

```
In[84]:= ClearAll["Global`*"];
```

```
In[85]:= y1 := L1 Cos[φ1[t]] ;
x1 := L1 Sin[φ1[t]] ;
y2 := y1 + L2 Cos[φ2[t]] ;
x2 := x1 + L2 Sin[φ2[t]] ;
x1dot := D[x1, t];
y1dot := D[y1, t];
x2dot := D[x2, t];
y2dot := D[y2, t];
T := (m1 / 2) (x1dot^2 + y1dot^2) + (m2 / 2) (x2dot^2 + y2dot^2)
FullSimplify[T]
```

$$\text{Out[94]} = \frac{1}{2} \left(L1^2 (m1 + m2) \phi_1'[t]^2 + 2 L1 L2 m2 \cos[\phi_1[t] - \phi_2[t]] \phi_1'[t] \phi_2'[t] + L2^2 m2 \phi_2'[t]^2 \right)$$

```
In[95]:= (* note that I defined y axis to point downward *)
```

```
U := -m1 g y1 - m2 g y2
```

```
L := T - U
```

```
FullSimplify[L]
```

$$\text{Out[97]} = \frac{1}{2} \left(2 g (L1 (m1 + m2) \cos[\phi_1[t]] + L2 m2 \cos[\phi_2[t]]) + L1^2 (m1 + m2) \phi_1'[t]^2 + 2 L1 L2 m2 \cos[\phi_1[t] - \phi_2[t]] \phi_1'[t] \phi_2'[t] + L2^2 m2 \phi_2'[t]^2 \right)$$

```
In[98]:= eomexpr1 := D[D[L, φ1'[t]], t] == D[L, φ1[t]]
```

```
eom1 := FullSimplify[eomexpr1]
```

```
eom1
```

$$\text{Out[100]} = L1 \left(L2 m2 \sin[\phi_1[t] - \phi_2[t]] \phi_2'[t]^2 + (m1 + m2) (g \sin[\phi_1[t]] + L1 \phi_1''[t]) + L2 m2 \cos[\phi_1[t] - \phi_2[t]] \phi_2''[t] \right) == 0$$

```
In[101]:= eomexpr2 := D[D[L, φ2'[t]], t] == D[L, φ2[t]]
```

```
eom2 := FullSimplify[eomexpr2]
```

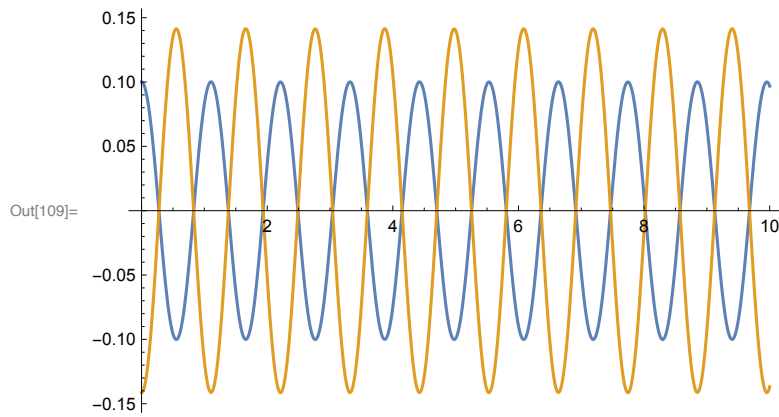
```
eom2
```

$$\text{Out[103]} = L2 m2 \left(g \sin[\phi_2[t]] - L1 \sin[\phi_1[t] - \phi_2[t]] \phi_1'[t]^2 + L1 \cos[\phi_1[t] - \phi_2[t]] \phi_1''[t] + L2 \phi_2''[t] \right) == 0$$

```

In[104]:= L1 = 1; L2 = 1; m1 = 1; m2 = 1; g = 9.8;
a = 0.1; b = -Sqrt[2]; tmax = 10;
soln = NDSolve[
  {eom1, eom2,
    $\phi_1'[0] == 0$ ,  $\phi_2'[0] == 0$ ,
    $\phi_1[0] == a$ ,  $\phi_2[0] == a b$ },
  { $\phi_1[t]$ ,  $\phi_2[t]$ },
  {t, 0, tmax}];
phi1[t_] := Evaluate[ $\phi_1[t]$  /. First[soln]]
phi2[t_] := Evaluate[ $\phi_2[t]$  /. First[soln]]
Plot[{phi1[t], phi2[t]}, {t, 0, tmax}]

```

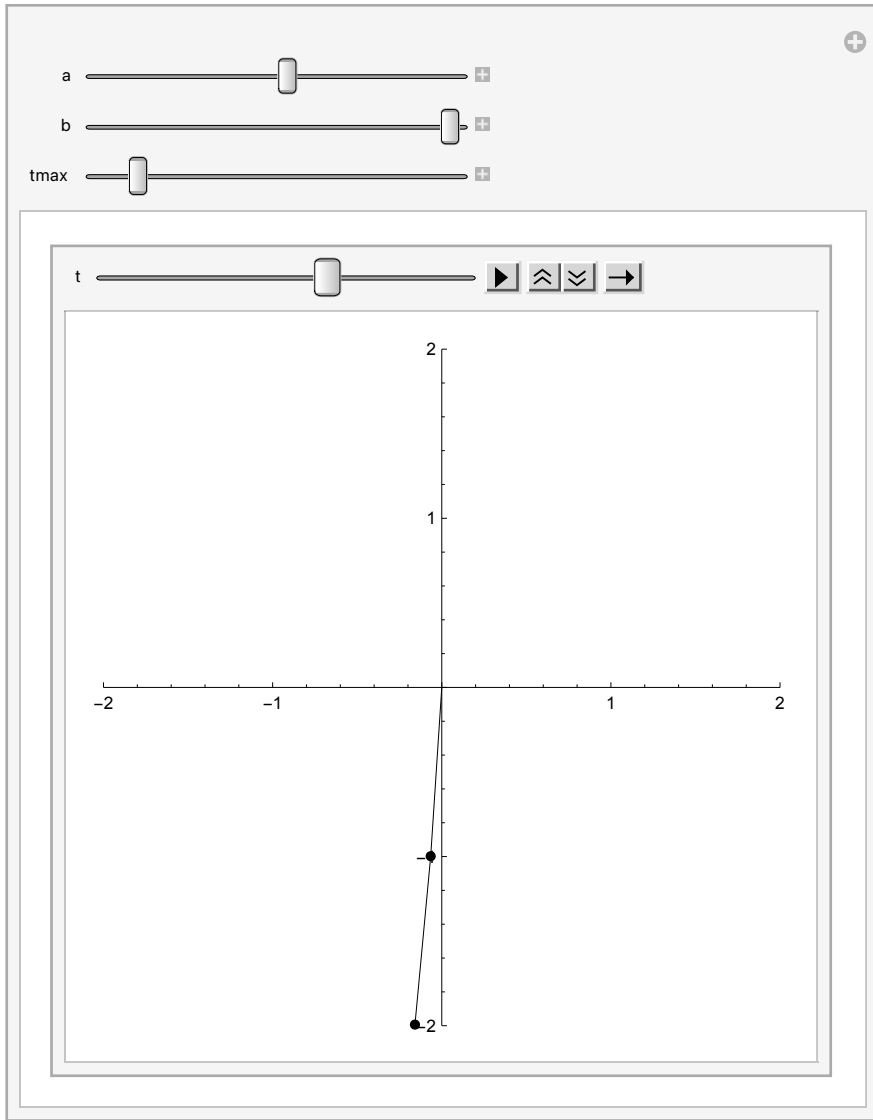


```

In[110]:= xx1[t_] := L1 Sin[phi1[t]];
yy1[t_] := -L1 Cos[phi1[t]];
xx2[t_] := xx1[t] + L1 Sin[phi2[t]];
yy2[t_] := yy1[t] - L2 Cos[phi2[t]];
bobradius = 0.03;
Manipulate[
  L1 = 1; L2 = 1; m1 = 1; m2 = 1; g = 9.8;
  soln = NDSolve[
    {eom1, eom2,
     phi1'[0] == 0, phi2'[0] == 0,
     phi1[0] == a, phi2[0] == b},
    {phi1[t], phi2[t]},
    {t, 0, tmax}];
  phi1[t_] := Evaluate[phi1[t] /. First[soln]];
  phi2[t_] := Evaluate[phi2[t] /. First[soln]];
  Animate[
    Graphics[
      {
        Line[{{0, 0}, {xx1[t], yy1[t]}}],
        Disk[{xx1[t], yy1[t]}, bobradius],
        Line[{{xx1[t], yy1[t]}, {xx2[t], yy2[t]}}],
        Disk[{xx2[t], yy2[t]}, bobradius]
      },
    Axes -> True, PlotRange -> {{-2, 2}, {2, -2}}
  ], {t, 0, tmax}],
  {{a, 0.1}, -1.5, 1.5},
  {{b, Sqrt[2]}, -Sqrt[2], +Sqrt[2]},
  {{tmax, 10}, 0, 100}]

```

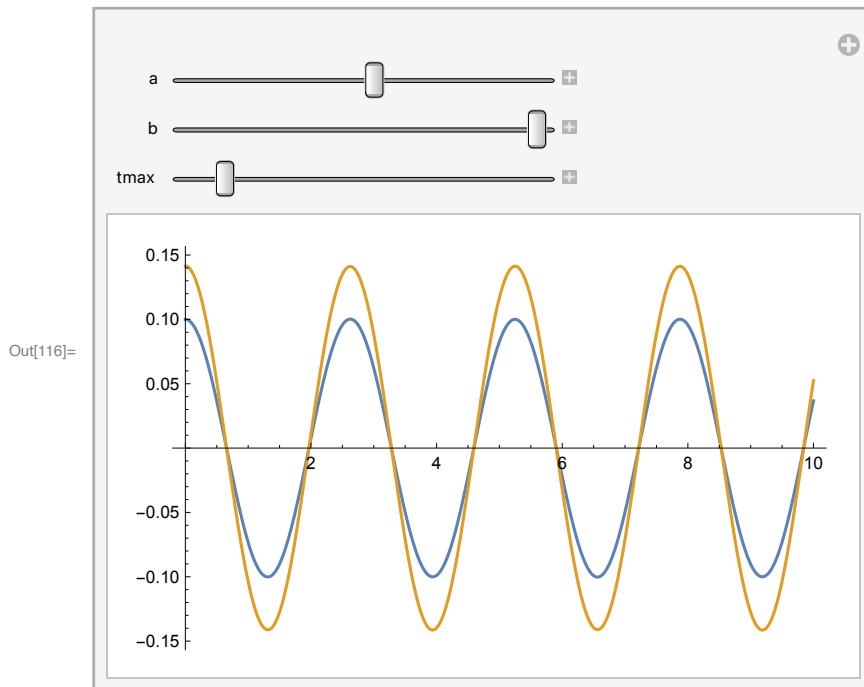
Out[115]=



```

In[116]:= Manipulate[
  L1 = 1; L2 = 1; m1 = 1; m2 = 1; g = 9.8;
  soln = NDSolve[
    {eom1, eom2,
      $\phi_1'[0] == 0$ ,  $\phi_2'[0] == 0$ ,
      $\phi_1[0] == a$ ,  $\phi_2[0] == a b$ },
    { $\phi_1[t]$ ,  $\phi_2[t]$ },
    {t, 0, tmax}];
  phi1[t_] := Evaluate[ $\phi_1[t]$  /. First[soln]];
  phi2[t_] := Evaluate[ $\phi_2[t]$  /. First[soln]];
  Plot[{phi1[t], phi2[t]}, {t, 0, tmax},
    {{a, 0.1}, -1.5, 1.5},
    {{b, Sqrt[2]}, -Sqrt[2], +Sqrt[2]},
    {{tmax, 10}, 0, 100}]

```



```

In[80]:= Clear[m, a, b, c, d, e, f, g, h, i];
m = {{a, b, c}, {d, e, f}, {g, h, i}};
MatrixForm[m]
Det[m]

```

Out[82]//MatrixForm=

$$\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}$$

Out[83]= $-c e g + b f g + c d h - a f h - b d i + a e i$