Physics 351 — Monday, February 26, 2018

- You just read the first half (§10.1–10.7) of Chapter 10, which we'll probably start to discuss this Friday.
- The midterm exam (**March** 26) will cover (only!) chapters 7,8,9. But we will be working on Ch 10 by then.

Start to work out the Coriolis deflection of an object dropped (from rest) from a height h above Earth's surface. For simplicity, let the starting point be directly above the equator.

Work to first order in Ω. Since the Coriolis force depends on velocity, your evaluating the Coriolis force will require you first to plug in the "zeroth order" expression [i.e. the "freshman physics" expression] for $v_z(t)$, where z is altitude above Earth's surface. Let $\hat{x}$ point east, let $\hat{y}$ point north, and let $\hat{z}$ point up.

$$\boldsymbol{F}_{\text{coriolis}} = 2m\,\boldsymbol{v} \times \boldsymbol{\Omega}$$

The cross-product of (downward toward Earth's center) with $\boldsymbol{\Omega}$ points

(A) toward the east

(B) toward the west

Let's work out together the Coriolis deflection of an object dropped (from rest) from a height h above Earth's surface. For simplicity, let the starting point be directly above the equator.

$\boxed{\text{drop}}$ object from height h above equator

$$\underset{\sim}{F}_{coriolis} = 2m\,\underset{\sim}{v} \times \underset{\sim}{\Omega}$$

let $\hat{x}$ = east $\quad \hat{y}$ = north, $\quad \hat{z}$ = up

at equator, $\underset{\sim}{\Omega} = \Omega\hat{y}$

$$F_{coriolis} = 2m\left(v_x\Omega\hat{z} - v_z\Omega\hat{x}\right)$$

$$m\ddot{x} = -2mv_z\Omega$$

$$\ddot{x} = -2(-gt)\Omega = 2g\Omega t$$

0th order: $v_z = -gt$

$$t_f = \sqrt{\frac{2h}{g}}$$

$$\dot{x} = g\Omega t^2$$

$$x_f = \frac{1}{3}g\Omega t^3 = \frac{1}{3}g\Omega\left(\frac{2h}{g}\right)^{3/2} = \boxed{\frac{2}{3}\Omega h\sqrt{\frac{2h}{g}}}$$

speed of zero) gives $v_{\text{east}} = \omega g t^2 \sin\theta$. Integrating again to obtain the eastward deflection (with an initial eastward deflection of zero) gives $d_{\text{east}} = \omega g t^3 \sin\theta/3$. Plugging in $t = \sqrt{2h/g}$ gives

$$d_{\text{east}} = \frac{2\omega h \sin\theta}{3}\sqrt{\frac{2h}{g}}. \qquad (10.18)$$

The frequency of the earth's rotation is $\omega \approx 7.3 \cdot 10^{-5}\,\text{s}^{-1}$, so if we pick $\theta = \pi/2$ and $h = 100\,\text{m}$, for example, then we have $d_{\text{east}} \approx 2\,\text{cm}$.

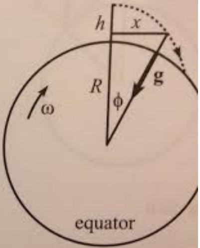
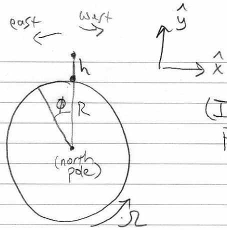


Fig. 10.8

REMARK: We can also solve this problem by working in an inertial frame; see Stirling (1983). Figure 10.8 shows the setup where a ball is dropped from a tower of height h located at the equator (the view is from the south pole). The earth is rotating in the inertial frame, so the initial sideways speed of the ball, $(R+h)\omega$, is larger than the sideways speed of the base of the tower, $R\omega$. This is the basic cause of the eastward deflection.

However, after the ball has moved to the right, the gravitational force on it picks up a component pointing to the left, and this slows down the sideways speed. If the ball has moved a distance x to the right, then the leftward component of gravity equals $g\sin\phi \approx g(x/R)$. Now, to leading order we have $x = R\omega t$, so the sideways acceleration of the ball is $a = -g(R\omega t/R) = -\omega g t$. Integrating this, and using the initial speed of $(R+h)\omega$, gives a rightward speed of $(R+h)\omega - \omega g t^2/2$. Integrating again gives a rightward distance of $(R+h)\omega t - \omega g t^3/6$. Subtracting off the rightward position of the base of the tower (namely $R\omega t$), and using $t \approx \sqrt{2h/g}$ (neglecting higher-order effects such as the curvature of the earth and the variation of g with altitude), we obtain an eastward deflection of $\omega h\sqrt{2h/g}(1-1/3) = (2/3)\omega h\sqrt{2h/g}$, relative to the base of the tower. If the ball is dropped at a polar angle θ instead of at the equator, then the only modification is that all speeds are decreased by a factor of $\sin\theta$, so we obtain the result in Eq. (10.18). ♣

Here is the same problem worked out from the inertial frame, instead of the Earth frame. I don't think it's worth going through in class, but here it is. Note that the sketch is looking down from the north pole.



(I chose $\hat{x}$ to point west here, confusingly.)

$$\underset{\sim}{F}_{grav} = -mg\hat{r} = -mg\left(\hat{y}\cos\Omega t - \hat{x}\sin\Omega t\right)$$

$$\simeq -mg\hat{y} + mg\Omega t\,\hat{x}$$

$$\ddot{y} = -g \qquad \ddot{x} = \Omega g t$$

$$\dot{x} = v_{x0} + \tfrac{1}{2}\Omega g t^2$$

$$x = v_{x0}t + \tfrac{1}{6}\Omega g t^3$$

$$= -(R+h)\Omega t + \tfrac{1}{6}\Omega g t^3$$

But bottom of tower has moved by

$$\Delta x = -R\Omega t$$

So (to first order in Ω) deflection of dropped object w.r.t. earth's surface is eastward by

$$h\Omega t - \tfrac{1}{6}\Omega g t^3 \qquad \left(t \simeq \sqrt{\tfrac{2h}{g}}\right)$$

$$= \left(\Omega\sqrt{\tfrac{2h}{g}}\right)\left(h - \tfrac{1}{6}g\left(\tfrac{2h}{g}\right)\right) = \tfrac{2}{3}\Omega h\sqrt{\tfrac{2h}{g}}$$

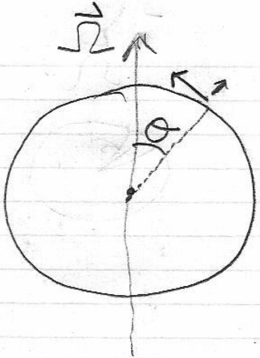
$$\boldsymbol{F}_{\text{coriolis}} = 2m\,\boldsymbol{v} \times \boldsymbol{\Omega}$$

A projectile of mass m is fired with initial speed v_0 horizontally and due north from a position of colatitude θ. Find the direction and magnitude of the Coriolis force in terms of m, v_0, θ, and Earth's angular velocity Ω.

How does the Coriolis force compare with the projectile's weight if $v_0 = 1000\ \mathrm{m/s}$ and $\theta = 60°$? $\quad(\Omega \approx 7.3 \times 10^{-5}/\mathrm{s}, \quad g \approx 10\ \mathrm{m/s^2})$

(1000 m/s is around Mach 3, i.e. $3\times$ the speed of sound. In more familiar units, it's 3600 kph, or 2240 mph. Wikipedia says this is the right order of magnitude for modern artillery cannons. I imagine (just guessing) that the Napoleonic-era cannon that severely wounded Prince Andrei in *War and Peace* was sub-sonic.)

$$\vec{F}_{\text{coriolis}} = 2m\vec{v} \times \vec{\Omega}$$



$$\vec{\Omega} = \Omega \sin\theta\,(\widehat{\text{north}}) + \Omega\cos\theta\,(\widehat{\text{up}})$$

$$(\widehat{\text{north}}) \times (\widehat{\text{north}}) = 0$$

$$(\widehat{\text{north}}) \times (\widehat{\text{up}}) = (\widehat{\text{east}})$$

$$\Rightarrow \underset{(\text{initial})}{\vec{F}_{\text{coriolis}}} = 2mv_0\Omega\cos\theta\,(\widehat{\text{east}})$$

if $\theta = 60°$ then $\cos\theta = \frac{1}{2}$ $\quad \Omega \approx \frac{2\pi}{86400} = 7.3 \times 10^{-5}/s$

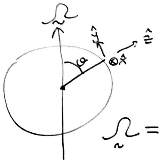
$$\frac{2mv_0\Omega\cos\theta}{mg} = \frac{v_0\Omega}{g} \simeq \frac{(1000)(7.3\times10^{-5})}{10} = .007$$

(somewhat under 1% of mg)

From spring 2015 midterm: At a polar angle θ (colatitude), a projectile is fired due north with initial velocity v_0 at an inclination angle α above the ground. Working to first order in Earth's rotational velocity Ω, show that the eastward deflection due to the Coriolis force is (vs. time t since the projectile was fired)

$$x(t) = \Omega v_0(\cos\alpha\cos\theta - \sin\alpha\sin\theta)t^2 + \frac{1}{3}\Omega g t^3 \sin\theta$$

[Assume that air resistance is negligible and that $\boldsymbol{g}$ is a constant throughout the flight. I recommend first neglecting the Coriolis force and writing the "zeroth order" $y(t)$ (northward) and $z(t)$ (upward) for ordinary projectile motion; then use this zeroth-order trajectory to calculate the first-order Coriolis deflection, after evaluating $\boldsymbol{\Omega}$ in terms of the local $\hat{\boldsymbol{x}}$, $\hat{\boldsymbol{y}}$, $\hat{\boldsymbol{z}}$ axes.]



$$\underset{\sim}{\Omega} = (\cos\theta)\hat{z} + (\sin\theta)\hat{y}$$

0th order in Ω:

$$z(t) = 0 + (V_0 \sin\alpha)t - \tfrac{1}{2}gt^2 \Rightarrow V_z = V_0 \sin\alpha - gt$$

$$y(t) = 0 + (V_0\cos\alpha)t \quad \Rightarrow V_y = V_0 \cos\alpha$$

1st order in Ω:

$$\underset{\sim}{F}_{cor} = 2m\,\underset{\sim}{v} \times \underset{\sim}{\Omega}$$

$$m\ddot{x} = 2m\,(\underset{\sim}{v}\times\underset{\sim}{\Omega})_x = 2m\,(V_y\Omega_z - V_z\Omega_y) = 2m\left((V_0\cos\alpha)(\Omega\cos\theta) - (V_0\sin\alpha - gt)(\Omega\sin\theta)\right)$$

$$\ddot{x} = (2V_0\Omega\cos\alpha\cos\theta - 2V_0\Omega\sin\alpha\sin\theta) + 2gt\,\Omega\sin\theta$$

$$\dot{x} = 2V_0\Omega(\cos\alpha\cos\theta - \sin\alpha\sin\theta)t + gt^2\Omega\sin\theta$$

$$x = (V_0\Omega t^2)(\cos\alpha\cos\theta - \sin\alpha\sin\theta) + \left(\tfrac{1}{3}gt^3\right)\Omega\sin\theta$$

Here's a slightly trickier Coriolis projectile problem. What makes this problem tricky is that the Coriolis force also has a first-order effect on the projectile's flight time.

At a polar angle θ, a projectile is fired eastward with speed v_0 at an angle α above the ground. Let $\hat{x}$ point east, let $\hat{y}$ point north, and let $\hat{z}$ point "up." To first order in Ω, work out the effects, Δx and Δy, of the Coriolis force on the projectile's landing point. Ignore air resistance and ignore Earth's curvature.

$$m\boldsymbol{a} \;=\; \boldsymbol{F} + 2m\boldsymbol{v} \times \boldsymbol{\Omega} + m\,\Omega^2\rho\,\hat{\boldsymbol{\rho}}$$

0^{th} order in Ω:

$$x = (v_0 \cos\alpha)t \qquad y = 0 \qquad z = (v_0 \sin\alpha)t - \frac{1}{2}gt^2$$

$$z_f = 0 \Rightarrow t_f = \frac{2v_0 \sin\alpha}{g} \Rightarrow x_f = \frac{2v_0^2 \sin\alpha\cos\alpha}{g}$$

1^{st} order in Ω:

$$\hat{\Omega} = \hat{z}\cos\theta + \hat{y}\sin\theta$$

$$\underset{\sim}{F}_{\text{coriolis}} = 2m\underset{\sim}{v} \times \underset{\sim}{\Omega}$$

$$m\ddot{x} = 2m(\cancel{v_y}\Omega_z - v_z\Omega_y) = -2m(\Omega\sin\theta)(v_0\sin\alpha - gt)$$

$$m\ddot{y} = 2m(v_z\cancel{\Omega_x} - v_x\Omega_z) = -2m(\Omega\cos\theta)(v_0\cos\alpha)$$

$$m\ddot{z} = -mg + 2m(v_x\Omega_y - \cancel{v_y\Omega_x}) = -mg + 2m(\Omega\sin\theta)(v_0\cos\alpha)$$

$$\ddot{z} = -g + 2V_0\Omega\sin\theta\cos\alpha$$
$$V_z = V_0\sin\alpha + (2V_0\Omega\sin\theta\cos\alpha - g)\,t$$
$$z = (V_0\sin\alpha)\,t + \tfrac{1}{2}(2V_0\Omega\sin\theta\cos\alpha - g)\,t^2$$

$$z_f = 0 \Rightarrow t_f = \frac{2V_0\sin\alpha}{g - 2V_0\Omega\sin\theta\cos\alpha} \simeq \frac{2V_0\sin\alpha}{g} + \frac{4V_0^2\Omega\sin\theta\sin\alpha\cos\alpha}{g^2}$$

$$\ddot{y} = -2V_0\Omega\cos\theta\cos\alpha$$
$$V_y = (-2V_0\Omega\cos\theta\cos\alpha)\,t$$
$$y = \tfrac{1}{2}(-2V_0\Omega\cos\theta\cos\alpha)\,t^2 = -(V_0\Omega\cos\theta\cos\alpha)\,t^2$$
$$y_f^{(\text{1st order})} = -(V_0\Omega\cos\theta\cos\alpha)\left(\frac{2V_0\sin\alpha}{g}\right)^2$$
$$\Delta y \equiv y_f^{(1)} - y_f^{(0)} = -\frac{4V_0^3\Omega\cos\theta}{g^2}\cos\alpha\sin^2\alpha$$

$$t_f = \frac{2v_0 \sin\alpha}{g - 2v_0 \Omega \sin\theta \cos\alpha} \simeq \left[t_f\right]_{\Omega=0} + \Omega \left[\frac{dt_f}{d\Omega}\right]_{\Omega=0}$$

$$\left[\frac{dt_f}{d\Omega}\right]_{\Omega=0} = \left[\frac{-2v_0 \sin\alpha\,(-2v_0 \sin\theta \cos\alpha)}{(g - 2v_0 \Omega \sin\theta \cos\alpha)^2}\right]_{\Omega=0} = \frac{4v_0^2 \sin\theta \sin\alpha \cos\alpha}{g^2}$$

$$t_f = \frac{2v_0 \sin\alpha}{g} + \frac{4v_0^2 \Omega \sin\theta \sin\alpha \cos\alpha}{g^2} + \mathcal{O}(\Omega^2)$$

$$\ddot{x} = -2v_0\Omega\sin\theta\sin\alpha + 2(\Omega\sin\theta)gt$$

$$v_x = v_0\cos\alpha - 2(v_0\Omega\sin\theta\sin\alpha)t + (\Omega\sin\theta)gt^2$$

$$x = (v_0\cos\alpha)t - (v_0\Omega\sin\theta\sin\alpha)t^2 + \frac{1}{3}(\Omega\sin\theta)gt^3$$

$$x_f^{(1st\ order)} = (v_0\cos\alpha)\left(\frac{2v_0\sin\alpha}{g} + \frac{4v_0^2\Omega\sin\theta\sin\alpha\cos\alpha}{g^2}\right)$$

$$-(v_0\Omega\sin\theta\sin\alpha)\left(\frac{2v_0\sin\alpha}{g}\right)^2$$

$$+\frac{1}{3}(\Omega\sin\theta)g\left(\frac{2v_0\sin\alpha}{g}\right)^3$$

$$-\frac{12}{3} + \frac{8}{3} = \underbrace{-\frac{4}{3}}$$

$$= \frac{2v_0^2\sin\alpha\cos\alpha}{g} + \frac{4v_0^3\Omega}{g^2}\sin\theta\sin\alpha\cos^2\alpha + \frac{v_0^3\Omega}{g^2}\sin\theta\sin^3\alpha\left(-4 + \frac{8}{3}\right)$$

$$\Delta x \equiv x_f^{(1)} - x_f^{(0)} = \frac{4v_0^3\Omega}{g^2}\sin\theta\left(\sin\alpha\cos^2\alpha - \frac{1}{3}\sin^3\alpha\right)$$

$$\begin{aligned} m\boldsymbol{a} &= \boldsymbol{F} + 2m\boldsymbol{v}\times\boldsymbol{\Omega} + m(\boldsymbol{\Omega}\times\boldsymbol{r})\times\boldsymbol{\Omega} \\ m\boldsymbol{a} &= \boldsymbol{F} + 2m\boldsymbol{v}\times\boldsymbol{\Omega} + m\,\Omega^2\rho\,\hat{\boldsymbol{\rho}} \end{aligned}$$

Let's go through two more examples to try to gain more insight into the less intuitive Coriolis term.

First, a quick question: for an object in the northern hemisphere moving due north, the Coriolis force points due ... ?

$$m\boldsymbol{a} \;=\; \boldsymbol{F} + 2m\boldsymbol{v}\times\boldsymbol{\Omega} + m(\boldsymbol{\Omega}\times\boldsymbol{r})\times\boldsymbol{\Omega}$$

$$m\boldsymbol{a} \;=\; \boldsymbol{F} + 2m\boldsymbol{v}\times\boldsymbol{\Omega} + m\,\Omega^2\rho\,\hat{\boldsymbol{\rho}}$$

Let's go through two more examples to try to gain more insight into the less intuitive Coriolis term.

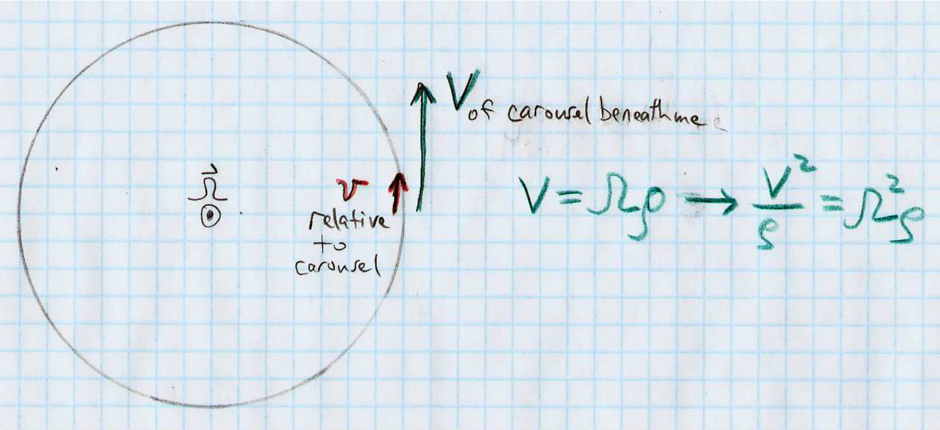
First, a quick question: for an object in the northern hemisphere moving due north, the Coriolis force points due ... ?

$$\hat{\boldsymbol{e}}_{\rm north} \;=\; \hat{\boldsymbol{\Omega}}\sin\theta - \hat{\boldsymbol{\rho}}\cos\theta$$

$$\hat{\boldsymbol{\Omega}}\times\hat{\boldsymbol{\Omega}} \;=\; 0 \qquad\qquad (-\hat{\boldsymbol{\rho}})\times\hat{\boldsymbol{\Omega}} \;=\; \hat{\boldsymbol{e}}_{\rm east}$$

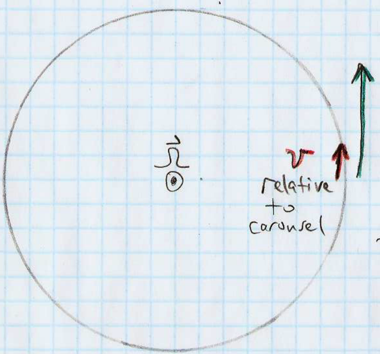
$$\hat{\boldsymbol{e}}_{\rm north}\times\hat{\boldsymbol{\Omega}} \;=\; (\cos\theta)\;\hat{\boldsymbol{e}}_{\rm east} \;=\; (\cos\theta)\;\hat{\boldsymbol{\phi}}$$

Centrifugal force only cares about ρ and always points in the $\hat{\boldsymbol{\rho}}$ direction. Coriolis force looks at $\hat{\boldsymbol{\rho}}$ and $\hat{\boldsymbol{\phi}}$ components of $\boldsymbol{v}$ and points the plane perpendicular to $\hat{\boldsymbol{\Omega}}$. So let's work in the $\perp$ plane, e.g. on a carousel.



If I am standing on the carousel and I want to move tangentially at constant speed v w.r.t. the rotating frame of the carousel (so I'm at constant radius ρ), how big must be the radial force of friction between the carousel and my feet?

Evaluate this in the inertial frame, in terms of v, V, and ρ.



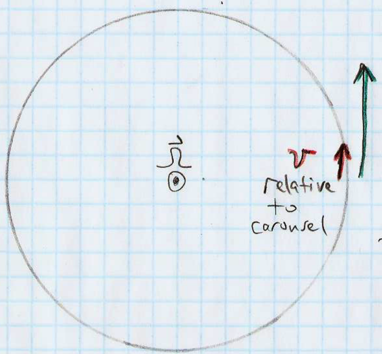
$$V = \Omega\rho \rightarrow \frac{V^2}{\rho} = \Omega^2\rho$$

$$\frac{mv_0^2}{\rho} = \frac{m(v+V)^2}{\rho} = \frac{mv^2}{\rho} + \frac{2mvV}{\rho} + \frac{mV^2}{\rho}$$

$$= \frac{mv^2}{\rho} + 2mv\Omega + m\Omega^2\rho$$

If I am standing on the carousel and I want to move tangentially at constant speed v w.r.t. the rotating frame of the carousel (so I'm at constant radius ρ), how big must be the radial force of friction between the carousel and my feet? ("v_0" is w.r.t. inertial frame.)

Now interpret each of the three terms above, from the perspective of the rotating frame.



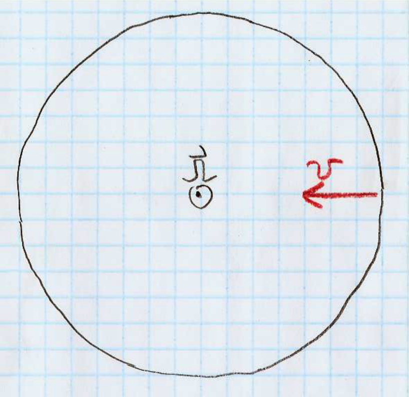
$$V=\Omega\rho \rightarrow \frac{V^2}{\rho}=\Omega^2\rho$$

$$\frac{mv_0^2}{\rho}=\frac{m(v+V)^2}{\rho}=\frac{mv^2}{\rho}+\frac{2mvV}{\rho}+\frac{mV^2}{\rho}$$

$$=\frac{mv^2}{\rho}+2mv\Omega+m\Omega^2\rho$$

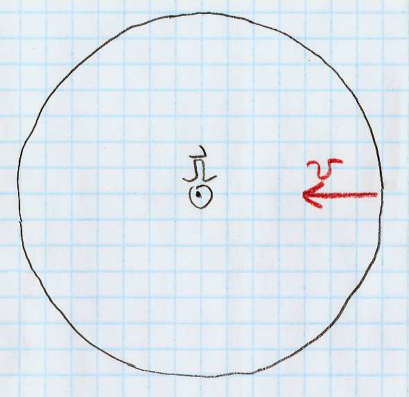
If I am standing on the carousel and I want to move tangentially at constant speed v w.r.t. the rotating frame of the carousel (so I'm at constant radius ρ), how big must be the radial force of friction between the carousel and my feet? (“v_0” is w.r.t. inertial frame.)

How would the result change if I were instead walking tangentially at (relative) speed v opposite the carousel's direction of rotation?



Now suppose I want to walk radially inward in the frame of the carousel, i.e. I want to walk along a radial line painted on the floor of the carousel.

Before I start to walk, when I'm just standing "still" at the outer radius of the carousel, what is the magnitude of the frictional force between the carousel floor and my feet? How does the balance of forces look in the rotating frame?



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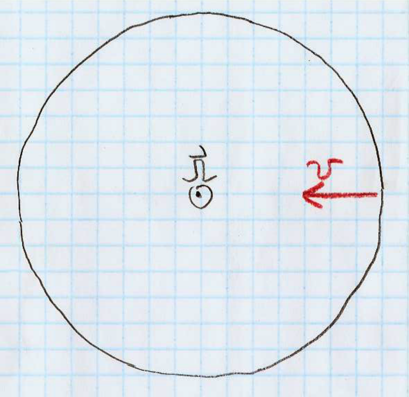
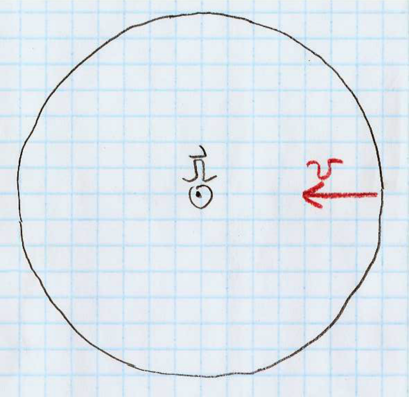
From perspective of carousel's rotating frame,

$$m\boldsymbol{a} = \boldsymbol{F}_{\text{friction}} + \boldsymbol{F}_{\text{centrifugal}} = 0$$



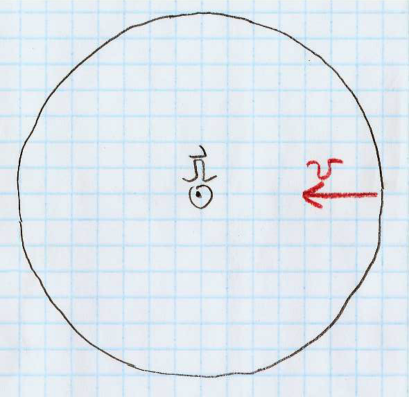
Now suppose I want to walk radially inward in the frame of the carousel, i.e. I want to walk along a radial line painted on the floor of the carousel.

Let my mass be m and let my radial position be ρ. Before I start to walk, what is my angular momentum (which you should evaluate in the inertial frame)?



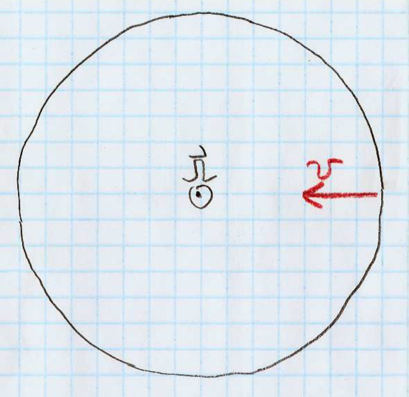
Now suppose I want to walk radially inward in the frame of the carousel, i.e. I want to walk along a radial line painted on the floor of the carousel.

Let my mass be m and let my radial position be ρ. As I walk radially inward with constant speed v (in carousel frame), what is the rate of change of my angular momentum (which you should evaluate in the inertial frame)?



Now suppose I want to walk radially inward in the frame of the carousel, i.e. I want to walk along a radial line painted on the floor of the carousel.

What [real] force (acting on my feet!) provides the torque that must equal the rate of change of my angular momentum as I walk inward? In what direction does that force point?



Now suppose I want to walk radially inward in the frame of the carousel, i.e. I want to walk along a radial line painted on the floor of the carousel.

In the rotating frame of the carousel, my tangential acceleration is zero. As I look from the frame of the carousel, what pseudoforce balances the tangential frictional force such that the net tangential acceleration is zero?

http://positron.hep.upenn.edu/p351/files/0223_morin_coriolis.pdf

A puck slides with speed v on frictionless ice. The surface is "level" in the sense that it is orthogonal to the effective (gravitational + centrifugal) $\boldsymbol{g}$ at all points. Show that the puck moves in a circle, as seen in Earth's rotating frame. (Assume that v is small enough that the radius of the circle is much smaller than the radius of Earth, so that the colatitude θ is essentially constant throughout the motion.) What is the radius of the circle? What is the frequency of the motion?

Let $\hat{x}$ point east, $\hat{y}$ point north, $\hat{z}$ point "up" so that $\vec{g} = -g\hat{z}$.

Earth's rotation vector is $\vec{\Omega}$ where $|\Omega| \simeq \frac{2\pi}{86400\,s} \simeq 7.3 \times 10^{-5}\,s^{-1}$, and

$$\hat{\Omega} = \cos\theta\,\hat{z} + \sin\theta\,\hat{y}$$

$$\vec{F}_{\text{coriolis}} = 2m\vec{v} \times \vec{\Omega}$$

$$= 2m\left[(V_y\Omega_z - \overset{0}{V_z}\Omega_y)\hat{x} + (\overset{0}{V_z}\Omega_x - V_x\Omega_z)\hat{y} + (V_x\Omega_y - V_y\overset{0}{\Omega_x})\hat{z}\right]$$

Ignore F_z, since we're on frictionless ice, and since we'll assume $|V_x\Omega_y| << g$, so puck will not go airborne.

$$m\ddot{x} = 2m v_y \Omega_z = 2m\Omega\cos\theta\, v_y = m\dot{v}_x$$
$$m\ddot{y} = -2m v_x \Omega_z = -2m\Omega\cos\theta\, v_x = m\dot{v}_y$$

$$\dot{v}_x = (2\Omega\cos\theta)\, v_y \quad , \quad \dot{v}_y = -(2\Omega)\cos\theta\, v_x$$

let $\eta = v_x + i v_y = A e^{i\omega t} \longrightarrow \dot{\eta} = i\omega\eta$

$$\dot{v}_x + i\dot{v}_y = i\omega v_x - \omega v_y \longrightarrow \dot{v}_x = -\omega v_y \,,\; \dot{v}_y = \omega v_x$$
$$\Longrightarrow \boxed{\omega = -2\Omega\cos\theta}$$

$$x + iy = \frac{A}{i\omega}(\cos\omega t + i\sin\omega t) = \frac{|A|e^{i\delta}}{i\omega}(\cos\omega t + i\sin\omega t)$$

$$x = \frac{|A|}{\omega}\sin(\omega t+\delta) \qquad y = -\frac{|A|}{\omega}\cos(\omega t+\delta)$$

which describes a circle of radius $\dfrac{|A|}{|\omega|} = \boxed{\dfrac{v}{2\Omega\cos\theta}}$
and angular frequency $2\Omega\cos\theta$

If $\theta = 45^\circ$, $v = 1\,\frac{m}{s}$, then $R \simeq 10\,\text{km}$! (small effect!)

Physics 351 — Monday, February 26, 2018

- You just read the first half (§10.1–10.7) of Chapter 10, which we'll probably start to discuss this Friday.
- The midterm exam (**March** 26) will cover (only!) chapters 7,8,9. But we will be working on Ch 10 by then.