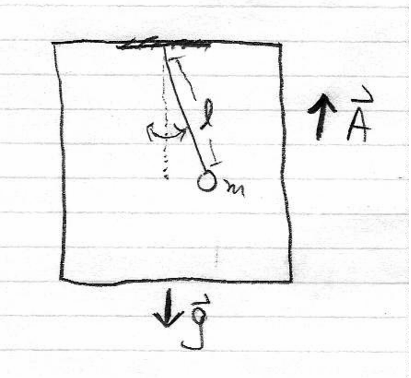
Physics 351 — Wednesday, February 21, 2018

- HW5 due Friday. For HW help, Bill is in DRL 3N6 Wed 4–7pm. Grace is in DRL 2C2 Thu 5:30–8:30pm.

It is often convenient to treat the "inertial (pseudo)force" $\boldsymbol{F}_{\text{inertial}} = -m\boldsymbol{A}$ as if it were just another component of gravity. For instance, the natural period of oscillation of a pendulum suspended from the ceiling of an elevator that is **accelerating upward** with constant acceleration $\boldsymbol{A}$ is



(a) $\text{period} = 2\pi\sqrt{\ell/g}$

(b) $\text{period} = 2\pi\sqrt{\ell/(g+A)}$

(c) $\text{period} = 2\pi\sqrt{\ell/(g-A)}$

(d) $\text{period} = 2\pi\sqrt{\ell/\sqrt{g^2+A^2}}$

(e) $\text{period} = 2\pi\sqrt{\ell/\sqrt{g^2-A^2}}$

(f) none of the above

As you know from experience riding in cars, trains, airplanes, etc., if you observe the world from the perspective of a non-inertial reference frame (i.e. a frame that is accelerating or rotating w.r.t. the distant stars), you perceive "pseudo-forces" that modify the usual results of Newton's 2nd law.

The simplest case is that your frame of reference $\mathcal{S}$ is accelerating (but not rotating) w.r.t. inertial frame $\mathcal{S}_0$.

Let the acceleration of your frame $\mathcal{S}$ w.r.t. $\mathcal{S}_0$ be $\boldsymbol{A}$. Then from your perspective, an object of mass m experiences an **inertial force**

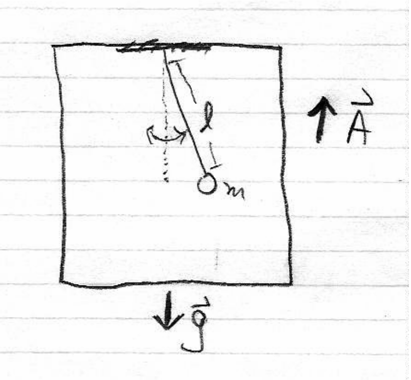
$$\boldsymbol{F}_{\text{inertial}} = -m\boldsymbol{A}$$

If your elevator accelerates upward, you feel an additional **downward** force $-m\boldsymbol{A}$, so your apparent weight is $m(g + A)$.

If your elevator accelerates downward, you feel an additional **upward** force $-m\boldsymbol{A}$, so your apparent weight is $m(g - A)$.

In GR, Einstein's principle of equivalence assumes "the complete physical equivalence of a gravitational field and a corresponding acceleration of the reference system."

It is often convenient to treat this "inertial force" $\boldsymbol{F}_{\text{inertial}} = -m\boldsymbol{A}$ as if it were just another component of gravity. For instance, the natural period of oscillation of a pendulum suspended from the ceiling of an elevator that is **accelerating upward** with constant acceleration $\boldsymbol{A}$ is



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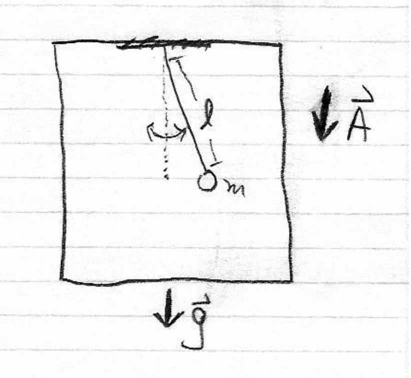
(f) none of the above

It is often convenient to treat this "inertial force" $\boldsymbol{F}_{\text{inertial}} = -m\boldsymbol{A}$ as if it were just another component of gravity.

The natural period of oscillation of a pendulum suspended from the ceiling of an elevator that is **accelerating downward** with constant acceleration $\boldsymbol{A}$ is



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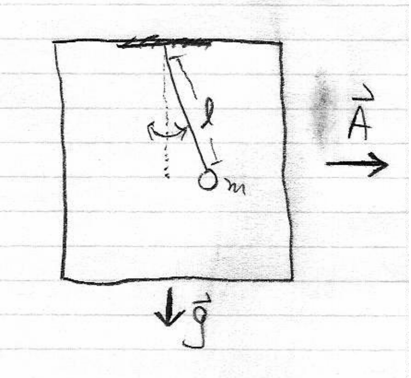
It is often convenient to treat this "inertial force" $\boldsymbol{F}_{\text{inertial}} = -m\boldsymbol{A}$ as if it were just another component of gravity.

The natural period of oscillation of a pendulum suspended from the ceiling of a train car that is **accelerating to the right** with constant acceleration $\boldsymbol{A}$ is



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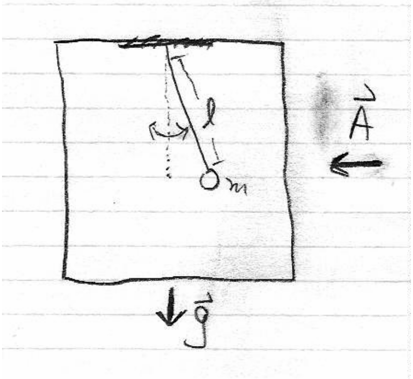
(f) none of the above

It is often convenient to treat this "inertial force" $\boldsymbol{F}_{\text{inertial}} = -m\boldsymbol{A}$ as if it were just another component of gravity.

The natural period of oscillation of a pendulum suspended from the ceiling of a train car that is **accelerating to the left** with constant acceleration $\boldsymbol{A}$ is



(a) $\text{period} = 2\pi\sqrt{\ell/g}$

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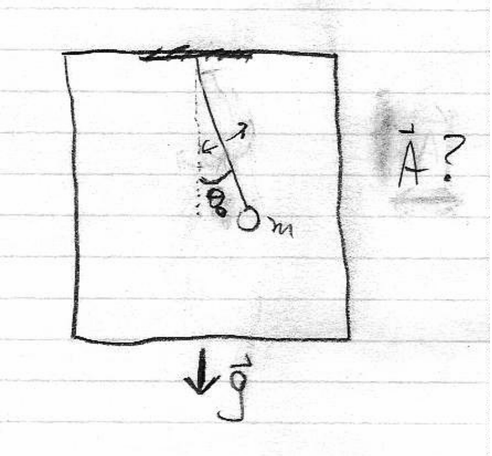
(d) $\text{period} = 2\pi\sqrt{\ell/\sqrt{g^2+A^2}}$

(e) $\text{period} = 2\pi\sqrt{\ell/\sqrt{g^2-A^2}}$

(f) none of the above

(That last one was intended to make you laugh.)

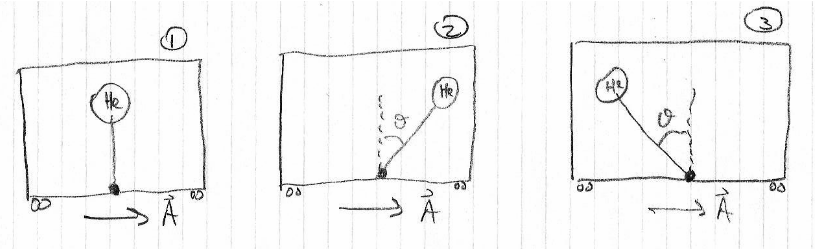
If the indicated angle θ_0 is in fact the **equilibrium** position of the pendulum in the figure, in which direction is the train car (from whose ceiling it is suspended) accelerating?



(a) $\vec{A}$ points to the right

(b) $\vec{A}$ points to the left

(HW problem from 2014.) A helium balloon is anchored by a massless string to the floor of a car that is accelerating **to the right** with (horizontal) acceleration $\boldsymbol{A}$. At equilibrium, in which direction does the balloon string tilt (w.r.t. vertical)?



(a) vertical. (b) balloon tilts right. (c) balloon tilts left.

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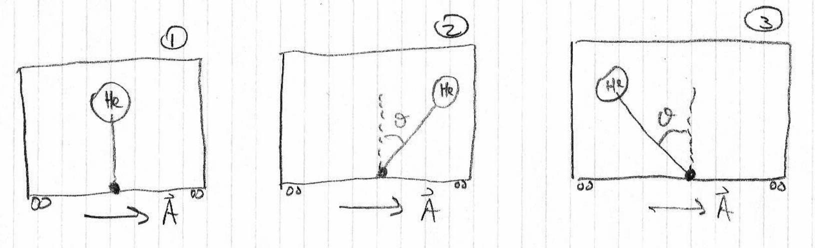


(a) vertical. (b) balloon tilts right. (c) balloon tilts left.

$\boldsymbol{A}$ is horizontal and has magnitude A. What is $\tan(\theta)$?

Balloon-in-car video:

`https://www.youtube.com/watch?v=y8mzDvpKzfY`

While we normally use the more sophisticated $\frac{\mathrm{d}}{\mathrm{d}t} \to \frac{\mathrm{d}}{\mathrm{d}t} + \boldsymbol{\Omega}\times$ method for rotating frames, you can use the simpler $-m\boldsymbol{A}$ method for instantaneous centripetal acceleration. For instance ...

Another homework problem from 2014: A donut-shaped space station (outer radius R) arranges for artificial gravity by spinning on the axis of the donut with angular velocity ω. Sketch the forces on, and accelerations of, an astronaut standing in the station (a) as seen from an inertial frame outside the station, and (b) as seen in the astronaut's personal rest frame.

(c) How would you calculate the required ω to simulate Earth's gravity?

(d) If $R = 100\ \mathrm{m}$, by about what fraction does "g" differ between the astronaut's head and feet? (In astronaut's CM frame, this difference = centrifugal force.)

A rotation about some origin $\mathcal{O}$ requires us to specify a plane of rotation and an angle. In two spatial dimensions, there is only one plane, so we need only one number to specify a rotation.

In four spatial dimensions, there would be

$$\frac{4!}{2!\,2!} = 6$$

possible planes of rotation, which is more than you can specify with a single 4-dimensional vector.

So it's a peculiarity of our 3-dimensional world that we can use (3D) vectors to specify (3D) rotations. But in 3D, it is extremely convenient to write

$$\boldsymbol{\omega} = \omega\,\hat{\boldsymbol{u}}$$

where $\hat{\boldsymbol{u}}$ points in the direction of the rotation axis, using the right-hand rule to pick the sign.

Remarkably, angular velocities (about a given origin) add in the same way as translational velocities. Works because infinitesimal rotations commute, while finite rotations do not commute.

You can see by staring at a globe that a point $\boldsymbol{r}$ measured w.r.t. the center of the spinning globe has instantaneous velocity

$$\boldsymbol{v} = \boldsymbol{\omega} \times \boldsymbol{r}$$

whose magnitude is $\omega r \sin\theta = \omega\rho$.

What's a bit more remarkable is that this works for a unit vector $\boldsymbol{e}$ fixed on the spinning globe:

$$\frac{\mathrm{d}\boldsymbol{e}}{\mathrm{d}t} = \boldsymbol{\omega} \times \boldsymbol{e}$$

This gives you a much quicker way to derive Chapter 1's results for rotating unit vectors in cylindrical coordinates (take $\boldsymbol{\omega} = \dot{\phi}\hat{\boldsymbol{z}}$)

$$\frac{\mathrm{d}\hat{\boldsymbol{r}}}{\mathrm{d}t} = \boldsymbol{\omega} \times \hat{\boldsymbol{r}} = +\omega\hat{\boldsymbol{\phi}}$$

$$\frac{\mathrm{d}\hat{\boldsymbol{\phi}}}{\mathrm{d}t} = \boldsymbol{\omega} \times \hat{\boldsymbol{\phi}} = -\omega\hat{\boldsymbol{r}}$$

Imagine a globe spinning with angular velocity $\boldsymbol{\Omega}$.

If you observe **the same vector** $\boldsymbol{Q}$ from two different frames:

- A "space" frame $\mathcal{S}_0$ that is stationary in space.
- A "body" frame $\mathcal{S}$ that is attached to the spinning globe.

You can relate $\mathrm{d}\boldsymbol{Q}/\mathrm{d}t$ as observed in the two frames using this incredibly useful identity:

$$\left(\frac{\mathrm{d}\boldsymbol{Q}}{\mathrm{d}t}\right)_{\mathcal{S}_0} = \left(\frac{\mathrm{d}\boldsymbol{Q}}{\mathrm{d}t}\right)_{\mathcal{S}} + \boldsymbol{\Omega} \times \boldsymbol{Q}$$

In Chapter 10 (in a week or so), we'll rephrase this as:

$$\left(\frac{\mathrm{d}\boldsymbol{Q}}{\mathrm{d}t}\right)_{\text{space}} = \left(\frac{\mathrm{d}\boldsymbol{Q}}{\mathrm{d}t}\right)_{\text{body}} + \boldsymbol{\Omega} \times \boldsymbol{Q}$$

$$\left(\frac{\mathrm{d}\boldsymbol{Q}}{\mathrm{d}t}\right)_{\mathcal{S}_0} = \left(\frac{\mathrm{d}\boldsymbol{Q}}{\mathrm{d}t}\right)_{\mathcal{S}} + \boldsymbol{\Omega} \times \boldsymbol{Q}$$

Applying this (twice) to Newton's second law (inertial frame $\mathcal{S}_0$):

$$m\left(\frac{\mathrm{d}^2\boldsymbol{r}}{\mathrm{d}t^2}\right)_{\mathcal{S}_0} = \boldsymbol{F}$$

gives Newton's 2nd law as seen in rotating frame $\mathcal{S}$

$$m\ddot{\boldsymbol{r}} = \boldsymbol{F} + 2m\dot{\boldsymbol{r}} \times \boldsymbol{\Omega} + m(\boldsymbol{\Omega} \times \boldsymbol{r}) \times \boldsymbol{\Omega}$$

and two well-known pseudo-forces appear. (This assumes $\dot{\boldsymbol{\Omega}} = 0$.)

The pseudo-force due to the second (red) term is called ...

The pseudo-force due to the third (violet) term is called ...

(If $\dot{\boldsymbol{\Omega}} \neq 0$, then you get one more term, called the "azimuthal force." See HW06/q9.)

The **centrifugal force** (which our high-school physics teachers regarded as an abomination) is

$$\boldsymbol{F}_{\text{centrifugal}} = m(\boldsymbol{\Omega} \times \boldsymbol{r}) \times \boldsymbol{\Omega}$$

(Use right-hand rule to see the various vectors on globe.)

Its magnitude is . . . ?

The **centrifugal force** (which our high-school physics teachers regarded as an abomination) is

$$\boldsymbol{F}_{\text{centrifugal}} = m(\boldsymbol{\Omega} \times \boldsymbol{r}) \times \boldsymbol{\Omega}$$

(Use right-hand rule to see the various vectors on globe.)

Its magnitude is . . . ?

This is easier to see using $\rho = r \sin\theta$:

$$\boldsymbol{F}_{\text{centrifugal}} \;=\; m\Omega^2 \,\rho\, \hat{\boldsymbol{\rho}} \;=\; m\Omega^2 r \sin\theta\, \hat{\boldsymbol{\rho}}$$

$$\boldsymbol{F}_{\text{centrifugal}} = m(\boldsymbol{\Omega} \times \boldsymbol{r}) \times \boldsymbol{\Omega} \;=\; m\Omega^2\,\rho\,\hat{\boldsymbol{\rho}}$$

Another HW question from 2014: I am spinning a bucket of water about its vertical axis with angular velocity Ω. Once the water has settled in equilibrium (relative to the bucket), what will be the shape of the water surface? (Let ρ be the (horizontal) distance from the axis of rotation.)

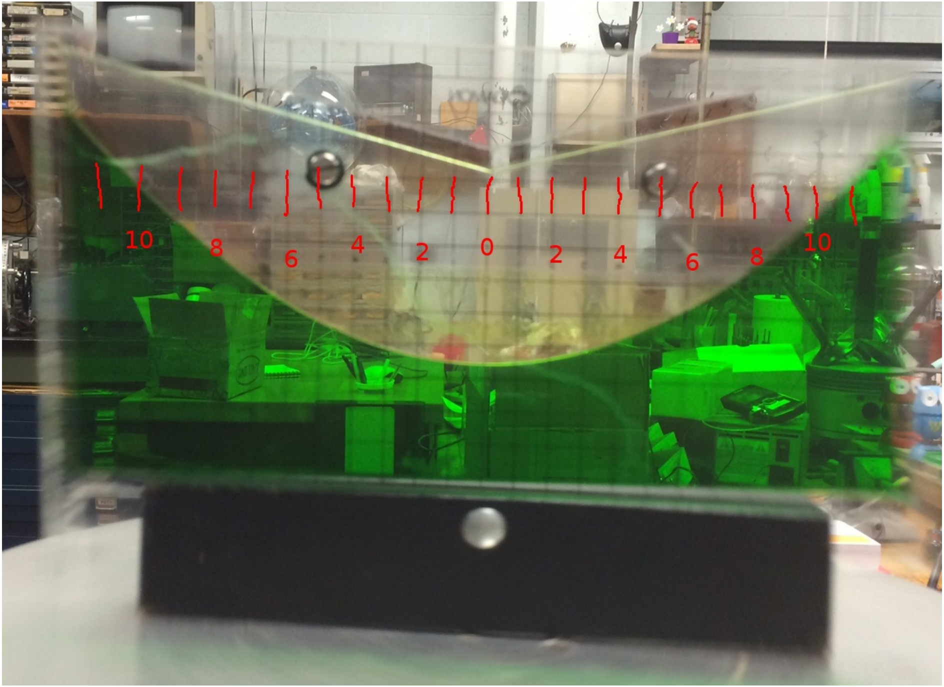
(A) $y \sim 1/\rho^2$

(B) $y \sim 1/\rho$

(C) $y \sim \rho$

(D) $y \sim \rho^2$

(E) $y \sim \rho^3$


10 8 6 4 2 0 2 4 6 8 10

Unlike the inertial force and the centrifugal force, which you experience when traveling in cars, airplanes, etc., the **Coriolis force** is much less familiar from everyday experience:

$$\boldsymbol{F}_{\text{coriolis}} = 2m\,\boldsymbol{v} \times \boldsymbol{\Omega}$$

(Use right-hand rule to see the various vectors on globe.) One handy mnemonic is the analogy with the magnetic force law.

One more HW problem from a past year: What are the directions of the centrifugal and Coriolis forces on a person moving

(1) south near the North Pole?

(2) east on the equator?

(3) west on the equator?

(4) south across the equator?

(5) east near Philadelphia?

(6) west near Philadelphia?

(7) north near Philadelphia?

If you look down at Earth from space (e.g. from above the equator), and you somehow manage to hover at rest w.r.t. Earth's CoM, as if you were watching a spinning globe, in which direction do you see the land beneath you moving?

(A) toward the east (from west to east)

(B) toward the west (from east to west)

(C) It depends on whether you're looking down at the northern or the southern hemisphere.

In the same scenario as the last page (hovering as if watching a spinning globe), if I look down at Earth, then blink my eyes for a moment, then look down again at Earth, the land that I see after blinking is

(A) east of the land that I saw before blinking (I see Philly, then a while later I see England)

(B) west of the land that I saw before blinking (I see Philly, then a while later I see San Francisco)

If I drop an object straight down from the top of a tall building, the Coriolis force will deflect the falling object

(A) toward the east

(B) toward the west

$$\boldsymbol{F}_{\text{coriolis}} = 2m\,\boldsymbol{v} \times \boldsymbol{\Omega}$$

The cross-product of (downward toward Earth's center) with $\boldsymbol{\Omega}$ points

(A) toward the east

(B) toward the west

speed of zero) gives $v_{\text{east}} = \omega g t^2 \sin\theta$. Integrating again to obtain the eastward deflection (with an initial eastward deflection of zero) gives $d_{\text{east}} = \omega g t^3 \sin\theta/3$. Plugging in $t = \sqrt{2h/g}$ gives

$$d_{\text{east}} = \frac{2\omega h \sin\theta}{3}\sqrt{\frac{2h}{g}}. \tag{10.18}$$

The frequency of the earth's rotation is $\omega \approx 7.3 \cdot 10^{-5}\,\text{s}^{-1}$, so if we pick $\theta = \pi/2$ and $h = 100$ m, for example, then we have $d_{\text{east}} \approx 2$ cm.

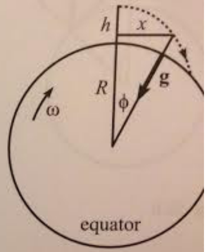


Fig. 10.8

REMARK: We can also solve this problem by working in an inertial frame; see Stirling (1983). Figure 10.8 shows the setup where a ball is dropped from a tower of height h located at the equator (the view is from the south pole). The earth is rotating in the inertial frame, so the initial sideways speed of the ball, $(R+h)\omega$, is larger than the sideways speed of the base of the tower, $R\omega$. This is the basic cause of the eastward deflection.

However, after the ball has moved to the right, the gravitational force on it picks up a component pointing to the left, and this slows down the sideways speed. If the ball has moved a distance x to the right, then the leftward component of gravity equals $g\sin\phi \approx g(x/R)$. Now, to leading order we have $x = R\omega t$, so the sideways acceleration of the ball is $a = -g(R\omega t/R) = -\omega g t$. Integrating this, and using the initial speed of $(R+h)\omega$, gives a rightward speed of $(R+h)\omega - \omega g t^2/2$. Integrating again gives a rightward distance of $(R+h)\omega t - \omega g t^3/6$. Subtracting off the rightward position of the base of the tower (namely $R\omega t$), and using $t \approx \sqrt{2h/g}$ (neglecting higher-order effects such as the curvature of the earth and the variation of g with altitude), we obtain an eastward deflection of $\omega h\sqrt{2h/g}(1 - 1/3) = (2/3)\omega h\sqrt{2h/g}$, relative to the base of the tower. If the ball is dropped at a polar angle θ instead of at the equator, then the only modification is that all speeds are decreased by a factor of $\sin\theta$, so we obtain the result in Eq. (10.18). ♣

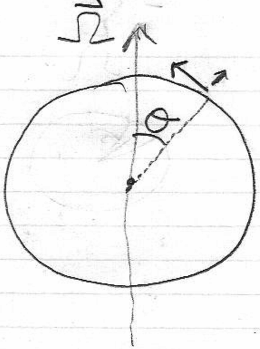
$$\boldsymbol{F}_{\text{coriolis}} = 2m\,\boldsymbol{v} \times \boldsymbol{\Omega}$$

A projectile of mass m is fired with initial speed v_0 horizontally and due north from a position of colatitude θ. Find the direction and magnitude of the Coriolis force in terms of m, v_0, θ, and Earth's angular velocity Ω.

How does the Coriolis force compare with the projectile's weight if $v_0 = 1000\ \text{m/s}$ and $\theta = 60°$? $\quad (\Omega \approx 7.3 \times 10^{-5}/\text{s}, \quad g \approx 10\ \text{m/s}^2)$

(1000 m/s is around Mach 3, i.e. $3\times$ the speed of sound. In more familiar units, it's 3600 kph, or 2240 mph. Wikipedia says this is the right order of magnitude for modern artillery cannons. I imagine (just guessing) that the cannon that severely wounded Prince Andrei in *War and Peace* was sub-sonic.)

$$\vec{F}_{coriolis} = 2m\vec{v} \times \vec{\Omega}$$



$$\vec{\Omega} = \Omega \sin\theta\,(\hat{north}) + \Omega\cos\theta\,(\hat{up})$$

$$(\hat{north}) \times (\hat{north}) = 0$$

$$(\hat{north}) \times (\hat{up}) = (\hat{east})$$

$$\Rightarrow \vec{F}_{\substack{coriolis \\ (initial)}} = 2mv_0\Omega\cos\theta\,(\hat{east})$$

if $\theta = 60°$ then $\cos\theta = \frac{1}{2}$

$$\Omega \approx \frac{2\pi}{86400} = 7.3\times10^{-5}/s$$

$$\frac{2mv_0\Omega\cos\theta}{mg} = \frac{v_0\Omega}{g} \simeq \frac{(1000)(7.3\times10^{-5})}{10} = .007$$

(somewhat under 1% of mg)

For spinning Earth, $\boldsymbol{\Omega}$ points up out of the north pole. This rotation gives position $\vec{r}$ on Earth's surface the velocity

$$\boldsymbol{v} = \boldsymbol{\Omega} \times \boldsymbol{r}$$

That is, as seen from space,

$$\frac{\mathrm{d}\boldsymbol{r}}{\mathrm{d}t} = \boldsymbol{\Omega} \times \boldsymbol{r}$$

This also works for a unit vector $\boldsymbol{e}$ fixed on the spinning globe:

$$\frac{\mathrm{d}\boldsymbol{e}}{\mathrm{d}t} = \boldsymbol{\Omega} \times \boldsymbol{e}$$

At any given instant, you can project a given vector $\boldsymbol{Q}$ onto two different sets of axes: the "space" axes $\hat{\boldsymbol{x}}$, $\hat{\boldsymbol{y}}$, $\hat{\boldsymbol{z}}$ that are **not** rotating (they're fixed in space somewhere), and the "body" axes $\hat{e}_1$, $\hat{e}_2$, $\hat{e}_3$ that are rotating with the globe (a.k.a. the rigid body).

If you observe **the same vector $\boldsymbol{Q}$** from two different frames:

- A "space" frame $\mathcal{S}_0$ that is stationary in space.
- A "body" frame $\mathcal{S}$ that is attached to the spinning globe.

$$\boldsymbol{Q} = Q_x\hat{\boldsymbol{x}} + Q_y\hat{\boldsymbol{y}} + Q_z\hat{\boldsymbol{z}} = Q_1\hat{\boldsymbol{e}}_1 + Q_2\hat{\boldsymbol{e}}_2 + Q_3\hat{\boldsymbol{e}}_3$$

Then we use the product rule for the RHS to account for

$$\frac{\mathrm{d}\hat{\boldsymbol{e}}_1}{\mathrm{d}t} = \boldsymbol{\Omega} \times \hat{\boldsymbol{e}}_1$$

which gives this incredibly useful identity:

$$\left(\frac{\mathrm{d}\boldsymbol{Q}}{\mathrm{d}t}\right)_{\mathcal{S}_0(\text{space})} = \left(\frac{\mathrm{d}\boldsymbol{Q}}{\mathrm{d}t}\right)_{\mathcal{S}(\text{body})} + \boldsymbol{\Omega} \times \boldsymbol{Q}$$

You'll use this often to relate $\mathrm{d}\boldsymbol{Q}/\mathrm{d}t$ as observed in the two frames.

$$\left(\frac{\mathrm{d}\boldsymbol{Q}}{\mathrm{d}t}\right)_{\mathcal{S}_0} = \left(\frac{\mathrm{d}\boldsymbol{Q}}{\mathrm{d}t}\right)_{\mathcal{S}} + \boldsymbol{\Omega} \times \boldsymbol{Q}$$

Applying this (twice) to Newton's second law (inertial frame $\mathcal{S}_0$):

$$m\left(\frac{\mathrm{d}^2\boldsymbol{r}}{\mathrm{d}t^2}\right)_{\mathcal{S}_0} = \boldsymbol{F}$$

gives Newton's 2nd law as seen in rotating frame $\mathcal{S}$

$$\boxed{m\ddot{\boldsymbol{r}} \ = \ \boldsymbol{F} + 2m\dot{\boldsymbol{r}} \times \boldsymbol{\Omega} + m(\boldsymbol{\Omega} \times \boldsymbol{r}) \times \boldsymbol{\Omega}}$$

and two well-known pseudo-forces appear. (This assumes $\dot{\boldsymbol{\Omega}} = 0$.)

2nd term is "Coriolis force." 3rd term is "centrifugal force."

(If $\dot{\boldsymbol{\Omega}} \neq 0$, then you get one more term, $m\boldsymbol{r} \times \dot{\boldsymbol{\Omega}}$, called the "azimuthal force" or "Euler force." See HW06/q9.)

Pause for two points: how to evaluate each quantity in the boxed equation; and which directions all the various vectors point.

The **centrifugal force** (which our high-school physics teachers regarded with abomination) is

$$\boldsymbol{F}_{\text{centrifugal}} = m(\boldsymbol{\Omega} \times \boldsymbol{r}) \times \boldsymbol{\Omega}$$

(Use right-hand rule to see the various vectors on globe.)

Its magnitude is . . . ?

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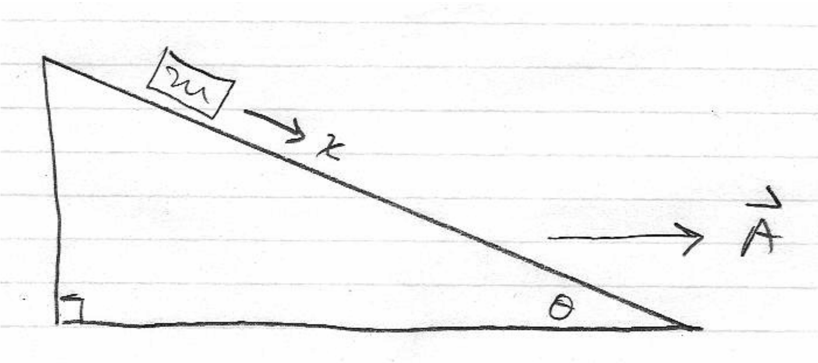
$$\boldsymbol{F}_{\text{centrifugal}} = m(\boldsymbol{\Omega} \times \boldsymbol{r}) \times \boldsymbol{\Omega}$$

(Use right-hand rule to see the various vectors on globe.)

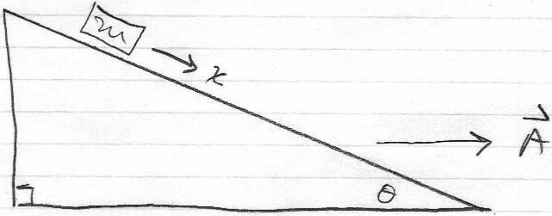
Its magnitude is ... ?

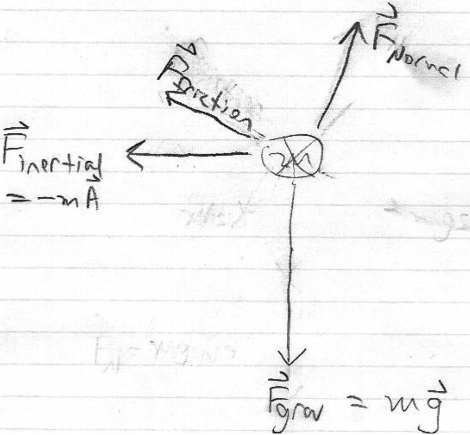
This is easier to see using $\rho = r \sin \theta$:

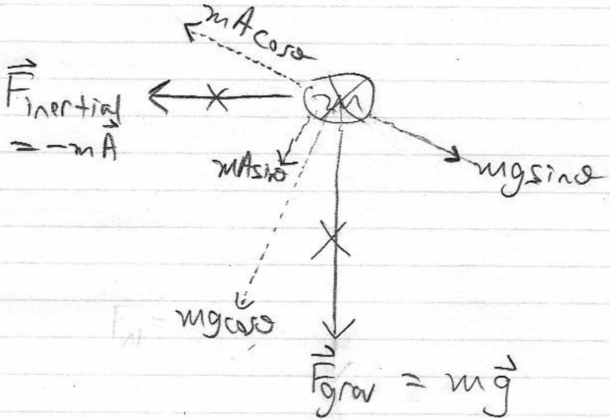
$$\boldsymbol{F}_{\text{centrifugal}} \;=\; m\Omega^2 \, \rho \, \hat{\boldsymbol{\rho}} \;=\; m\Omega^2 r \sin \theta \, \hat{\boldsymbol{\rho}}$$



What is smallest value of static friction coefficient μ for which the block can stand still on the ramp? (What's the familiar answer if $A = 0$?)


m
x
A
θ


F Normal
F friction
F inertial = −m A
m
F grav = m g



$$F_N = mg\cos\theta + mA\sin\theta$$

$$F_F \leq \mu F_N = \mu m(g\cos\theta + A\sin\theta)$$

$$m\ddot{x} = m(g\sin\theta - A\cos\theta) - F_F$$

$$F_N = mg\cos\theta + mA\sin\theta$$

$$F_F \le \mu F_N = \mu m(g\cos\theta + A\sin\theta)$$

$$m\ddot{x} = m(g\sin\theta - A\cos\theta) - F_F$$

block just starts to slip on ramp if

$$g\sin\theta - A\cos\theta = \mu(g\cos\theta + A\sin\theta)$$

$$\mu = \frac{g\sin\theta - A\cos\theta}{g\cos\theta + A\sin\theta} = \frac{\cos\beta\sin\theta - \sin\beta\cos\theta}{\cos\beta\cos\theta + \sin\beta\sin\theta}$$

$$= \tan(\theta - \beta) \quad \text{where} \quad \tan\beta = A/g$$

Note that

$$\tan(\theta - \beta) = \frac{\cos\beta\sin\theta - \sin\beta\cos\theta}{\cos\beta\cos\theta + \sin\beta\sin\theta}$$

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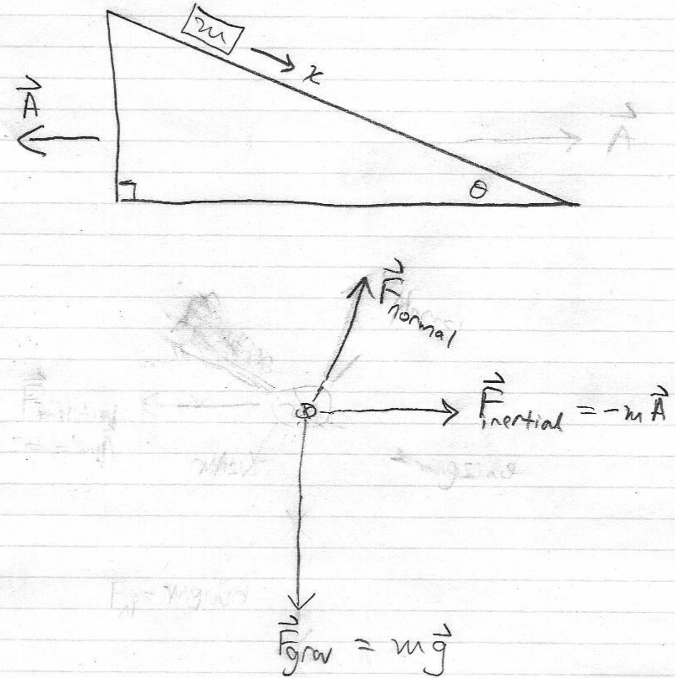
```
In[6]:= Factor[TrigExpand[Tan[a-b]]]

        Cos[b] Sin[a] - Cos[a] Sin[b]
Out[6]= -----------------------------
        Cos[a] Cos[b] + Sin[a] Sin[b]
```



No friction, but wedge is recoiling with recoil acceleration $\vec{A}$ (to the left)

$$m\ddot{x} = m(g\sin\theta + A\cos\theta)$$

linear momentum conservation:

$$m(\ddot{x}\cos\theta - A) = MA \longrightarrow A = \frac{m\ddot{x}\cos\theta}{M+m}$$

$$m\ddot{x} = mg\sin\theta + \frac{m^2}{M+m}\ddot{x}\cos^2\theta$$

$$\ddot{x}\left(1 - \frac{m}{m+M}\cos^2\theta\right) = g\sin\theta$$

$$\ddot{x} = \frac{g\sin\theta}{1 - \frac{m}{m+M}\cos^2\theta}$$

Physics 351 — Wednesday, February 21, 2018

- You read Ch 9 (mechanics in non-inertial frames) last weekend. The midterm exam (**March** 26) will cover (only!) chapters 7,8,9. But we will be working on Ch 10 by then.
- HW5 due Friday. For HW help, Bill is in DRL 3N6 Wed 4–7pm. Grace is in DRL 2C2 Thu 5:30–8:30pm.