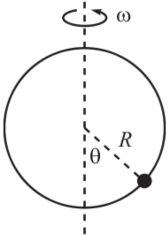
Physics 351 — Wednesday, April 25, 2018

- Final exam: Thursday, May 3, 9am–11am, DRL A2. Covers chapters 7,9,10,13. One hand-written 3×5 card OK.
- Last day to turn in XC is Sunday, May 6 (three days after the exam). For the few people who did Perusall (sorry!), I will factor that in as XC.
- If you felt that you just could not get enough of Feynman's introduction to quantum ideas, and if you have free time in the coming 1.5 weeks, feel free to read for XC (and summarize in writing) the first 3 or 4 chapters of Volume III (QM) of the Feynman Lectures on Physics.
- There is interest in forming study groups to help review or catch up on material from this course, as the semester winds down. Learning physics really is a lot more fun when it is done cooperatively. To try to faciliate this, I created a Canvas discussion area, but so far **only 3 people** have followed up.
- Do you want a review session?

Question:

- When is $\mathcal{H}$ conserved (i.e. a constant of the motion)?
- When does $\mathcal{H}$ equal the total energy?
- Notice that these are two different questions.

Morin 15.11. A bead is free to slide along a frictionless hoop of radius R. The hoop is forced to rotate with constant angular speed ω around a vertical diameter. Find $\mathcal{H}$ in terms of θ and p_θ, then write down Hamilton's equations. Is $\mathcal{H}$ the energy? Is $\mathcal{H}$ conserved?



[My solution is here in the notes, just FYI.]

$\phi = \omega t \rightarrow \dot{\phi} = \omega$ $\quad$ $(x,y) = R(\cos\omega t, \sin\omega t)$ which depends on time explicitly

$T = \frac{m}{2}\left(R^2\dot{\theta}^2 + (R\sin\theta)^2\omega^2\right)$ $\qquad U = -mgR\cos\theta$

$\mathcal{L} = \frac{1}{2}mR^2\dot{\theta}^2 + \frac{1}{2}mR^2\omega^2\sin^2\theta + mgR\cos\theta$

$p = \frac{\partial \mathcal{L}}{\partial \dot{\theta}} = mR^2\dot{\theta} \Rightarrow \dot{\theta} = \frac{p}{mR^2}$

$T = \frac{1}{2}mR^2\left(\frac{p}{mR^2}\right)^2 + \frac{1}{2}mR^2\omega^2\sin^2\theta = \frac{p^2}{2mR^2} + \frac{1}{2}mR^2\omega^2\sin^2\theta$

$\mathcal{H} = p\dot{\theta} - \mathcal{L} = \frac{p^2}{mR^2} - \frac{p^2}{2mR^2} - \frac{1}{2}mR^2\omega^2\sin^2\theta - mgR\cos\theta$

$\mathcal{H} = \frac{p^2}{2mR^2} - \frac{1}{2}mR^2\omega^2\sin^2\theta - mgR\cos\theta$

$\dot{\theta} = \frac{\partial \mathcal{H}}{\partial p} = \frac{p}{mR^2}$ $\quad\Big\}\quad$ $\dot{p} = -\frac{\partial \mathcal{H}}{\partial \theta} = \frac{\partial}{\partial \theta}\left(\frac{mR^2\omega^2}{2}\sin^2\theta + mgR\cos\theta\right)$

$\dot{p} = mR^2\omega^2\sin\theta\cos\theta - mgR\sin\theta$

$\rightarrow \ddot{\theta} = \omega^2\sin\theta\cos\theta - \frac{g}{R}\sin\theta$

What is the energy? $\frac{p^2}{2mR^2} + \frac{1}{2}mR^2\omega^2\sin^2\theta - mgR\cos\theta \neq \mathcal{H}$

It seems that $\mathcal{H}$ = energy $- mR^2\omega^2\sin^2\theta$

$\frac{\partial \mathcal{L}}{\partial t} = -\frac{\partial \mathcal{H}}{\partial t} = 0 \Rightarrow \mathcal{H}$ is conserved!
(but it's not the energy)

$\rightarrow$ energy $=$ const. $+ mR^2\omega^2\sin^2\theta$

(We can show by explicit calculation that $\frac{dH}{dt} = 0$)

$$mR^2\ddot{\theta}\,\boxed{\dot{\theta}} = mR^2\omega^2\sin\theta\cos\theta\,\boxed{\dot{\theta}} - mgR\sin\theta\,\boxed{\dot{\theta}}$$

$d\theta = \dot{\theta}\,dt$

$d\dot{\theta} = \ddot{\theta}\,dt$

$\frac{1}{2}d(\dot{\theta}^2) = \dot{\theta}\ddot{\theta}\,dt$

$$\frac{d}{dt}\left(\frac{1}{2}mR^2\dot{\theta}^2\right) = mR^2\omega^2\sin\theta\cos\theta\,\dot{\theta} - mgR\sin\theta\,\dot{\theta}$$

$$= \frac{d}{dt}\left(\frac{p^2}{2mR^2}\right)$$

$$\rightarrow \frac{dH}{dt} = \frac{d}{dt}\left(\frac{p^2}{2mR^2} - \frac{1}{2}mR^2\omega^2\sin^2\theta - mgR\cos\theta\right)$$

$$= \frac{d}{dt}\left(\frac{p^2}{2mR^2}\right) - mR^2\omega^2\sin\theta\cos\theta\,\dot{\theta} + mgR\sin\theta\,\dot{\theta} = 0 \quad \checkmark$$

The quantity $mR^2\omega^2\sin^2\theta$ turns out to be the work done on the bead by the force of constraint that enforces $\phi = \omega t$.

$$\tau_z = \frac{dL_z}{dt} = \frac{d}{dt}\left(m(R\sin\theta)^2\omega\right) = 2mR^2\omega\sin\theta\cos\theta\,\dot{\theta}$$

Power delivered by this torque $= \frac{d(\text{work})}{dt} = \omega\tau_z = 2mR^2\omega^2\sin\theta\cos\theta\,\dot{\theta}$

$$= \frac{d}{dt}\left(mR^2\omega^2\sin^2\theta\right)$$

Question from Feynman/Hibbs reading: "In what way do the classical laws of motion arise from the quantum laws?"

One answer: In the classical approximation, $S \gg \hbar$ so the phase contribution is very large. So the action S is an extremum for the special path $\overline{x}(t)$. All the contributions for the paths in this region are nearly in phase at $S_{\text{classical}}/\hbar$ and do not cancel out. So, in the classical limit we only need to consider the paths in the vicinity of $\overline{x}(t)$ as giving important contributions [to the quantum-mechanical amplitude]. So in this way, the classical laws of motion arise from the quantum laws.

[I elaborate on this in the next few slides, which we'll skip. They're here for reference, if you're curious.]

In the reading, Feynman argued that the classical action

$$S_{\mathrm{cl}} = \int_{t_i}^{t_f} \mathcal{L}(\dot{x}(t), x(t), t)\,\mathrm{d}t$$

is proportional to the trajectory's quantum-mechanical phase:

$$\text{phase} = S_{\mathrm{cl}}/\hbar$$

Many of you noticed that Feynman's Problems 2-4 and 2-5 suggest a way to prove, using calculus of variations, that

$$(p)_{x=x_f} = \left(\frac{\partial \mathcal{L}}{\partial \dot{x}}\right)_{x=x_f} = +\frac{\partial S_{\mathrm{cl}}}{\partial x_f} \qquad \text{and} \qquad E = \mathcal{H} = -\frac{\partial S_{\mathrm{cl}}}{\partial t_f}$$

Here's another route to that result: Remember that

$$\mathcal{H} = p\dot{x} - \mathcal{L} \qquad \Rightarrow \qquad \mathcal{L} = p\dot{x} - \mathcal{H}$$

So we can rewrite the classical action as

$$S_{\mathrm{cl}} = \int_{t_i}^{t_f} (p\dot{x} - \mathcal{H})\,\mathrm{d}t = \int_{t_i}^{t_f} p\dot{x}\,\mathrm{d}t - \int_{t_i}^{t_f} \mathcal{H}\,\mathrm{d}t = \int_{x_i}^{x_f} p\,\mathrm{d}x - \int_{t_i}^{t_f} \mathcal{H}\,\mathrm{d}t$$

$$S = \int_{t_i}^{t} \mathcal{L}\,\mathrm{d}t = \int_{t_i}^{t} (p\dot{x} - \mathcal{H})\,\mathrm{d}t = \int_{x_i}^{x} p\,\mathrm{d}x - \int_{t_i}^{t} \mathcal{H}\,\mathrm{d}t$$

Therefore,

$$\left(\frac{\partial S}{\partial t}\right)_{\text{fixed x}} = -\mathcal{H} \qquad \text{and} \qquad \left(\frac{\partial S}{\partial x}\right)_{\text{fixed t}} = p$$

$\partial S/\partial t + \mathcal{H} = 0$ is the "Hamilton-Jacobi equation." If we plug in

$$\mathcal{H} = \frac{p^2}{2m} + U(x)$$

we can write this differential equation for the classical action:

$$\frac{\partial S}{\partial t} + \frac{1}{2m}\left(\frac{\partial S}{\partial x}\right)^2 + U(x) = 0$$

or in three dimensions,

$$\frac{\partial S}{\partial t} + \frac{1}{2m}(\nabla S)^2 + U(\boldsymbol{r}) = 0$$

What is this telling us?

$$\frac{\partial S}{\partial t} = -E \qquad \frac{\partial S}{\partial x} = p_x \qquad \frac{\partial S}{\partial y} = p_y \qquad \frac{\partial S}{\partial z} = p_z \qquad \nabla S = \boldsymbol{p}$$

For constant energy, an action of the form

$$S(\boldsymbol{r}, t) = \boldsymbol{p} \cdot \boldsymbol{r} - Et$$

satisfies these equations. Notice that momentum $\boldsymbol{p}$ is $\perp$ to surface of constant S. Near the classical path, moving $\perp$ to the trajectory does not change the action — as we expect from the "principle of stationary action."

In Physics 250, you may have described "matter waves" using the de Broglie relations $\boldsymbol{p} = \hbar \boldsymbol{k}$ and $E = \hbar\omega$. This suggests

$$S(\boldsymbol{r}, t)/\hbar = \boldsymbol{k} \cdot \boldsymbol{r} - \omega t$$

which describes the phase of a plane wave.

Meanwhile, the Hamilton-Jacobi equation

$$\frac{\partial S}{\partial t} + \frac{1}{2m}(\nabla S)^2 + U(\boldsymbol{r}) = 0$$

is starting to seem vaguely similar to Schrödinger's equation:

$$i\hbar \frac{\partial \psi(\boldsymbol{r},t)}{\partial t} = \left(-\frac{\hbar^2}{2m}\nabla^2 + U(\boldsymbol{r})\right) \psi(\boldsymbol{r},t)$$

Let's try plugging (into Schrödinger) a wavefunction

$$\psi(x,t) = \psi_0(x,t)\, e^{i\Sigma(x,t)/\hbar}$$

where $\psi_0(x,t)$ and $\Sigma(x,t)$ are real (i.e. not complex) functions.
So $|\psi_0|^2$ tells us about probability, and $\Sigma/\hbar$ tells us about phase.

$$\frac{\partial \psi}{\partial t} = \frac{\partial \psi_0}{\partial t}\, e^{i\Sigma/\hbar} + \left(\frac{i}{\hbar}\frac{\partial \Sigma}{\partial t}\right) \psi_0\, e^{i\Sigma/\hbar}$$

$$\frac{\partial^2 \psi}{\partial x^2} = \frac{\partial^2 \psi_0}{\partial x^2} e^{i\Sigma/\hbar} + \frac{2i}{\hbar}\frac{\partial \Sigma}{\partial x}\frac{\partial \psi_0}{\partial x} e^{i\Sigma/\hbar} + \frac{i}{\hbar}\frac{\partial^2 \Sigma}{\partial x^2}\psi_0 e^{i\Sigma/\hbar} - \frac{1}{\hbar^2}\left(\frac{\partial \Sigma}{\partial x}\right)^2 \psi_0 e^{i\Sigma/\hbar}$$

Plugging in and canceling common factor $\psi_0 e^{i\Sigma/\hbar}$ gives real part

$$\frac{\partial \Sigma}{\partial t} + \frac{1}{2m}\left(\frac{\partial \Sigma}{\partial x}\right)^2 + U = \frac{\hbar^2}{2m}\frac{1}{\psi_0}\frac{\partial^2 \psi_0}{\partial x^2}$$

which equals the Hamilton-Jacobi equation, "in $\hbar \to 0$ limit." So evidently in some classical limit, the phase Σ of Schrödinger's $\psi(x,t)$ satisfies the same diffeq. as does the classical action S.

The imaginary part gives (skip the math here)

$$\frac{\partial \psi_0}{\partial t} + \frac{1}{m}\frac{\partial \Sigma}{\partial x}\frac{\partial \psi_0}{\partial x} + \frac{1}{2m}\psi_0 \frac{\partial^2 \Sigma}{\partial x^2} = 0$$

which can be turned into (multiply by $2\psi_0$, use $\partial\Sigma/\partial x \to p$ if $\Sigma \to S$)

$$\frac{\partial}{\partial t}(\psi_0^2) + \frac{\partial}{\partial x}(\psi_0^2 \frac{1}{m}\frac{\partial \Sigma}{\partial x}) = 0$$

$$\frac{\partial}{\partial t}(\psi_0^2) + \nabla \cdot (\psi_0^2\, \boldsymbol{v}) = 0$$

which is (Taylor 16.130) just the continuity equation expressing conservation of probability (ψ_0^2) as the particle travels.

The Schrödinger equation gives us

$$\frac{\partial \Sigma}{\partial t} + \frac{1}{2m}\left(\frac{\partial \Sigma}{\partial x}\right)^2 + U = \frac{\hbar^2}{2m}\frac{1}{\psi_0}\frac{\partial^2 \psi_0}{\partial x^2}$$

while the classical Hamilton-Jacobi equation gave us

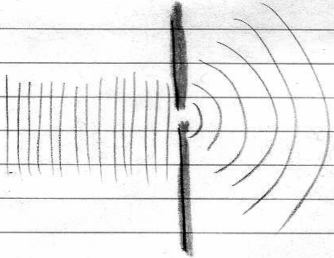
$$\frac{\partial S}{\partial t} + \frac{1}{2m}\left(\frac{\partial S}{\partial x}\right)^2 + U = 0$$

What does it mean for $\frac{\hbar^2}{2m}\frac{1}{\psi_0}\frac{\partial^2 \psi_0}{\partial x^2}$ to be "small?" Consider a gaussian distribution $\psi_0(x) \sim e^{-x^2/2\sigma^2}$. Then

$$\left(\frac{1}{\psi_0}\frac{\partial^2 \psi_0}{\partial x^2}\right)_{x=0} = -\frac{1}{\sigma^2} \sim \frac{1}{L^2}$$

where L is the length over which the probability for finding the particle varies appreciably, e.g. slit size, or distance over which $U(x)$ varies considerably. Then classical limit means

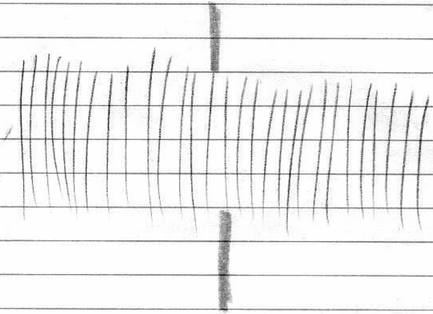
$$\frac{p^2}{2m} \gg \frac{\hbar^2}{2mL^2} \qquad p \gg \frac{\hbar}{L} \qquad L \gg \lambda_{\text{de Broglie}}$$



$L \sim \lambda$

(diffraction)

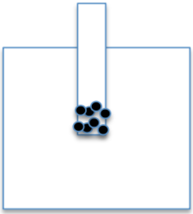
wave optics



$L \gg \lambda$

(ray optics is OK)

A glass tube whose horizontal cross-section is $1.0\,\mathrm{cm}^2$ floats vertically in water (as shown). What mass of lead pellets would you need to add to the tube in order to make it sink by $2\,\mathrm{cm}$?



How does pressure P vary with depth z beneath water surface?

What is the difference in pressure forces exerted on the bottom of the tube by the water vs. on the top of the tube by the atmosphere, as a function of immersed depth z?

What is the condition for static equilibrium?

depth z below water surface

$$P(z) = P_{atm} + \rho g z$$

A = tube's cross-sectional area

force on top $= P_{atm} \cdot A$ (downward)

force on bottom $= (P_{atm} + \rho g z) \cdot A$ (upward)

gravitational force on lead pellets $= Mg$ (downward)

$$(P_{atm} + \rho g z) A - P_{atm} \cdot A - Mg = 0 \Rightarrow \rho g z A = Mg$$

$$(\rho_{water})(Az) = M_{lead}$$

Az ← volume of displaced water

$(\rho_{water})(Az)$ ← mass of displaced water

$$(\rho_{water})(A \cdot \Delta z) = \Delta M_{lead} = \left(1 \frac{g}{cc}\right)(1\text{cm}^2 \;\; 2\text{cm})$$

= 2 grams

A sphere floats in water with 60% of its volume submerged. The same sphere floats in oil with 70% of its volume submerged. What is the density of the oil?

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$$(0.6\,V)\,\rho_{water} = M_{sphere} = (0.7V)\,\rho_{oil}$$

$$\rightarrow \rho_{oil} = \frac{6}{7}\,\rho_{water} \simeq 0.86\ \text{g/cc}$$

By the way, what is the density of water in SI units?

The “gauge pressure” (i.e. pressure w.r.t. $P_{\rm atm}$) in the tires of a car is 200 kPa. (That’s plausible: about 2 atm.) The area of each tire in contact with the road is $120\ \mathrm{cm}^2$. What is the mass of the car? (We’re assuming that the tire is initially perfectly round, then deforms until the pressure force balances the weight. I’m not sure how realistic this is.)

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$$Mg = 4\,(.0120\,\mathrm{m}^2)\left(200\times10^3\ \frac{\mathrm{N}}{\mathrm{m}^2}\right)$$

$$M = \frac{4\,(12\times10^{-3}\,\mathrm{m}^2)\left(200\times10^3\ \frac{\mathrm{N}}{\mathrm{m}^2}\right)}{9.8\ \mathrm{m/s^2}} = 980\ \mathrm{kg}$$

$(\sim 1\ \text{tonne})$

By the way, what is 1 atm in SI units? What's the density of mercury? How tall a column of mercury has its weight-per-area balanced by atmospheric pressure?

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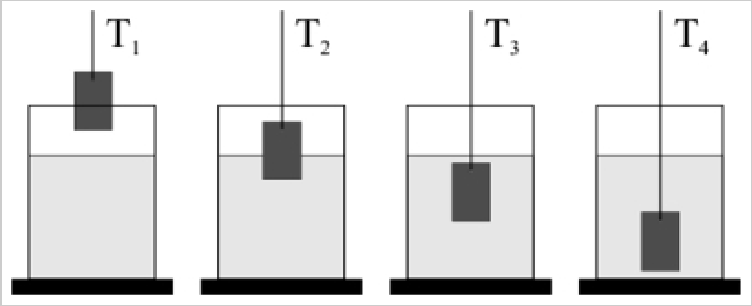
1 atm $= 101325$ Pa. $\rho_{\mathrm{Hg}} \approx 13.6\rho_{\mathrm{H_2O}} \approx 13600\ \mathrm{kg/m^2}$.

$101325/(13600\times9.8) = 0.760\ \mathrm{m} = 760\ \mathrm{mm}$.

A container of liquid (on Earth) has a vertical acceleration of a upward. (Maybe it's on an elevator that is just starting upward, or maybe it's on a rocket that was just launched.) How does the absolute pressure vary as a function of depth in the container?

A brick sinks when it is dropped into a bucket of water. Suppose that the same brick is supported by a string and slowly lowered (at constant speed) into a bucket of water. How do the tensions in the string compare at the four positions shown?



(A) $T_1 > T_2 > T_3 > T_4$

(B) $T_1 < T_2 < T_3 < T_4$

(C) $T_1 > T_2 > T_3 = T_4$

(D) $T_1 = T_2 = T_3 = T_4$

A boat carrying a large boulder is floating on a small lake. The boulder is thrown overboard and sinks. As a result, the water level (with respect to the bottom of the lake)

(A) rises

(B) drops

(C) remains the same

Do you want a hint? (Hint on next page.)

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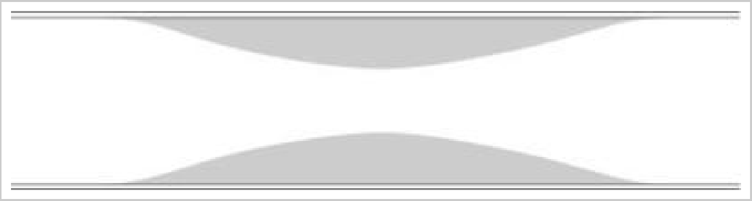
(A) rises

(B) drops

(C) remains the same

Hint: What volume of water is displaced by floating the boulder inside the boat? What volume of water is displaced by sinking the boulder into the lake?

Water flows through an old plumbing pipe that is partially blocked by mineral deposits along the wall of the pipe. Through which part of the pipe is the fluid speed largest?

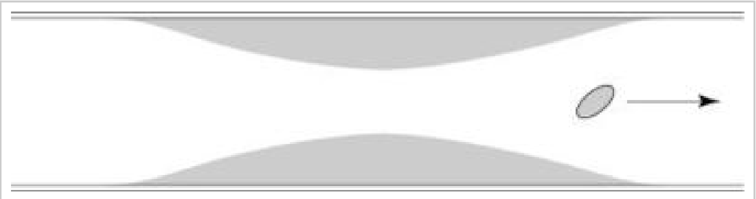


(A) Fastest in the narrow part.

(B) Fastest in the wide part.

(C) The speed is the same in both parts.

A water-borne insect drifts along with the flow of water through a pipe that is partially blocked by deposits. As the insect drifts from the narrow region to the wider region, it experiences

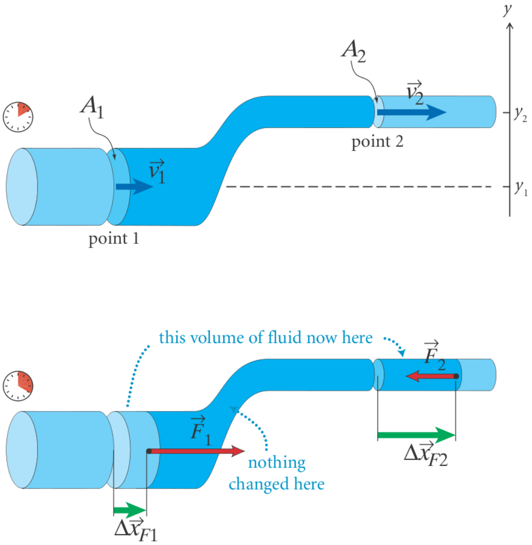


(A) an increase in pressure ($P_{\text{wide}} > P_{\text{narrow}}$)

(B) no change in pressure ($P_{\text{wide}} = P_{\text{narrow}}$)

(C) a decrease in pressure ($P_{\text{wide}} < P_{\text{narrow}}$)

Hint: is the speed the same or different? Is there a net force (per unit area) that causes this change in speed?

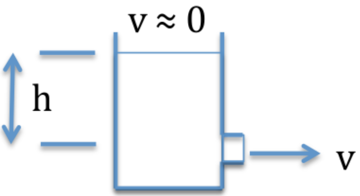


$$(P_1A_1)(v_1\Delta t) - (P_2A_2)(v_2\Delta t) = \frac{1}{2}m(v_2^2 - v_1^2) + mg(y_2 - y_1)$$

$$(P_1 - P_2)(\text{Volume}) = (P_1 - P_2)\frac{m}{\rho} = \frac{1}{2}m(v_2^2 - v_1^2) + mg(y_2 - y_1)$$

4. Water emerges from a small hole at the bottom of a large tank, as shown in the figure. Assume that the downward velocity of the water at the top of the tank is essentially zero. If the depth of the water is h, show that the water emerges from the hole with velocity

That is, the speed of the emerging fluid is the same as that of a particle that fell freely through the same distance. This surprising result is known as "Torricelli's theorem".



4. Water emerges from a small hole at the bottom of a large tank, as shown in the figure. Assume that the downward velocity of the water at the top of the tank is essentially zero. If the depth of the water is h, show that the water emerges from the hole with velocity

$$v = \sqrt{2gh}\,.$$

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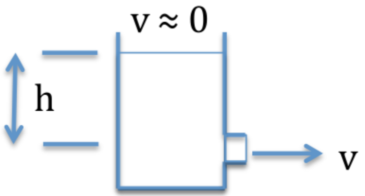
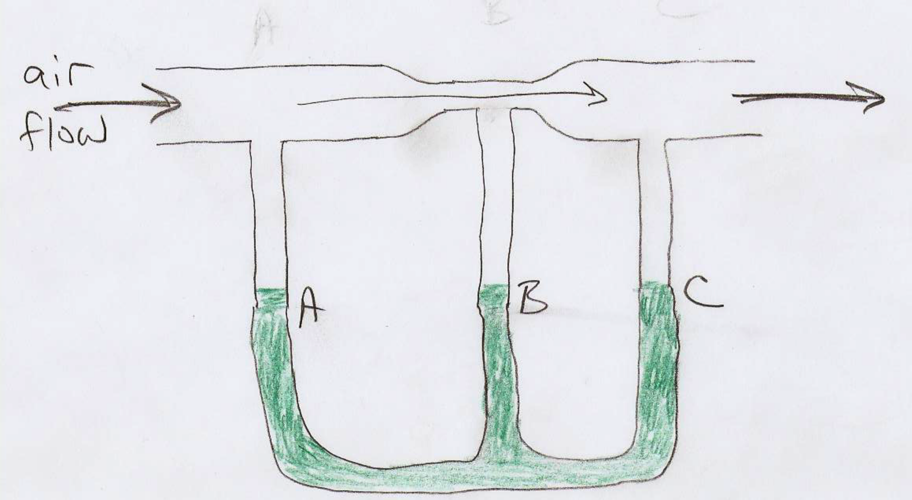
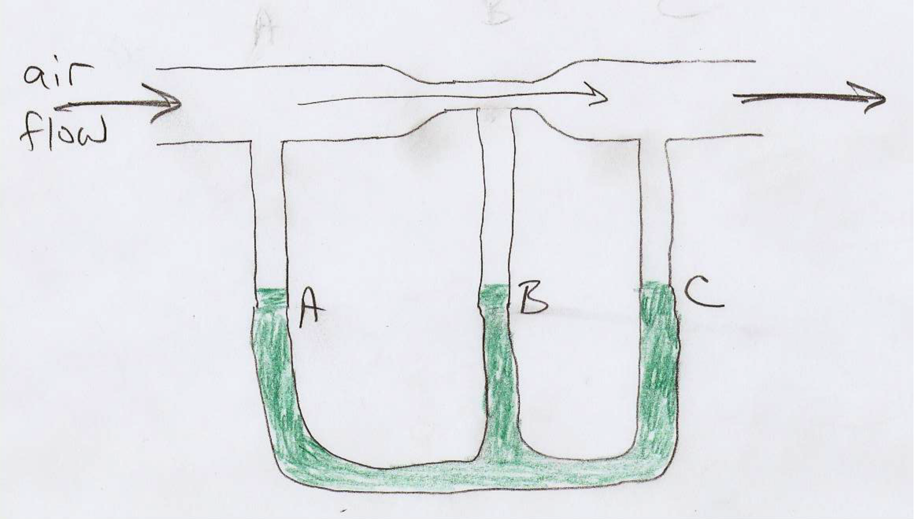


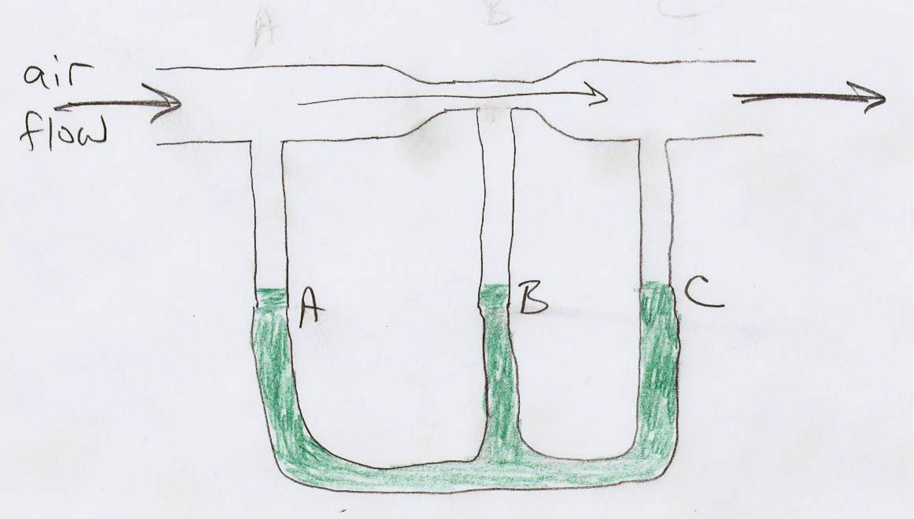
This experiment combines the "equation of continuity" with Bernoulli's equation. When I open the valve, compressed air will flow from left to right through the horizontal tube. The tube is wide on the left and right, but narrow in the middle. Where is the speed largest? Where is the pressure lowest? How will height of the green liquid respond to changes in pressure in the horizontal tube?





Once I turn on the air flow, the horizontal speed of the flowing air will be

(A) fastest above tube B
(B) slowest above tube B
(C) the same above all tubes



Once I turn on the air flow, the height of the green liquid will be

(A) lowest in tube B

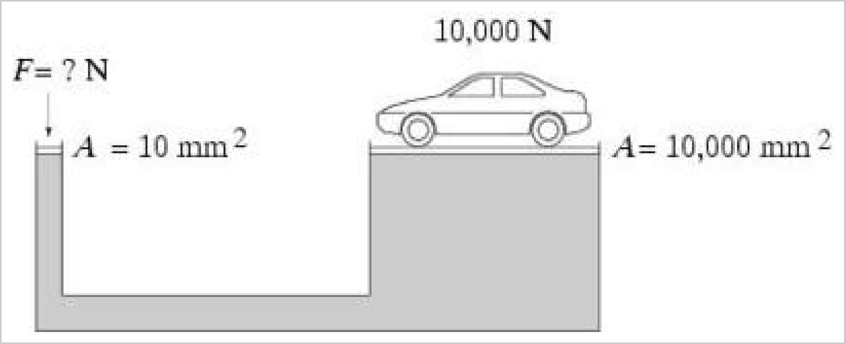
(B) highest in tube B

(C) the same in all tubes

“Pascal’s principle:” a pressure change applied to a confined incompressible fluid is transmitted undiminished throughout the fluid and to the walls of the container in contact with the fluid.

Have you ever used one of these handy devices? You can get a “mechanical advantage” that is huge compared with what you can easily get with e.g. a block and tackle.

A container is filled with oil and fitted on both ends with pistons. The area of the left piston is $10\ \mathrm{mm}^2$. The area of the right piston is $10000\ \mathrm{mm}^2$. What force must be exerted on the left piston to keep the 10000 N car on the right at the same height?



(A) 10 N

(B) 100 N

(C) 1000 N

(D) 10000 N

- In these past few months, you've learned a whole new method of solving the Newtonian mechanics problems that are likely what sparked your interest in physics as a high-school student.
- You've deepened your understanding of conservation laws and their connection to symmetries (invariance under spatial translation, rotation, time translation).
- You've learned how to work with the fictitious forces that arise when working in non-inertial reference frames.
- Turning "moment of inertia" into a 3×3 matrix has disrupted, in yet one more way, your high-school belief that you knew everything that mattered about classical mechanics.
- I think that for as long as you live, you'll never forget $\boxed{\frac{\partial \mathcal{L}}{\partial x} = \frac{\mathrm{d}}{\mathrm{d}t} \frac{\partial \mathcal{L}}{\partial \dot{x}}}$ or $\boxed{\left(\frac{\mathrm{d}}{\mathrm{d}t}\right)_{\mathrm{space}} = \left(\frac{\mathrm{d}}{\mathrm{d}t}\right)_{\mathrm{body}} + \boldsymbol{\omega} \times}$.
- Along the way, you learned how to use the first of those two equations to solve a whole new class of calculus problem.
- You've learned the Hamiltonian formalism, which I hope will make quantum mechanics easier to assimilate.

- You've read (not just scanned for equations while doing HW) 13 chapters of Taylor's book, plus a few gems from Feynman.
- You've dabbled in a few other topics that may be less familiar, like drag forces, coupled oscillators, chaos, and fluids.
- You've solved 107 HW problems, plus some XC problems. Believe it nor not, this number was 128 (145!) in 2017 (2015). This is the first year that people have consistently told me that the length of the homeworks has been about right.
- Thanks to Taylor's gently weaving math-methods review into many chapters of his book, you've reviewed and practiced, perhaps in a new context, a lot of the math you've learned in recent years. ("Vegetable curry?")
- You've made so many small-amplitude approximations to various equations of motion this semester that I doubt you'll ever need reminding of why Hooke's law is so ubiquitous.

- I'd like your feedback on what to change or not to change if I teach this course again two years from now.
- While Taylor's writing is a bit verbose, the book is very clear and easy to learn from. Keep it, yes?
- With a highly readable book like Taylor's, I think required reading really facilitates your learning. Then, when we discuss a topic in class, you're not seeing it for the first time. (This idea is still controversial in physics education.) Does it help?
- Using Perusall as an alternative to my reading questions would be a distaster, right?

- I think Taylor's brief review of Phys 150/170 topics (ch1–4) is worth reading, but quickly. This year, I had you start working Lagrangian mechanics problems 1–2 weeks earlier in the semester than in past years.
  - spread the many Lagrangian problems out over more weeks;
  - reduced number of 150/170 repeat problems in early weeks;
  - in turn, reduced overall number of HW problems a bit;
  - this let me budget more time later for Ch10 (rotation);
  - far fewer complaints this year about length of homeworks!
- I am tempted to write up a "Phys 351 compendium" of key results, chapter by chapter, along the lines of Prof. Mele's Phys 361/362 "equation sheet." [Yes!]
- Start working Lagrangian problems **even earlier**, before seeing where it comes from? Read ch1–4 even faster? [No!]
- (I try to plan the semester around what you read when.)

- Is our mainly working through problems together in class a better use of class time than going through all of the book's derivations, as if you hadn't read the book?
- Do you have ideas for getting more benefit, during class time, from the fact that your neighbors are there to help you to think things through? I think the class time is more fruitful when you're interacting with each other or with me, not just watching me. I'd like to know how to do that better.
- I enjoy our classroom time most when we all seem to be on the same page, when there is some time for you to talk with each other, and where we go slowly, step-by-step, through a manageable list of materials, without trying to hurry.

- Does making exam problems closely resemble HW problems have the intended effect of motivating you to make sure you use your own brain to solve each HW problem?
- Have two midterms instead of one? Have quizzes, at least for the first half of the semester, to give you more feedback? Make the midterm earlier (which the structure of Taylor's book makes difficult to do)?

- Does the predictable pace of the course work help you to plan your week, know what you're responsible for learning, and avoid falling behind? (I know at times the reading gets a few days ahead of what we do in class; I try to balance that against keeping the reading/homework schedule predictable.)
- Was learning to use Mathematica worthwhile?
- Should physics majors be encouraged (but not required) to take Phys 351 as sophomores, and preferably before you learn quantum mechanics? By contrast, should 351 be optional?
- If you have opinions on any of these points (or others!), I'd love to hear your thoughts either openly (by email or in person) or anonymously (via course review). This course is here for your benefit. Your input can improve Phys 351 for future students. For anonymous input, perhaps type it up now, then paste it in when you enter your course comments.

- My tentative plan (pending your feedback), if I teach this course again in the coming years, is:
- Keep the midterm late, to cover chapters 7,8,9.
- Do weekly quizzes, up until midterm week, but scale them generously, such that getting 66% is enough for full credit.
- Type up a compendium, to make it easier to digest Taylor's long chapters, and to make it easier for you to see the few key points in each chapter.
- Work some of the easier Lagrangian problems into the earliest homeworks, even before you see the formal derivation; spend even less time on ch1–4 review. [No! No!! No!!!]
- Try to avoid our falling behind around midterm & chapter 10.
- Find some convincing examples of the usefulness of the Hamiltonian formalism.
- Plan out each class so that the hour feels relaxed and is more interactive, so that you have a couple of occasions per class to see if you and your neighbor are on the same page.

From Richard Feynman (Feynman Lectures):

- “I think, however that there isn't any solution to this problem of education other than to realize that the best teaching can be done only when there is a direct individual relationship between a student and a good teacher — a situation in which the student discusses the ideas, thinks about the things, and talks about the things. It's impossible to learn very much by simply sitting in a lecture . . . .”

From Mary Boas (Mathematical Methods):

- “One point about your study of this material cannot be emphasized too strongly: To use mathematics effectively in applications, you need not just knowledge but skill. Skill can be obtained only through practice. You can obtain a certain superficial knowledge of mathematics by listening to lectures, but you cannot obtain skill this way. How many students have I heard say, ‘It looks so easy when you do it,’ or ‘I understand it but I can't do the problems!’ Such statements show lack of practice . . . .”

I've tried to focus your time in this course on solving (worthwhile) problems, and I've tried to facilitate your “talking about the things” with me, Grace, and each other as much as possible.

- Since so much knowledge can be found online now, the role of the college teacher has shifted from being "the sage on the stage" (lecturing at a podium) to being "the guide on the side" (coaching you to develop your own problem-solving skills).
- My role this term, as I see it, was to "coach" you as you worked your way through Lagrangian mechanics, fictitious forces, rigid-body rotation, Hamiltonian mechanics, and so on.
- The right coach, trainer, piano teacher, etc. will help you to make efficient use of the time that you invest. But it's still up to you to do the work to build your own skills. It's **you** who own what you learn here.
- You've learned a whole new way to solve mechanics problems, and solved 107+ homework problems along the way. I'm very happy with the hard work you've done this term. I hope this has been a fun and useful course for you, and generally not a source of stress.
- I've had great fun this semester working with you, discussing physics in person or by email, getting to know most of you, and being your "coach" this term for Phys 351. I've enjoyed every moment of it. Best wishes on your final exams, summer jobs, and whatever you decide to pursue next!

Physics 351 — Wednesday, April 25, 2018

- Final exam: Thursday, May 3, 9am–11am, DRL A2. Covers chapters 7,9,10,13. One hand-written 3×5 card OK.
- Last day to turn in XC is Sunday, May 6 (three days after the exam). For the few people who did Perusall (sorry!), I will factor that in as XC.
- If you felt that you just could not get enough of Feynman's introduction to quantum ideas, and if you have free time in the coming 1.5 weeks, feel free to read for XC (and summarize in writing) the first 3 or 4 chapters of Volume III (QM) of the Feynman Lectures on Physics.
- There is interest in forming study groups to help review or catch up on material from this course, as the semester winds down. Learning physics really is a lot more fun when it is done cooperatively. To try to faciliate this, I created a Canvas discussion area, but so far **only 3 people** have followed up.
- Do you want a review session?