Physics 351 — Monday, February 5, 2018

- HW3 due Friday. You finished reading ch7 for today.
- Tomorrow, Feb 6, undergrad groups WiP and SPS can meet with Musk Public Lecture speaker, Dr. Jim Gates, for an informal lunch: Tue 1:30pm–2:30pm in DRL 4N12.
- Public lecture Tue 5pm, Meyerson B1: "Will Evolution and Information Theory Provide the Fundamentals of Physics?"
- Gates was awarded the National Medal of Science from President Barack Obama in 2013. His work has produced first introductions of concepts in physics including the complete extension of Einstein's geometrical concepts to the theory of supergravity, genuine four-dimensional string theory based on Standard Model concepts, maximal SUSY Chern-Simons theories, and more. His co-authored book, *Superspace, or One Thousand and One Lessons in Supersymmetry,* is the first published comprehensive book on supersymmetry.
- Dr. Sylvester James Gates, Jr., is currently Ford Foundation Professor of Physics at Brown University.

8. A rigid T consists of a long rod glued perpendicular to another rod of length l that is pivoted at the origin. The T rotates around in a horizontal plane with constant frequency ω. A mass m is free to slide along the long rod and is connected to the intersection of the rods by a spring with spring constant k and relaxed length zero (see figure). Find $r(t)$, where r is the position of the mass along the long rod. There is a special value of ω. What is it, and why is it special?



[If you're feeling clever, you might want to solve Problem 9 first and then just set $g = 0$ to solve Problem 8.]

9. Consider the setup in Problem 8, but now let the T swing around in a vertical plane with constant frequency ω. Find $r(t)$. There is a special value of ω. What is it, and why is it special? (You may assume $\omega < \sqrt{k/m}$.) [You will find that whereas Problem 8 was a "free oscillation" problem, instead

Problem 9 is a "forced oscillation" problem. So you get to use what we learned in Chapter 5, after using the Lagrangian approach to write the equation of motion.]

Note: in problems like this, if you find a clever way to evaluate the kinetic energy, you can save yourself a huge amount of algebra.

```
ClearAll["Global`*"];
y[t_] := r[t] Cos[ω t] + l Sin[ω t];
x[t_] := l Cos[ω t] - r[t] Sin[ω t];
y'[t]
```

$$l\,\omega \operatorname{Cos}[t\,\omega] - \omega\, r[t] \operatorname{Sin}[t\,\omega] + \operatorname{Cos}[t\,\omega]\, r'[t]$$

```
x'[t]
```

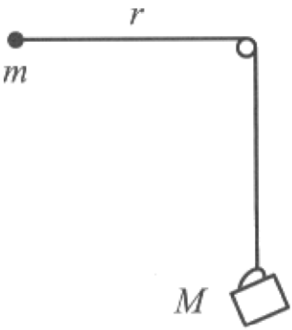
$$-\omega \operatorname{Cos}[t\,\omega]\, r[t] - l\,\omega \operatorname{Sin}[t\,\omega] - \operatorname{Sin}[t\,\omega]\, r'[t]$$

```
FullSimplify[x'[t]^2 + y'[t]^2]
```

$$\omega^2 r[t]^2 + (l\,\omega + r'[t])^2$$

One problem from HW1 (Q10) illustrated an interesting idea that reappears this week on HW3 (Q10): work done against (or by) the centripetal force of an object in circular motion of changing radius.

A coffee cup of mass M is connected to a mass m by a string. The coffee cup hangs over a frictionless pulley of negligible size, and the mass m is initially held with the string horizontal, as shown in the figure. The mass m is then released. (a) Find the EOM for r (the length of string between m and the pulley) and θ (the angle that the string to m makes with the horizontal). Assume that m somehow doesn't run into the string holding the cup up. The coffee cup will initially fall, but it turns out that it will reach a lowest point and then rise back up. (b) Use Mathematica (or similar) to determine numerically the ratio of the r at this lowest point to the r at the start, as a function of the value of m/M. (To check your computation, a value of $m/M = 1/10$ yields a ratio of about 0.208.)

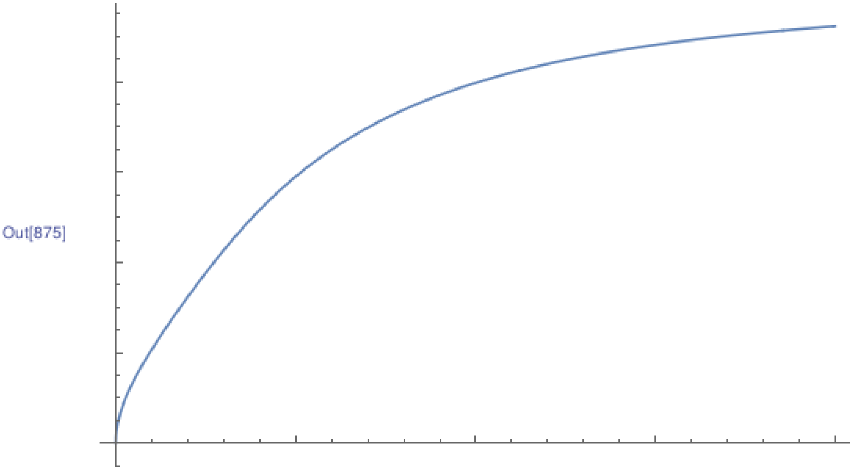


Crucial hint: the two coupled EOM can't be solved analytically. Use `NDSolveValue` then `FindMinimum` in Mathematica.

I defined $\mu = m/M$, let $r_0 = 1$, then let eq1 and eq2 be the EOM for $\ddot{r}$ and $\ddot{\theta}$ respectively, in terms of μ. Then `NDSolveValue` to numerically solve for $r(t)$ and $\theta(t)$, then `FindMinimum` (with a starting point of $t \approx 0.01$) to find r (which is same r/r_0, since $r_0 = 1$) at its turn-around point. It's also fun to graph $r(t)$.

```
ClearAll["global`*"];
mu = 0.1;
g = 9.8;
eq1 := (mu + 1) r''[t] == mu r[t] theta'[t]^2 +    (censored)    ;
eq2 :=  (censored)  + r[t] theta''[t] == g Cos[theta[t]];
Plot[
 rsoln =
  NDSolveValue[{eq1, eq2, theta[0] == 0, theta'[0] == 0, r[0] == 1, r'[0] == 0},
    {r[t], theta[t]}, {t, 0, 2}][[1]];
 rmin = FindMinimum[rsoln, {t, 0.01}][[1]],
 {mu, 0, 2}]
```

Here's my graph of r/r_0 (at turnaround point) vs. m/M (with axis scales censored).



Checking that $r_{\min}/r_0 = 0.208$ for $m/M = 1/10$

```
ClearAll["global`*"];
mu = 0.1;
g = 9.8;
eq1 := (mu + 1) r''[t] == mu r[t] theta'[t]^2 + (censored) ;
eq2 := (censored) + r[t] theta''[t] == g Cos[theta[t]];
bothsolns =
 NDSolveValue[{eq1, eq2, theta[0] == 0, theta'[0] == 0, r[0] == 1, r'[0] == 0},
  {r[t], theta[t]}, {t, 0, 2}]
```

```
{InterpolatingFunction[ Domain: {{0., 2.}} Output: scalar ][t],

 InterpolatingFunction[ Domain: {{0., 2.}} Output: scalar ][t]}
```

```
rsoln = bothsolns[[1]]
```

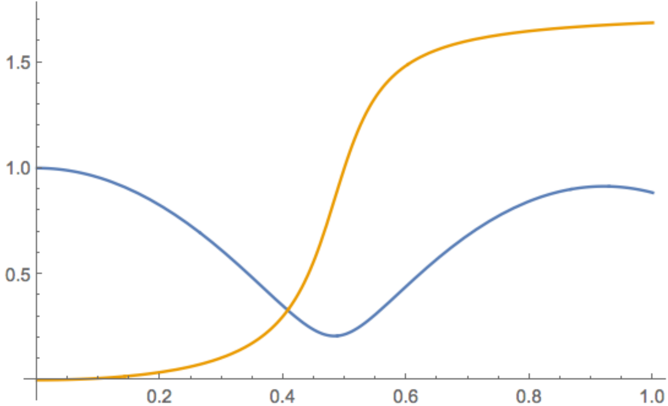
```
InterpolatingFunction[ Domain: {{0., 2.}} Output: scalar ][t]
```

```
FindMinimum[rsoln, {t, 0.01}]
```

```
{0.207629, {t → 0.483041}}
```

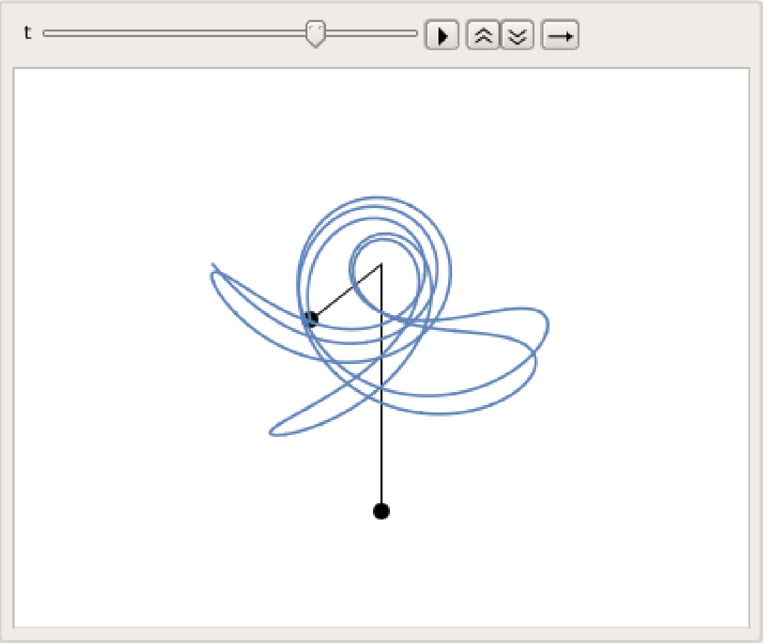
Graphing $r(t)$ and $\frac{1}{2\pi}\theta(t)$ for the $m/M = 1/10$ case

```
bothsolns =
  NDSolveValue[{eq1, eq2, theta[0] == 0, theta'[0] == 0,
    r[0] == 1, r'[0] == 0}, {r[t], theta[t]}, {t, 0, 2}];
rsoln = bothsolns[[1]];
thetasoln = bothsolns[[2]];
Plot[{rsoln, thetasoln / (2 Pi)}, {t, 0, 1}]
```



Non-linear behavior is evident at large amplitude!!

Out[10]=



Here was HW1/q10, containing a similar idea:

A particle of mass m is moving on a frictionless horizontal table and is attached to a massless string, whose other end passes through a hole in the table, where I am holding it. (a) Initially the particle is moving in a circle of radius r_0 with angular velocity ω_0, but I now pull the string down through the hole until a length r remains between the hole and the particle. What is the particle's angular velocity now? (b) Now let's see what happens during the pull described in part (a). Initially the particle is moving in a circle of radius r_0 with angular velocity ω_0. Starting at $t = 0$, I pull the string with constant velocity v so that the radial distance (r) to the mass decreases. Draw a force diagram for the mass and find a differential equation for $\omega(t)$. Find $\omega(t)$ and also find the force $F(t)$ that I need to exert on the string. [Hint: one component of the force exerted on m by the string is always zero.]

(a) $L_{\text{final}} = L_{\text{initial}}$

$$I_f \omega_f = I_i \omega_i \Rightarrow m r^2 \omega = m r_0^2 \omega_0$$

$$\Rightarrow \omega = \frac{\omega_0 r_0^2}{r^2}$$

"force diagr..."

(b) $r(t) = r_0 - vt$

Since angular momentum L is constant, I wrote

$$0 = \frac{dL}{dt} = \frac{d}{dt}\left(mr^2\omega\right)$$

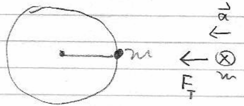
$$\Rightarrow 2r\dot{r}\omega + r^2\dot{\omega} = 0$$

$$\frac{\dot{\omega}}{\omega} = -\frac{2\dot{r}}{r}$$

$$\log\left(\frac{\omega}{\omega_0}\right) = -2\log\left(\frac{r}{r_0}\right)$$

$$\frac{\omega}{\omega_0} = \left(\frac{r_0}{r}\right)^2$$

$$\frac{\omega}{\omega_0} = \left(\frac{r_0}{r_0 - vt}\right)^2$$



The only force is tension, which acts radially.

$$F_r = \underbrace{\cancel{m\ddot{r}}}_{0} - m\omega^2 r$$

$$F_T = m\omega^2 r$$
(tension)

$$F_\phi = 0$$

Same equation we got in part (a)

(We're not going to go through this again! But here it is.)

The force I exert on string equals tension, which equals centripetal force $m\omega^2 r$.

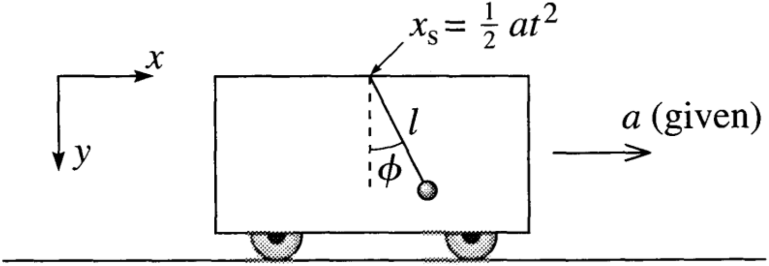
$$F_T = m\omega^2 r = m\left[\omega_0\left(\frac{r_0}{r}\right)^2\right]^2 r = \frac{m\omega_0^2 r_0^4}{r^3} = \frac{m\omega_0^2 r_0^4}{(r_0 - vt)^3}$$

check: $K.E. = \frac{1}{2} I\omega^2 = \frac{1}{2}(mr^2)\left[\omega_0\left(\frac{r_0}{r}\right)^2\right]^2$

$$= \frac{m\omega_0^2 r_0^4}{2r^2} \qquad -F_T = \frac{d(\text{work})}{dr} = \frac{d(K.E.)}{dr} = -\frac{m\omega_0^2 r_0^4}{r^3} \quad \checkmark$$

(You can look at this if you're interested, but we're not going to go through it. I just thought it was interesting to notice that the change in K.E. of the mass-on-string equals the work done by whatever force is pulling the string beneath the table.) The relevance for the coffee-cup problem is that as the mass-on-string gains angular velocity, the tension in the string increases.

This problem will reappear in the text of Taylor's Ch9 ("mechanics in non-inertial frames"), so let's work through it by writing the Lagrangian w.r.t. an inertial frame.



(7.30) A pendulum is suspended inside a railroad car that is forced to accelerate at constant acceleration a.

(a) Write down $\mathcal{L}$ and find EOM for ϕ.

(b) Let $\tan\beta \equiv a/g$, so $g = \sqrt{g^2+a^2}\cos\beta$, $a = \sqrt{g^2+a^2}\sin\beta$. Simplify using $\sin(\phi+\beta) = \cos\beta\sin\phi + \sin\beta\cos\phi$.

(c) Find equilibrium angle ϕ_0. Use EOM to show $\phi = \phi_0$ is stable. Find frequency of small oscillations about ϕ_0.

$$U = -mgy_m = -mgl\cos\phi$$

$$x_m = \frac{1}{2}at^2 + l\sin\phi \qquad y_m = l\cos\phi$$

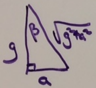
$$\dot{x}_m = at + l\dot{\phi}\cos\phi \qquad \dot{y}_m = -l\dot{\phi}\sin\phi$$

$$\dot{x}_m^2 + \dot{y}_m^2 = (at)^2 + (l\dot{\phi})^2(\cos^2\phi + \sin^2\phi) + 2atl\dot{\phi}\cos\phi$$

$$\mathcal{L} = \frac{1}{2}ma^2t^2 + \frac{1}{2}ml^2\dot{\phi}^2 + matl\dot{\phi}\cos\phi + mgl\cos\phi$$

$$\frac{\partial \mathcal{L}}{\partial \phi} = -(\cancel{matl\dot{\phi}} + mgl)\sin\phi = \frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{\phi}}\right) = \frac{d}{dt}\left(ml^2\dot{\phi} + matl\cos\phi\right) = ml^2\ddot{\phi} + mal\cos\phi - \cancel{matl\dot{\phi}}\sin\phi$$

$$ml^2\ddot{\phi} + mal\cos\phi + mgl\sin\phi = 0 \Rightarrow \ddot{\phi} + \frac{a}{l}\cos\phi + \frac{g}{l}\sin\phi = 0$$



$$g = (\cos\beta)\sqrt{g^2+a^2}$$
$$a = (\sin\beta)\sqrt{g^2+a^2}$$

$$\ddot{\phi} + \left(\frac{\sqrt{g^2+a^2}}{l}\right)\left(\sin\beta\cos\phi + \cos\beta\sin\phi\right) = 0$$

$$\ddot{\phi} + \frac{\sqrt{g^2+a^2}}{l}\sin(\phi+\beta) = 0$$

let $\varepsilon = \phi + \beta \Rightarrow \ddot{\varepsilon} = \ddot{\phi} \Rightarrow \ddot{\varepsilon} + \frac{\sqrt{g^2+a^2}}{l}\sin\varepsilon = 0$

for $\varepsilon \ll 1$ we get $\ddot{\varepsilon} = -\frac{\sqrt{g^2+a^2}}{l}\varepsilon$

$$x_m = \frac{1}{2}at^2 + l\sin\phi \qquad y_m = l(1-\cos\phi)$$

$$\dot{x}_m = at + l\cos\phi\,\dot{\phi} \qquad \dot{y}_m = l\sin\phi\,\dot{\phi}$$

$$\dot{x}_m^2 + \dot{y}_m^2 = a^2t^2 + l^2\cos^2\phi\,\dot{\phi}^2 + 2atl\cos\phi\,\dot{\phi} + l^2\sin^2\phi\,\dot{\phi}^2$$

$$= a^2t^2 + l^2\dot{\phi}^2 + 2atl\cos\phi\,\dot{\phi}$$

$$\boxed{U = mgl(1-\cos\phi)}$$

$$T = \frac{1}{2}m\left(a^2t^2 + l^2\dot{\phi}^2\right) + matl\cos\phi\,\dot{\phi}$$

$$L = \frac{m}{2}\left(a^2t^2 + l^2\dot{\phi}^2\right) + matl\cos\phi\,\dot{\phi} + mgl(\cos\phi - 1)$$

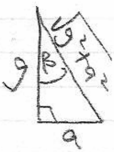
$$\frac{\partial L}{\partial \phi} = -matl\sin\phi\,\dot{\phi} - mgl\sin\phi$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\phi}}\right) = \frac{d}{dt}\left(ml^2\dot{\phi} + matl\cos\phi\right) = ml^2\ddot{\phi} + mal\cos\phi - matl\sin\phi\,\dot{\phi}$$

$$ml^2\ddot{\phi} + mal\cos\phi = -mgl\sin\phi \Rightarrow l\ddot{\phi} = -g\sin\phi - a\cos\phi$$

$$ml^2\ddot{\phi} + mal\cos\phi = -mgl\sin\phi \Rightarrow l\ddot{\phi} = -g\sin\phi - a\cos\phi$$

$$l\ddot{\phi} = -\sqrt{g^2+a^2}\left(\frac{g}{\sqrt{g^2+a^2}}\sin\phi + \frac{a}{\sqrt{g^2+a^2}}\cos\phi\right)$$



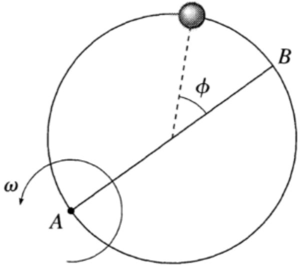
$$= -\sqrt{g^2+a^2}\left(\cos\beta\sin\phi + \sin\beta\cos\phi\right)$$

$$l\ddot{\phi} = -\sqrt{g^2+a^2}\sin(\phi+\beta)$$

let $\varepsilon = \phi - (-\beta) = \phi - \phi_0$

$$\ddot{\varepsilon} = \ddot{\phi} \qquad \ddot{\varepsilon} = -\frac{\sqrt{g^2+a^2}}{l}\sin\varepsilon$$

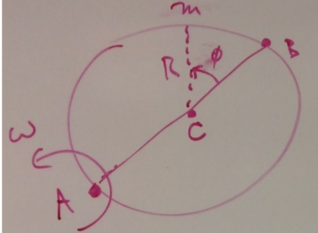
$$\rightarrow \omega_0 = \sqrt{\frac{\sqrt{g^2+a^2}}{l}}$$



The above-right figure is a bird's-eye view of a smooth horizontal wire hoop that is forced to rotate at a fixed angular frequency ω about a vertical axis through the point A. A bead of mass m is threaded on the hoop and is free to move around it, with its position specified by the angle ϕ that it makes at the center with the diameter AB. Find the Lagrangian for this system using ϕ as your generalized coordinate. Use the Lagrange EOM to show that the bead oscillates about the point B exactly like a simple pendulum. What is the frequency of these oscillations if their amplitude is small?

Next slide shows a handy trick that is helpful when you're able to write $\vec{x}_m = \vec{x}_{\text{point}} + \vec{x}_{\text{relative}}$.

Next-next slide shows how to use Mathematica to eliminate the drudgery of calculating T.



$$\underset{\sim}{x}_c = R(\cos\omega t, \sin\omega t) \qquad \dot{\underset{\sim}{x}}_c = R\omega(-\sin\omega t, \cos\omega t)$$

$$\underset{\sim}{x}_m = \underset{\sim}{x}_c + \boxed{R\left(\cos(\phi+\omega t), \sin(\phi+\omega t)\right)}$$

$$\equiv \underset{\sim}{x}_c + \boxed{\underset{\sim}{x}_{relative}} \qquad \dot{\underset{\sim}{x}}_{rel} = R(\dot\phi+\omega)\left(-\sin(\phi+\omega t), \cos(\phi+\omega t)\right)$$

handy trick: $\left|\dot{\underset{\sim}{x}}_m\right|^2 = \left|\dot{\underset{\sim}{x}}_c + \dot{\underset{\sim}{x}}_{rel}\right|^2$

$$= \left|\dot{\underset{\sim}{x}}_c\right|^2 + \left|\dot{\underset{\sim}{x}}_{rel}\right|^2 + 2\dot{\underset{\sim}{x}}_c \cdot \dot{\underset{\sim}{x}}_{rel}$$

$$= (R\omega)^2 + (R(\dot\phi+\omega))^2 + 2R^2\omega(\dot\phi+\omega)\left(\sin(\omega t)\sin(\phi+\omega t) + \cos(\omega t)\cos(\phi+\omega t)\right)$$

$$= R^2\omega^2 + R^2(\dot\phi+\omega)^2 + 2R^2\omega(\dot\phi+\omega)\cos\phi$$

In[ ]:= ClearAll["Global`*"];

```
x[t_] := R Cos[ϕ[t] + ω t] + R Cos[ω t];
y[t_] := R Sin[ϕ[t] + ω t] + R Sin[ω t];
x'[t]^2 + y'[t]^2
```

Out[ ]= $(R\,\omega\,\mathrm{Cos}[t\,\omega] + R\,\mathrm{Cos}[t\,\omega + \phi[t]]\,(\omega + \phi'[t]))^2 + (-R\,\omega\,\mathrm{Sin}[t\,\omega] - R\,\mathrm{Sin}[t\,\omega + \phi[t]]\,(\omega + \phi'[t]))^2$

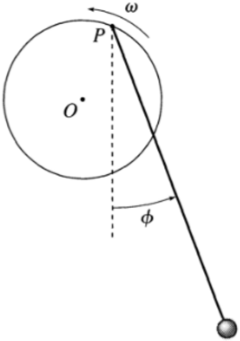
In[ ]:= FullSimplify[%]

Out[ ]= $R^2\left(2\,\omega^2\,(1 + \mathrm{Cos}[\phi[t]]) + 2\,\omega\,(1 + \mathrm{Cos}[\phi[t]])\,\phi'[t] + \phi'[t]^2\right)$

In[ ]:= Expand[%]

Out[ ]= $2\,R^2\,\omega^2 + 2\,R^2\,\omega^2\,\mathrm{Cos}[\phi[t]] + 2\,R^2\,\omega\,\phi'[t] + 2\,R^2\,\omega\,\mathrm{Cos}[\phi[t]]\,\phi'[t] + R^2\,\phi'[t]^2$

The figure shows a simple pendulum (mass m, length l) whose point of support P is attached to the edge of a wheel (center O, radius R) that is forced to rotate at a fixed angular velocity ω. At $t = 0$, the point P is level with O on the right. Write down the Lagrangian and find the EOM for the angle ϕ. [Hint: Be careful writing down T, the K.E. A safe way to get the velocity right is to write down the position of the bob at time t, and then differentiate.] Check that your answer makes sense in the special case $\omega = 0$.



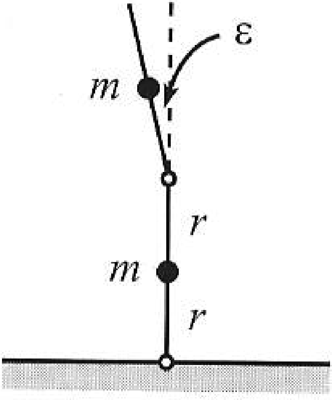
Next slide shows how to use Mathematica to eliminate the drudgery of calculating T.

In[6]:=
```
ClearAll["Global`*"];
x[t_] := R Cos[ω t] + l Sin[ϕ[t]];
y[t_] := R Sin[ω t] - l Cos[ϕ[t]];
x'[t]^2 + y'[t]^2
```

Out[9]=
```
(-R ω Sin[t ω] + l Cos[ϕ[t]] ϕ′[t])^2 +
  (R ω Cos[t ω] + l Sin[ϕ[t]] ϕ′[t])^2
```

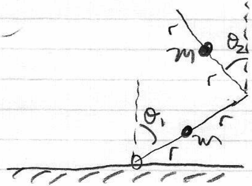
In[10]:=
```
FullSimplify[%]
```

Out[10]=
```
R^2 ω^2 + l ϕ′[t] (-2 R ω Sin[t ω - ϕ[t]] + l ϕ′[t])
```



HW3 XC7 is the "three sticks" generalization of this problem. Let's try the "two sticks" version.

Two massless sticks of length $2r$, each with a mass m fixed at its middle, are hinged at an end. One stands on top of the other. The bottom end of the lower stick is hinged on the ground. They are held such that the lower stick is vertical, and the upper one is tilted at a small angle ε w.r.t. vertical. They are then released. At the instant after release, what are the angular accelerations of the two sticks? Work in the approximation where $\varepsilon \ll 1$.



$$y_1 = r\cos\theta_1$$

$$y_2 = 2r\cos\theta_1 + r\cos\theta_2$$

$$U = mg(y_1 + y_2) = mgr\left(3\cos\theta_1 + \cos\theta_2\right)$$

$$x_1 = r\sin\theta_1 \rightarrow \dot{x}_1 = r\cos\theta_1\,\dot{\theta}_1$$

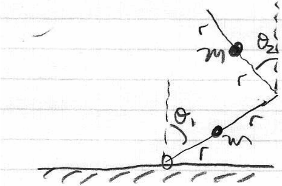
$$x_2 = 2r\sin\theta_1 - r\sin\theta_2 \rightarrow \dot{x}_2 = 2r\cos\theta_1\,\dot{\theta}_1 - r\cos\theta_2\,\dot{\theta}_2$$

$$\dot{y}_1 = -r\sin\theta_1\,\dot{\theta}_1 \qquad \dot{y}_2 = -2r\sin\theta_1\,\dot{\theta}_1 - r\sin\theta_2\,\dot{\theta}_2$$

$$T = \frac{m}{2}\left(\dot{x}_1^2 + \dot{y}_1^2 + \dot{x}_2^2 + \dot{y}_2^2\right) =$$

$$= \frac{mr^2}{2}\Big[\cos^2\theta_1\,\dot{\theta}_1^2 + \sin^2\theta_1\,\dot{\theta}_1^2 + 4\cos^2\theta_1\,\dot{\theta}_1^2 + \cos^2\theta_2\,\dot{\theta}_2^2 - 4\cos\theta_1\cos\theta_2\,\dot{\theta}_1\dot{\theta}_2$$
$$+ 4\sin^2\theta_1\,\dot{\theta}_1^2 + \sin^2\theta_2\,\dot{\theta}_2^2 + 4\sin\theta_1\sin\theta_2\,\dot{\theta}_1\dot{\theta}_2\Big]$$

$$T = \frac{mr^2}{2}\left[5\dot{\theta}_1^2 + \dot{\theta}_2^2 - 4\dot{\theta}_1\dot{\theta}_2\cos(\theta_1+\theta_2)\right] \simeq \frac{mr^2}{2}\left[5\dot{\theta}_1^2 + \dot{\theta}_2^2 - 4\dot{\theta}_1\dot{\theta}_2\left(1 - \frac{(\theta_1+\theta_2)^2}{2}\right)\right]$$



$$y_1 = r\cos\theta_1$$

$$y_2 = 2r\cos\theta_1 + r\cos\theta_2$$

$$U = mg(y_1 + y_2) = mgr(3\cos\theta_1 + \cos\theta_2)$$

$$x_1 = r\sin\theta_1 \rightarrow \dot{x}_1 = r\cos\theta_1\,\dot{\theta}_1$$

$$x_2 = 2r\sin\theta_1 - r\sin\theta_2 \rightarrow \dot{x}_2 = 2r\cos\theta_1\,\dot{\theta}_1 - r\cos\theta_2\,\dot{\theta}_2$$

$$\dot{y}_1 = -r\sin\theta_1\,\dot{\theta}_1 \qquad \dot{y}_2 = -2r\sin\theta_1\,\dot{\theta}_1 - r\sin\theta_2\,\dot{\theta}_2$$

$$U = mgr(3\cos\theta_1 + \cos\theta_2) \simeq mgr\left(3 - \frac{3\theta_1^2}{2} + 1 - \frac{\theta_2^2}{2}\right)$$

$$\mathcal{L} = T - U = \frac{mr^2}{2}\left(5\dot{\theta}_1^2 + \dot{\theta}_2^2 - 4\dot{\theta}_1\dot{\theta}_2\cos(\theta_1 + \theta_2)\right) - mgr(3\cos\theta_1 + \cos\theta_2)$$

$$\simeq \frac{mr^2}{2}\left(5\dot{\theta}_1^2 + \dot{\theta}_2^2 - 4\dot{\theta}_1\dot{\theta}_2\right) + mgr\left(\frac{3\theta_1^2}{2} + \frac{\theta_2^2}{2} - 4\right)$$

$$L \simeq \frac{mr^2}{2}\left(5\dot\theta_1^2 + \dot\theta_2^2 - 4\dot\theta_1\dot\theta_2\right) + mgr\left(\frac{3\theta_1^2}{2} + \frac{\theta_2^2}{2} - 4\right)$$

$$\frac{\partial L}{\partial \theta_1} = 3mgr\theta_1 \qquad \frac{\partial L}{\partial \theta_2} = mgr\theta_2$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot\theta_1}\right) = \frac{d}{dt}\left(\frac{mr^2}{2}\left(10\dot\theta_1 - 4\dot\theta_2\right)\right) = mr^2\left(5\ddot\theta_1 - 2\ddot\theta_2\right)$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot\theta_2}\right) = \frac{d}{dt}\left(\frac{mr^2}{2}\left(2\dot\theta_2 - 4\dot\theta_1\right)\right) = mr^2\left(\ddot\theta_2 - 2\ddot\theta_1\right)$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot\theta_1} = \frac{\partial L}{\partial \theta_1} \Rightarrow \boxed{5\ddot\theta_1 - 2\ddot\theta_2 = \frac{3g\theta_1}{r}}$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot\theta_2} = \frac{\partial L}{\partial \theta_2} \Rightarrow \boxed{\ddot\theta_2 - 2\ddot\theta_1 = \frac{g\theta_2}{r}}$$

Now plug in, at $t = 0$, given conditions $\theta_1 = 0$, $\theta_2 = \varepsilon$, and find initial angular accelerations $\ddot\theta_1$ and $\ddot\theta_2$.

$$\frac{d}{dt}\frac{\partial L}{\partial \dot\theta_1} = \frac{\partial L}{\partial \theta_1} \Rightarrow \boxed{5\ddot\theta_1 - 2\ddot\theta_2 = \frac{3g\theta_1}{r}}$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot\theta_2} = \frac{\partial L}{\partial \theta_2} \Rightarrow \boxed{\ddot\theta_2 - 2\ddot\theta_1 = \frac{g\theta_2}{r}}$$

at $\boxed{t=0,}$ plug in $\theta_1 = 0, \ \theta_2 = \varepsilon$

$$5\ddot\theta_1 = 2\ddot\theta_2$$

$$\rightarrow \ddot\theta_1 = \frac{2}{5}\ddot\theta_2$$

$$\ddot\theta_2 - 2\left(\frac{2}{5}\ddot\theta_2\right) = \frac{g\varepsilon}{r} = \frac{\ddot\theta_2}{5}$$

$$\rightarrow \boxed{\text{at } t=0, \quad \ddot\theta_2 = 5g\varepsilon/r, \quad \ddot\theta_1 = 2g\varepsilon/r}$$

Physics 351 — Monday, February 5, 2018

- HW3 due Friday. You finished reading ch7 for today.
- Tomorrow, Feb 6, undergrad groups WiP and SPS can meet with Musk Public Lecture speaker, Dr. Jim Gates, for an informal lunch: Tue 1:30pm–2:30pm in DRL 4N12.
- Public lecture Tue 5pm, Meyerson B1: "Will Evolution and Information Theory Provide the Fundamentals of Physics?"
- Gates was awarded the National Medal of Science from President Barack Obama in 2013. His work has produced first introductions of concepts in physics including the complete extension of Einstein's geometrical concepts to the theory of supergravity, genuine four-dimensional string theory based on Standard Model concepts, maximal SUSY Chern-Simons theories, and more. His co-authored book, *Superspace, or One Thousand and One Lessons in Supersymmetry,* is the first published comprehensive book on supersymmetry.
- Dr. Sylvester James Gates, Jr., is currently Ford Foundation Professor of Physics at Brown University.